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# Grouping tasks to save energy in a cyclic scheduling problem: a complexity study

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## 1 Motivation

In order to improve the functionality and efficiency of industrial monitoring and control systems, the industries are looking toward Wireless Sensor Networks (WSNs), especially in environments where electricity is not available. So, saving energy is an important tradeoff in such systems. The IEEE 802.15.4/ZigBee[2][6] standards are leading technologies for low-cost, low-power and low-rate WSNs.

In the beacon-enabled variant, a network of clusters in a tree-shape is considered, which leaves are sensors. The communications between sensors are given by a set of data flows through the network. The data flows repeat periodically, but as clusters cannot transfer more than one data at the same time, two independent flows may induce a collision in a cluster and one of them should be delayed. Thus, the key problem is to find a periodic schedule which specifies when the clusters are active and transfer data while avoiding possible inter-cluster collisions and meeting all flows end-to-end deadlines, expressed by a bound on the number of periods needed to achieve the data flow.

But we need to consider the energy efficiency. The Zigbee standard [6] specifies that each cluster is active only once during the period, so that during a period, all tasks of the data-flows passing through a cluster are grouped. Consequently not every instance has a feasible solution meeting all the requirements.

This motivating problem led us to address the more general problem setting where a cyclic dependence graph with uniform constraints is to be scheduled so that some tasks are grouped in order to save energy. Notice that the grouping constraints apply to the core of the periodic schedule, i.e. that they may involve tasks of different iterations. We consider the feasibility of such a system, and we prove that even if no resource constraints are considered, this problem is *NP*-complete. We then investigate the ZigBee special case.

## 2 Problem setting

We are given a graph  $G$  of uniform constraints with unit length and nonnegative heights : each arc  $(i, j)$  of  $G$  is characterized by an integer height  $h_{ij}$ , that indicates that for any integer  $k$  the  $k^{th}$  occurrence of  $i$  precedes the  $k + h_{ij}^{th}$  occurrence of  $j$ . Each task  $i$  has a type  $k_i \in 1, \dots, K$  and a unit processing time. A periodic schedule of period  $\lambda$  assigns to each task  $i$  a starting time  $s_i$  so that

$$\forall (i, j) \in G, s_j - s_i \geq 1 - \lambda h_{ij} \quad (1)$$

A periodic schedule can also be described by its core  $\sigma$  and its retiming  $\alpha$  as follows [3][7] :  $\alpha(i)$  is the largest integer such that  $s_i = \sigma(i) + \lambda\alpha(i)$ , with  $\sigma(i) \in [0, \lambda)$ .

A periodic schedule is said to be grouped if the tasks of the same type are executed close to each other in the core : each group is to be scheduled as a single task in the core schedule. So a grouped schedule associates a starting time  $t_r$  to each group  $r$  in the core. The task  $i$  of type  $r$  is scheduled at time  $\sigma(i) = t_r$  in the core.

We can now state the feasibility problem called *UGF* : Given a uniform graph and the tasks groups, Does it exist a grouped periodic schedule of  $G$  ?

### 3 *UGF* complexity

Grouping constraints might be considered as a special case of core precedence (see [5]) So, they can be easily embedded in an ILP formulation, even with additional resource constraints. However, this formulation does not give an insight on the complexity of the problem *UGF*, although it can provide solutions for small size problems [4].

But the structure of *UGF* allows us to formulate it as a graph problem :

For a given retiming  $\alpha$ , we define  $\mathcal{G}^\alpha$  the sub-graph of  $G$  composed with all arcs  $(i, j)$  such that  $h_{ij} + \alpha(j) - \alpha(i) = 0$ . It is well known [3][1] that  $\mathcal{G}^\alpha$  is an acyclic graph.

Let  $R^\alpha$  be the graph obtained from  $\mathcal{G}^\alpha$  by merging the nodes of the same group. An arc  $(i, j)$  of  $\mathcal{G}^\alpha$  results in  $R^\alpha$  in an arc  $(k_i, k_j)$ .

Notice that even if  $\mathcal{G}^\alpha$  is acyclic, by merging nodes,  $R^\alpha$  may contain circuits, and in this case, no feasible schedule can be found for this retiming.

So the feasibility problem is to find a retiming vector  $\alpha$  such that the resulting grouped graph  $R^\alpha$  is acyclic.

This property is used to state our main result :

**Theorem 1** *UGF is NP-Complete*

We prove this theorem by an original reduction from 3 – SAT. In the last section, we investigate the Zigbee special case, wher there is a special underlying structure for groups and for uniform constraints. We prove some properties of the structure of grouped schedules and provide a polynomial subcase. However, the complexity of this special case is still unknown.

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