Stabilization of nonlinear systems using event-triggered output feedback controllers

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***Abstract*—The objective is to design output feedback event-triggered controllers to stabilize a class of nonlinear systems. One of the main difficulties of the problem is to ensure the existence of a minimum amount of time between two consecutive transmissions, which is essential in practice. We solve this issue by combining techniques from event-triggered and time-triggered control. The idea is to turn on the event-triggering mechanism only after a fixed amount of time has elapsed since the last transmission. This time is computed based on results on the stabilization of time-driven sampled-data systems. The overall strategy ensures an asymptotic stability property for the closed-loop system. The results are proved to be applicable to linear time-invariant (LTI) systems as a particular case.**

I. Introduction

Networked control systems (NCS) are systems in which the communication between the plant and the controller occurs through a shared digital channel. Since the network has a limited bandwidth and is typically used by other tasks, it is essential to develop communication-aware control strategies. Event-triggered control is a relevant paradigm in this context as it adapts transmissions to the current state of the plant, see *e.g.* [1]–[5] and the references therein. In that way, transmissions only occur when it is needed according to the control objectives.

A fundamental issue in the implementation of event-triggered controllers is to ensure the existence of a minimum amount of time between two consecutive transmissions to respect the hardware limitations. This task becomes particularly challenging when we have to design the controller using only an output of the system and not the full state vector (see [6]), in particular when we aim to guarantee asymptotic stability properties. To the best of our knowledge, this problem has been first addressed in [7] and then in [6], [8]–[12] for linear time-invariant (LTI) systems and in [13] for nonlinear systems.

In this paper, we design output feedback event-triggered controllers for nonlinear systems which guarantee a (global) asymptotic stability property and the existence of a uniform strictly positive lower bound on the inter-transmission times. The proposed strategy combines the event-triggering condition of [3] adapted to output measurements and the results on time-driven sampled-data systems in [14]. Indeed, the event-triggering condition is only (continuously) evaluated after T units of times have elapsed since the last transmission, where T corresponds to the *maximum allowable sampling period* (MASP) given by [14]. This two-step procedure is justified by the fact that the adaptation of the event-triggering condition of [3] to output feedback on its own can lead to Zeno phenomenon (see [6]). Although the rationale of the approach is intuitive, the analysis is not trivial. Indeed, we need to construct a hybrid Lyapunov function which handles the features of both the time-triggering condition and of the event-triggering one to prove stability. The obtained function is only locally Lipschitz (and not differentiable everywhere). As a consequence, we need to invoke non-smooth analysis tools to conclude about the stability of the closed-loop system (like in [4]).

This type of triggering rules has been used in [15] to stabilize nonlinear singularly perturbed systems under a different set of assumptions. Note that the idea of enforcing a given time between two jumps is linked to time regularization techniques, see [16]. Similar approaches have been followed in [10], [11], [17], [18] in different contexts, mainly for linear systems. It is worth mentioning that our strategy is essentially different from the aforementioned techniques in the sense that the enforced lower bound on the inter-transmission times corresponds to the MASP for time-triggered controllers [14]. This is not the case in the previous works where the lower bound in [10] comes from the event-triggering condition, or taken small only to rule out the Zeno phenomenon [11], [17], [18]. This seems to be the first study where tools from time-triggered control and event-triggered control are combined to stabilize nonlinear systems with output feedback laws. The only result that addresses this class of systems is [13], to the best of our knowledge. Compared to [13], we propose a different approach and we rely on a different set of assumptions, which allows to consider classes of systems for which the results of [13] do not apply as we show in the paper.

Our results rely on similar assumptions as in [14], so that we can derive an explicit expression for the upper bound on the MASP. These conditions are shown to be verified by the nonlinear Lorenz model of thermal convection [19] and the nonlinear model of a single link robot arm [20]. Furthermore, the required conditions are always satisfied by LTI systems that are stabilizable and detectable, in which case these are reformulated as a linear matrix inequality (LMI). In the particular case of LTI systems, the proposed technique also appears to provide interesting features compared to [6], [10], [11]. Indeed, unlike [10], [11], our approach is not necessarily based on an observer. This has the advantage to potentially lighten the implementation since the triggering mechanism only needs to have access to an output of the plant, and not the controller state variable (but our results are also applicable in this case). Compared to [6], we ensure a global asymptotic stability property as opposed to ultimate boundedness. Finally, simulation results show that our technique may generate less transmissions than existing techniques for LTI systems.

It has to be noted that the event-triggering mechanism that we propose is different from the periodic event-triggered control (PETC) paradigm, see *e.g.* [21], [22], where the triggering condition is verified only at some periodic sampling instants. In our case, the triggering mechanism is *continuously* evaluated once T units of time have elapsed since the last transmission. The first results of this work have been presented in [20]. In comparison to our previous works, we provide all the proofs of the results. We also show how the proposed technique can be fruitfully employed in the context of state feedback control as a special case, to directly tune the lower bound on the inter-transmission times. Finally, we apply the results on a different physical nonlinear example to better motivate our results and we compare our event-triggered controllers with the existing results on linear examples.

II. Preliminaries

Let $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$ and $\mathbb{Z}_{\geq 0} := \{0, 1, 2, ..\}$. A continuous function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is zero at zero,

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strictly increasing, and it is of class $\mathcal{K}_\infty$ if in addition $\gamma(s) \to \infty$ as $s \to \infty$. A continuous function $\gamma : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for each $t \in \mathbb{R}_{\geq 0}$, $\gamma(\cdot, t)$ is of class $\mathcal{K}$, and, for each $s \in \mathbb{R}_{\geq 0}$, $\gamma(s, \cdot)$ is decreasing to zero. We denote the minimum and maximum eigenvalues of the symmetric matrix A as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively. We write A^T to denote the transpose of A. We use $\mathbb{I}_n$ to denote the identity matrix of dimension n. We write (x, y) to represent the vector $[x^T, y^T]^T$ for $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. For a vector $x \in \mathbb{R}^n$, we denote by $|x| := \sqrt{x^T x}$ its Euclidean norm and for a matrix $A \in \mathbb{R}^{n \times m}$, $|A| := \sqrt{\lambda_{\max}(A^T A)}$. We will consider locally Lipschitz Lyapunov functions (that are not necessarily differentiable everywhere), therefore we will use the generalized directional derivative of Clarke which is defined as follows. For a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ and a vector $\upsilon \in \mathbb{R}^n$, $V^\circ(x; \upsilon) := \limsup_{h \to 0^+,\, y \to x} (V(y + h\upsilon) - V(y))/h$. For a continuously differentiable function V, $V^\circ(x; \upsilon)$ reduces to the standard directional derivative $\langle \nabla V(x), \upsilon \rangle$, where $\nabla V(x)$ is the (classical) gradient. We will invoke the following result, see Lemma II.1 in [23].

Lemma 1 (Lemma II.1 [23]). *Consider two functions $U_1 : \mathbb{R}^n \to \mathbb{R}$ and $U_2 : \mathbb{R}^n \to \mathbb{R}$ that have well-defined Clarke derivatives for all $x \in \mathbb{R}^n$ and $\upsilon \in \mathbb{R}^n$. Introduce three sets $A := \{x : U_1(x) > U_2(x)\}$, $B := \{x : U_1(x) < U_2(x)\}$, $\Gamma := \{x : U_1(x) = U_2(x)\}$. Then, for any $\upsilon \in \mathbb{R}^n$, the function $U(x) := \max\{U_1(x), U_2(x)\}$ satisfies $U^\circ(x; \upsilon) = U_1^\circ(x; \upsilon)$ for all $x \in A$, $U^\circ(x; \upsilon) = U_2^\circ(x; \upsilon)$ for all $x \in B$ and $U^\circ(x; \upsilon) \leq \max\{U_1^\circ(x; \upsilon), U_2^\circ(x; \upsilon)\}$ for all $x \in \Gamma$.* □

In this paper, we consider hybrid systems of the following form using the formalism of [24]

$$\dot{x} = F(x) \quad x \in C, \qquad x^+ = G(x) \quad x \in D, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, F is the flow map, C is the flow set, G is the jump map and D is the jump set. The vector fields F and G are assumed to be continuous and the sets C and D are closed. The solutions to system (1) are defined on so-called hybrid time domains. A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is called a *compact hybrid time domain* if $E = \bigcup_{j \in \{0, \ldots, J-1\}} ([t_j, t_{j+1}], j)$ for some finite sequence of times $0 = t_0 \leq t_1 \leq \ldots \leq t_J$ and it is a *hybrid time domain* if for all $(T, J) \in E$, $E \cap ([0, T] \times \{0, 1, \ldots, J\})$ is a compact hybrid time domain. A function $\phi : E \to \mathbb{R}^n$ is a hybrid arc if E is a hybrid time domain and if for each $j \in \mathbb{Z}_{\geq 0}$, $t \mapsto \phi(t, j)$ is locally absolutely continuous on $I^j := \{t : (t, j) \in E\}$. A hybrid arc ϕ is a solution to system (1) if: (i) $\phi(0, 0) \in C \cup D$; (ii) for any $j \in \mathbb{Z}_{\geq 0}$, $\phi(t, j) \in C$ and $\dot{\phi}(t, j) = F(\phi(t, j))$ for almost all $t \in I^j$; (iii) for every $(t, j) \in \operatorname{dom} \phi$ such that $(t, j + 1) \in \operatorname{dom} \phi$, $\phi(t, j) \in D$ and $\phi(t, j + 1) = G(\phi(t, j))$. A solution ϕ to system (1) is *maximal* if it cannot be extended, *complete* if its domain, $\operatorname{dom} \phi$, is unbounded, and it is *Zeno* if it is complete and $\sup_t \operatorname{dom} \phi < \infty$.

III. Problem statement

Consider the nonlinear plant model

$$\dot{x}_p = f_p(x_p, u), \qquad y = g_p(x_p), \tag{2}$$

where $x_p \in \mathbb{R}^{n_p}$ is the plant state, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^{n_y}$ is the measured output of the plant. We focus on general dynamic controllers of the form

$$\dot{x}_c = f_c(x_c, y), \qquad u = g_c(x_c, y), \tag{3}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state. We emphasize that the x_c-system is not necessarily an observer. Moreover, (3) captures static feedbacks as a particular case by setting $u = g_c(y)$. The functions f_p, f_c are assumed to be continuous and the functions g_p, g_c are assumed to be continuously differentiable. We follow an emulation approach in this paper to design the event-triggered controllers. Hence, we assume that the controller (3) renders the origin of system (2)-(3) uniformly (globally) asymptotically stable in the absence of a network. Afterwards, we take into account the communication constraints and we synthesize the triggering condition. In particular, we consider the scenario where controller (3) communicates with the plant via a digital channel. Hence, the plant output and the control input are sent only at transmission instants $t_i, i \in \mathcal{I} \subseteq \mathbb{Z}_{\geq 0}$. We are interested in an event-triggered implementation in the sense that the sequence of transmission instants is determined by a criterion based on the output measurement and the control input, see Figure 1. At each transmission instant, the plant output is sent to the

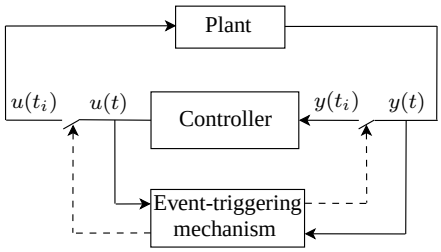

Fig. 1. Event-triggered control schematic [6]

controller, which computes a new control input that is instantaneously transmitted to the plant. We assume that this process is performed in a synchronous manner[1] and we ignore the computation times and the possible transmission delays. In that way, we obtain

$$\left.\begin{array}{rcll} \dot{x}_p &=& f_p(x_p, \hat{u}) & t \in [t_i, t_{i+1}] \\ \dot{x}_c &=& f_c(x_c, \hat{y}) & t \in [t_i, t_{i+1}] \\ u &=& g_c(x_c, \hat{y}) & \\ y &=& g_p(x_p) & \\ \dot{\hat{y}} &=& 0 & t \in [t_i, t_{i+1}] \\ \dot{\hat{u}} &=& 0 & t \in [t_i, t_{i+1}] \\ \hat{y}(t_i^+) &=& y(t_i) & \\ \hat{u}(t_i^+) &=& u(t_i), & \end{array}\right\} \tag{4}$$

where $\hat{y}$ and $\hat{u}$ respectively denote the last transmitted values of the plant output and the control input. We assume that zero-order-hold devices are used to generate the sampled values $\hat{y}$ and $\hat{u}$, which leads to $\dot{\hat{y}} = 0$ and $\dot{\hat{u}} = 0$. We introduce the network-induced error $e := (e_y, e_u) \in \mathbb{R}^{n_e}$, where $e_y := \hat{y} - y$ and $e_u := \hat{u} - u$ which are reset to 0 at each transmission instant.

Remark 1. *We can alternatively define the sampling-induced error e_u as $e_u := \hat{x}_c - x_c$, where $\hat{x}_c$ denotes the value of x_c at the last transmission instant. All the results presented hereafter apply in this case, provided the required conditions hold. This allows to encompass the case where the controller state, rather than the output of the controller, is used by the event-triggering mechanism which may help reducing the amount of transmissions, as shown in Example 2.* □

We model the event-triggered control system using the hybrid formalism of [24] as in, *e.g.*, [4], [6], [11], for which a jump corresponds to a transmission. In that way, we obtain

$$\begin{pmatrix} \dot{x} \\ \dot{e} \\ \dot{\tau} \end{pmatrix} = \begin{pmatrix} f(x, e) \\ g(x, e) \\ 1 \end{pmatrix} \quad q \in C, \qquad \begin{pmatrix} x^+ \\ e^+ \\ \tau^+ \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \quad q \in D, \tag{5}$$

[1] See Section VII for a discussion on the asynchronous case.

where $q := (x, e, \tau) \in \mathbb{R}^{n_x+n_e+1}$ with $x := (x_p, x_c) \in \mathbb{R}^{n_x}$ and $\tau \in \mathbb{R}$ is a clock variable which describes the time elapsed since the last jump, $f(x,e) := \Big(f_p(x_p, g_c(x_c, y+e_y)+e_u),\ f_c(x_c, y+e_y)\Big)$ and $g(x,e) := \Big(-\frac{\partial}{\partial x_p} g_p(x_p) f_p(x_p, g_c(x_c, y+e_y)+e_u),\ -\frac{\partial}{\partial x_c} g_c(x_c, y+e_y) f_c(x_c, y+e_y)\Big)$. The flow and jump sets of (5) are defined according to the triggering condition we will define. As long as the triggering condition is not violated, the system flows on C and a jump occurs when the state enters in D. When $(x, e, \tau) \in C \cap D$, the solution may flow only if flowing keeps (x, e, τ) in C, otherwise the system experiences a jump. The functions f and g are continuous (in view of the assumptions made on f_p, f_c, g_p and g_c), and the sets C and D will be closed (which ensure that system (5) is well-posed, see Chapter 6 in [24]).

The main objective of this paper is to design the flow and the jump sets of system (5), *i.e.* the triggering condition which involves e and y, to ensure a (global) asymptotic stability property for system (5).

IV. Main results

We first present the conditions that we impose on system (5), then we present the triggering technique and finally we state the main result. We make the following assumption on system (5), which is inspired by [14].

Assumption 1. *There exist $\Delta_x, \Delta_e > 0$, a locally Lipschitz function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, a locally Lipschitz positive semi-definite function $W : \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$, a continuous function $H : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, real numbers $\gamma, L \geq 0$, $\underline{\alpha}, \overline{\alpha}, \alpha \in \mathcal{K}_\infty$ and a continuous, nonnegative function $\delta : \mathbb{R}^{n_y} \to \mathbb{R}_{\geq 0}$ such that, for all $x \in \mathbb{R}^{n_x}$*

$$\underline{\alpha}(|x|) \leq V(x) \leq \overline{\alpha}(|x|), \tag{6}$$

for all $|e| \leq \Delta_e$ and almost all $|x| \leq \Delta_x$

$$\langle \nabla V(x), f(x,e) \rangle \leq -\alpha(|x|) - H^2(x) - \delta(y) + \gamma^2 W^2(e) \tag{7}$$

and for all $|x| \leq \Delta_x$ and almost all $|e| \leq \Delta_e$

$$\langle \nabla W(e), g(x,e) \rangle \leq L W(e) + H(x). \tag{8}$$

We say that Assumption 1 holds globally if (7) and (8) hold for almost all $x \in \mathbb{R}^{n_x}$ and $e \in \mathbb{R}^{n_e}$. □

Conditions (6)-(7) imply that the system $\dot{x} = f(x,e)$ is $\mathcal{L}_2$-gain stable from W to $(H, \sqrt{\delta})$. This property can be analysed by investigating the robustness property of the closed-loop system (2)-(3) with respect to input and/or output measurement errors in the absence of sampling. We also assume an exponential growth condition of the e-system on flows in (8) which is already used in [14], [25]. We provide an example of a nonlinear system which satisfies Assumption 1 at the end of this section and we can always guarantee it for any stabilizable and detectable linear system.

Remark 2. *Note that, since W is positive semi-definite and continuous (since it is locally Lipschitz), there exists $\chi \in \mathcal{K}_\infty$ such that $W(e) \leq \chi(|e|)$ (by following similar arguments as in the proof of Lemma 4.3 in [26]). Hence, conditions (6), (7) imply that the system $\dot{x} = f(x,e)$ is input-to-state stable (ISS), however the converse is not necessarily true. Although this requirement is stronger than ISS, it is satisfied by important classes of systems as we show in the paper and in [20] as well. We rely on Assumption 1 to design the MASP that we enforce as a lower bound on the inter-transmission times as we explain in the sequel.* □

Under Assumption 1, the adaptation of [3] leads to a triggering condition of the form $\gamma^2 W^2(e) \leq \delta(y)$. The problem is that Zeno phenomenon may occur with this type of triggering conditions. Indeed, when $y = 0$, an infinite number of jumps occurs for any value of x such that $g_p(x_p) = y = 0$. In [6], this issue is overcome by adding a constant to the triggering condition, which would lead to $\gamma^2 W^2(e) \leq \delta(y) + \varepsilon$ here for $\varepsilon > 0$, from which we can derive a practical stability property. The event-triggered mechanism that we propose allows us to guarantee an asymptotic stability property for the closed-loop while ensuring that the inter-transmission times are lower bounded by a strictly positive constant. The idea is to evaluate the event-triggering condition only after T units have elapsed since the last transmission, where T corresponds to the MASP given by [14]. In that way, we allow the user to directly tune the minimum inter-jump interval, up to a certain extent as explained in the following. We thus define the triggering condition as follows

$$\gamma^2 W^2(e) \leq \delta(y) \text{ or } \tau \in [0, T], \tag{9}$$

where we recall that $\tau \in \mathbb{R}_{\geq 0}$ is the clock variable introduced in (5). Consequently, the flow and jump sets of system (5) are

$$\begin{aligned} C &= \Big\{(x,e,\tau) : \gamma^2 W^2(e) \leq \delta(y) \text{ or } \tau \in [0,T]\Big\} \\ D &= \Big\{(x,e,\tau) : \gamma^2 W^2(e) \geq \delta(y) \text{ and } \tau \geq T\Big\}. \end{aligned} \tag{10}$$

Hence, the inter-jump times are uniformly lower bounded by T. This constant is selected such that $T < \mathcal{T}(\gamma, L)$, where

$$\mathcal{T}(\gamma, L) := \begin{cases} \frac{1}{Lr} \arctan(r) & \gamma > L \\ \frac{1}{L} & \gamma = L \\ \frac{1}{Lr} \operatorname{arctanh}(r) & \gamma < L \end{cases} \tag{11}$$

with $r := \sqrt{\left|(\frac{\gamma}{L})^2 - 1\right|}$ and L, γ come from Assumption 1.

Remark 3. *The triggering condition (10) requires to continuously monitor both the plant output and the control input which are needed to evaluate $W(e)$. This may be difficult to implement in practice. We have decided to present the triggering condition with $W(e)$ which depends on both e_y and e_u for the sake of generality. Indeed, this formulation encompasses the following important implementation scenarios as particular cases:*

- *when only the output measurement is sampled and the controller is directly connected to the actuators, in this case $e = e_y$;*
- *when only the control input is sampled but not y, in this case $e = e_u$. Note that in this case the event-triggering rule $\gamma^2 W^2(e_u) \leq \delta(y)$ depends on both y and u, i.e. the controller and the sensors need to be co-located.*

When the controller and the plant/sensors are not co-located and we need to sample both y and u, asynchronous event-triggered implementations are probably more relevant, like in [6], [10] for LTI systems. The proposed mechanism can be adapted to cover this implementation scenario for nonlinear systems, see [27]. □

We are ready to state the main result.

Theorem 1. *Suppose that Assumption 1 holds and consider system (5) with the flow and jump sets (10), where the constant T is such that $T \in (0, \mathcal{T}(\gamma, L))$. There exist $\Delta > 0$ and $\beta \in \mathcal{KL}$ such that any solution $\phi = (\phi_x, \phi_e, \phi_\tau)$ with $|(\phi_x(0,0), \phi_e(0,0))| \leq \Delta$ satisfies*

$$|\phi_x(t,j)| \leq \beta(|(\phi_x(0,0), \phi_e(0,0))|, t+j) \quad \forall (t,j) \in \operatorname{dom} \phi. \tag{12}$$

Moreover, the inter-transmission times are lower bounded by T, and if ϕ is maximal, then it is complete. If Assumption 1 holds globally, then (12) holds globally. □

Example 1. Consider the controlled Lorenz equations which model a thermal convection loop [19], $\dot{x}_1 = -a x_1 + a x_2$, $\dot{x}_2 = b x_1 - x_2 - x_1 x_3 + u$, $\dot{x}_3 = x_1 x_2 - c x_3$ and $y = x_1$, where $a, b, c > 0$. The static output feedback law $u = -(\frac{p_1}{p_2} a + b) x_1$, where $p_1, p_2 >$

0, globally stabilizes the origin. This can be proved by using the quadratic Lyapunov function $V(x) = p_1x_1^2 + p_2x_2^2 + p_2x_3^2$, which verifies condition (6) with $\underline{\alpha}(|x|) = \min\{p_1, p_2\}|x|^2$ and $\overline{\alpha}(|x|) = \max\{p_1, p_2\}|x|^2$. We take into account the network-induced error $e = \hat{y} - y$ (it is not necessary to consider the error in u as the controller is static) and we select $W(e) = |e|$. Hence, condition (8) is satisfied with $L = 0$, $H(x) = a(|x_1|+|x_2|)$. By taking $p_1 > 1$ and $p_2 > 2a$, condition (7) holds with $\alpha(|x|) = \min\{a(p_1 - 1), (p_2 - 2a), 2p_2c\}|x|^2$, $\delta(y) = a(p_1 - 1)y^2$ and $\gamma^2 = p_2(\frac{p_1}{p_2}a + b)^2$. For the parameter values $a = 10, b = 28, c = 8/3$ used in [19], we set $p_1 = 2$, $p_2 = 3a$ and we obtain $T = 0.01$. Figure 2 shows that the Zeno phenomenon occurs when we transmit only based on the event-triggering rule $\gamma^2W^2(e) \leq \delta(y)$, *i.e.* with $T = 0$ in (10), which supports the discussion above (9). We note that the results in [13] are not applicable to this system because condition (3) of Proposition 1 in [13] does not hold. □

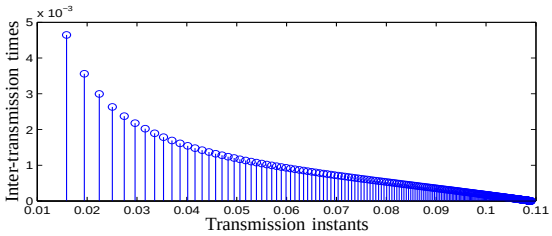

Fig. 2. Inter-transmission times with [3].

V. Linear systems

We now focus on the particular case of linear systems. Consider the LTI plant model

$$\dot{x}_p = A_px_p + B_pu, \qquad y = C_px_p, \tag{13}$$

where $x_p \in \mathbb{R}^{n_p}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and A_p, B_p, C_p are matrices of appropriate dimensions. We design the following dynamic controller to stabilize (13) in the absence of sampling

$$\dot{x}_c = A_cx_c + B_cy, \qquad u = C_cx_c + D_cy, \tag{14}$$

where $x_c \in \mathbb{R}^{n_c}$ and A_c, B_c, C_c, D_c are matrices of appropriate dimensions. Afterwards, we take into account the communication constraints. Then, the hybrid model (5) is

$$\begin{pmatrix} \dot{x} \\ \dot{e} \\ \dot{\tau} \end{pmatrix} = \begin{pmatrix} \mathcal{A}_1x + \mathcal{B}_1e \\ \mathcal{A}_2x + \mathcal{B}_2e \\ 1 \end{pmatrix} \quad q \in C, \qquad \begin{pmatrix} x^+ \\ e^+ \\ \tau^+ \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \quad q \in D, \tag{15}$$

where $q := (x, e, \tau)$, $\mathcal{A}_1 := \begin{pmatrix} A_p+B_pD_cC_p & B_pC_c \\ B_cC_p & A_c \end{pmatrix}$, $\mathcal{B}_1 := \begin{pmatrix} B_pD_c & B_p \\ B_c & 0 \end{pmatrix}$, $\mathcal{A}_2 := \begin{pmatrix} -C_p(A_p+B_pD_cC_p) & -C_pB_pC_c \\ -C_cB_cC_p & -C_cA_c \end{pmatrix}$ and $\mathcal{B}_2 := \begin{pmatrix} -C_pB_pD_c & -C_pB_p \\ -C_cB_c & 0 \end{pmatrix}$.

We obtain the following result.

Proposition 1. *Consider system (15). Suppose that there exist $\varepsilon_1, \varepsilon_2, \mu > 0$ and a positive definite symmetric real matrix P such that*

$$\begin{pmatrix} \mathcal{A}_1^TP + P\mathcal{A}_1 + \mathcal{A}_2^T\mathcal{A}_2 + \varepsilon_1\overline{C}_p^T\overline{C}_p + \varepsilon_2\mathbb{I}_{n_x} & P\mathcal{B}_1 \\ \mathcal{B}_1^TP & -\mu\mathbb{I}_{n_e} \end{pmatrix} \leq 0, \tag{16}$$

where $\overline{C}_p = [C_p \quad 0]$. Then Assumption 1 holds with $V(x) = x^TPx, \underline{\alpha}(|x|) = \lambda_{\min}(P)|x|^2, \overline{\alpha}(|x|) = \lambda_{\max}(P)|x|^2, W(e) = |e|, H(x) = |\mathcal{A}_2x|, L = |\mathcal{B}_2|, \gamma = \sqrt{\mu}, \alpha(|x|) = \varepsilon_2|x|^2, \delta(y) = \varepsilon_1|y|^2$, for any $x \in \mathbb{R}^{n_x}, y \in \mathbb{R}^{n_y}, e \in \mathbb{R}^{n_e}$. □

Proposition 1 provides a sufficient condition, namely (16), for the verification of Assumption 1, which thus allows us to apply the results of Section IV. It has to be noted that LMI (16) can always be satisfied when system (13) is stabilizable and detectable. Indeed, in this case, we can select the controller (14) such that $\mathcal{A}_1$ is Hurwitz. Noting that (16) is equivalent to the following inequalities, by using the Schur complement of (16) (see Section A.5.5 in [28]), $\mathcal{A}_1^TP + P\mathcal{A}_1 + \mathcal{A}_2^T\mathcal{A}_2 + \varepsilon_1\overline{C}_p^T\overline{C}_p + \varepsilon_2\mathbb{I}_{n_x} + \frac{1}{\mu}P\mathcal{B}_1\mathcal{B}_1^TP \leq 0$. We see that we can select the matrix P such that $\mathcal{A}_1^TP + P\mathcal{A}_1 + \mathcal{A}_2^T\mathcal{A}_2 + \varepsilon_1\overline{C}_p^T\overline{C}_p + \varepsilon_2\mathbb{I}_{n_x}$ is negative definite. It then suffices to choose μ sufficiently large to ensure the last inequality.

Example 2. We consider Example 1 in [10] where both the output measurement and the controller state are used to trigger transmissions. We thus redefine the sampling-induced error e_u to be $e_u = \hat{x}_c - x_c$, as explained in Remark 1, and we modify the matrices $\mathcal{A}_1, \mathcal{A}_2, \mathcal{B}_1, \mathcal{B}_2$ in (15) accordingly. In this case, $L = |\mathcal{B}_2| = 23.7340$. By solving LMI (16), we obtain $\varepsilon_1 = 86.1643$, $\varepsilon_2 = 15.1622$, $\gamma = 151.4831$. The guaranteed minimum inter-transmission time is $T = 0.009$, according to (11). We have run simulations for 100 randomly distributed initial conditions for 5 s such that $|x_p(0,0)| \leq 100, |x_c(0,0)| \leq 100$, $e_y(0,0) = 0$, $e_u(0,0) = (0,0)$, and $\tau(0,0) = 0$ and we compare our result with architecture II in [10]. Table I provides the obtained minimum and average inter-transmission times, respectively denoted as $\tau_{\min}$ and τ_{avg}. We notice that less transmissions are generated when $e_u = \hat{x}_c - x_c$. We also note that in this case our technique yields larger values of both $\tau_{\min}$ and τ_{avg} than [10]. □

	[10]	Proposed mechanism with $e_u = \hat{u} - u$	Proposed mechanism with $e_u = \hat{x}_c - x_c$
Guaranteed lower bound	0.57×10^{-3}	0.9029×10^{-3}	9.4476×10^{-3}
$\tau_{\min}$	0.57×10^{-3}	0.9029×10^{-3}	9.4476×10^{-3}
τ_{avg}	3.8310×10^{-3}	1.2207×10^{-3}	10.1955×10^{-3}

TABLE I
Simulation comparison with [10].

VI. State feedback controllers

The technique proposed in Section IV is also relevant in the context of state feedback control, *i.e.* when $y = x$, as the constant T in (10) can be used to directly tune the minimum inter-transmission time (up to $\mathcal{T}(\gamma, L)$ in (11)). It has to be noted that in this case, we can replace $\gamma^2W^2(e) \leq \delta(y)$ in (9) by $\gamma^2W^2(e) \leq (\alpha(|x|) + H^2(x) + \delta(x))$ when Assumption 1 holds. The following result is a direct consequence of Theorem 1.

Corollary 1. *Suppose that Assumption 1 holds and consider system (5) with $y = x$ and the flow and jump sets defined as*

$$C = \Big\{q : \gamma^2W^2(e) \leq \sigma(\alpha(|x|) + H^2(x) + \delta(x)) \text{ or } \tau \in [0, T]\Big\}$$
$$D = \Big\{q : \gamma^2W^2(e) \geq \sigma(\alpha(|x|) + H^2(x) + \delta(x)) \text{ and } \tau \geq T\Big\}, \tag{17}$$

where $q := (x, e, \tau)$, $\sigma \in (0, 1)$ and T is such that $T \in (0, \mathcal{T}(\gamma, L))$. Then, the conclusions of Theorem 1 hold. □

Example 3. Consider the dynamics of a single-link robot arm $\dot{x}_1 = x_1$, $\dot{x}_2 = -\sin(x_1) + u$, where x_1 denotes the angle, x_2 the rotational velocity and u the input torque. The system can be written as $\dot{x} = Ax + Bu - \phi(x)$ where $x := (x_1, x_2)$ and $A = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $\phi(x) = \left[\begin{smallmatrix} 0 \\ \sin(x_1) \end{smallmatrix}\right]$. We design $u = Kx + B^T\phi(x)$ such that $\bar{A} = A + BK$ is Hurwitz (which is possible since the pair (A, B) is controllable). We now take into account the effect of the network $e = \hat{x} - x$ and we obtain $\dot{x} = \bar{A}x + BKe + \phi(x + e) - \phi(x)$.

We take $W(e) = |e|$, $V(x) = x^T P x$, for all $(x, e) \in \mathbb{R}^{n_x+n_e}$, where P is a real positive definite symmetric matrix such that $\bar{A}^T P + P\bar{A} = -Q$. By following similar lines as in Section VI in [20], we deduce that conditions (7), (8) hold with $L = |BK| + 1, H(x) = |A + BK||x|, \gamma = \sqrt{\frac{2(|PBK|+|P|)^2}{\lambda_{\min}(Q)}}$, $\delta(x) = 0$ and $\alpha(|x|) = (\frac{\lambda_{\min}(Q)}{2} - |A+BK|^2)|x|^2$ with $\lambda_{\min}(Q) > 2|A+BK|^2$. We take $K = [-2 \ -3]$ and we obtain $L = 4.6056$ and $\gamma = 19.1361$ which yields $\mathcal{T} = 0.071485$ in view of (11). We set $T = 0.0714$ and $\sigma = 0.15$ in (17) and we run simulations for 100 randomly distributed initial conditions such that $|x(0,0)| \leq 100$, $e(0,0) = (0,0)$ and $\tau(0,0) = 0$ for 10 s. We obtain $\tau_{\min} = 0.0714 = T$ and $\tau_{\text{avg}} = 0.0778$ which indicate the interaction between the time-triggering and the event-triggering rules. To compare with [3], we set $T = 0$ and we have obtained $\tau_{\min} = 0.0192$ and $\tau_{\text{avg}} = 0.0727$. Hence, the proposed triggering condition generates less transmissions than [3] for this example. □

VII. Conclusion

We have developed output-based event-triggered controllers for the stabilization of nonlinear systems. The proposed technique ensures an asymptotic stability property and enforces a minimum amount of time between two consecutive transmission instants. The required conditions are shown to be satisfied by any stabilizable and detectable LTI systems.

We show in [29] that these results can be used as a starting point to address the challenging co-design problem for linear systems in which the output feedback law is not obtained by emulation but is jointly synthesized with the triggering condition. The proposed approach is relevant for perturbed systems as the enforced dwell-time prevents the occurrence of the Zeno phenomenon, which may occur otherwise, see [15], [30]. We address this problem in [31] for systems affected by plant disturbance, measurement errors and perturbation on the control input. We also study in [27] the extension of the presented approach to the asynchronous transmissions of the plant output and the control input.

Appendix

Proof of Theorem 1. First, we prove the result when Assumption 1 holds globally. Let $\zeta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ be the solution to

$$\dot{\zeta} = -2L\zeta - \lambda(\zeta^2 + 1) \qquad \zeta(0) = \theta^{-1}, \tag{18}$$

where $\theta \in (0,1)$, $\lambda := \sqrt{\gamma^2 + \eta}$ for some $\eta > 0$ and L, γ come from Assumption 1. We denote $\widetilde{\mathcal{T}}(\theta, \eta, \gamma, L)$ the time it takes for ζ to decrease from θ^{-1} to θ. This time $\widetilde{\mathcal{T}}(\theta, \eta, \gamma, L)$ is defined as (28) in [14] and is a continuous function of (θ, η) which is decreasing in θ and η. In addition, it holds that $\widetilde{\mathcal{T}}(\theta, \eta, \gamma, L) \to \mathcal{T}(\gamma, L)$ as (θ, η) tends to $(0,0)$ (where $\mathcal{T}(\gamma, L)$ is defined in Section IV), like in [14]. As a consequence, since $T < \mathcal{T}(\gamma, L)$, there exists (θ, η) such that $T < \widetilde{\mathcal{T}}(\theta, \eta, \gamma, L)$. We fix the couple (θ, η). Let $q := (x, e, \tau)$. We define for all $q \in C \cup D$, $R(q) := V(x) + \max\{0, \lambda\zeta(\tau)W^2(e)\}$. Let $q \in D$, we obtain, in view of (5) and the fact that W is positive semi-definite ,

$$R(G(q)) = V(x) + \max\{0, \lambda\zeta(0)W^2(0)\} = V(x) \leq R(q), \tag{19}$$

where $G(q) := (x, 0, 0)$. Let $q \in C$ and suppose that $\zeta(\tau) < 0$. As a consequence it holds that $\tau > T$. Indeed, $\zeta(\tau)$ is strictly decreasing in τ, in view of (18), and $\zeta(T) > \zeta(\widetilde{\mathcal{T}}(\theta, \eta, \gamma, L)) = \theta > 0$ as $T < \widetilde{\mathcal{T}}(\theta, \eta, \gamma, L)$. As a consequence $\zeta(\tau) < 0$ implies that $\tau > T$. Hence, $\gamma^2 W^2(e) \leq \delta(y)$ in view of (10) since $q \in C$. Consequently, in view of Lemma 1, Assumption 1 and the definition of the function R, $R^\circ(q; F(q)) = V^\circ(x; f(x,e)) \leq -\alpha(|x|)$, where $F(q) := (f(x,e), g(x,e), 1)$. Hence, by following similar arguments as in the proof of Theorem 1 in [14] since $\alpha \in \mathcal{K}_\infty$ and V is positive definite and radially unbounded, there exists a continuous positive definite function ρ_1 such that

$$R^\circ(q; F(q)) \leq -\rho_1(V(x)) = -\rho_1(R(q)). \tag{20}$$

When $q \in C$ and $\zeta(\tau) > 0$, we have $R(q) = V(x) + \lambda\zeta(\tau)W^2(e)$. As above, in view of Lemma 1, Assumption 1 and (18) and by following the same lines as in the proof of Theorem 1 in [14], we obtain $R^\circ(q; F(q)) \leq -\alpha(|x|) - H^2(x) - \delta(y) + \gamma^2 W^2(e) + 2\lambda\zeta(\tau)W(e)H(x) - \lambda^2\zeta^2(\tau)W^2(e) - \lambda^2 W^2(e)$. Using the fact that $2\lambda\zeta(\tau)W(e)H(x) \leq \lambda^2\zeta^2(\tau)W^2(e) + H^2(x)$, $R^\circ(q; F(q)) \leq -\alpha(|x|) - \delta(y) + \gamma^2 W^2(e) - \lambda^2 W^2(e) \leq -\alpha(|x|) + \gamma^2 W^2(e) - \lambda^2 W^2(e)$. Recall that $\lambda^2 = \gamma^2 + \eta$, it holds that $R^\circ(q; F(q)) \leq -\alpha(|x|) - \eta W^2(e)$. By using the same argument as in (20), we derive that $R^\circ(q; F(q)) \leq -\rho_1(V(x)) - \eta W^2(e) = -\rho_1(V(x)) - \frac{\eta\theta}{\lambda}\lambda\theta^{-1}W^2(e) = -\rho_1(V(x)) - \rho_2(\lambda\theta^{-1}W^2(e))$, where $\rho_2 : s \mapsto \frac{\eta\theta}{\lambda}s \in \mathcal{K}_\infty$. Since $\zeta(\tau) \leq \theta^{-1}$ for all $\tau \geq 0$ in view of (18), it holds that $R^\circ(q; F(q)) \leq -\rho_1(V(x)) - \rho_2(\lambda\zeta(\tau)W^2(e))$. We deduce that there exists a continuous positive definite function ρ_3 such that $R^\circ(q; F(q)) \leq -\rho_3(V(x) + \lambda\zeta(\tau)W^2(e)) = -\rho_3(R(q))$. In view of the last inequality, (20) and Lemma 1, when $\zeta(\tau) = 0$, $R^\circ(q; F(q)) \leq \max\{-\rho_1(R(q)), -\rho_3(R(q))\}$. Consequently, it holds that, for all $q \in C$, $R^\circ(q; F(q)) \leq -\rho(R(q))$. where $\rho := \min\{\rho_1, \rho_3\}$ is continuous and positive definite. Let ϕ be a solution to (5), (10). By definition of the Clarke's derivative (see Section II) and page 100 in [32], it holds that, for all j and for almost all $t \in I^j$ (where $I^j = \{t : (t, j) \in \operatorname{dom}\phi\}$)

$$\dot{R}(\phi(t,j)) \leq R^\circ(\phi(t,j); F(\phi(t,j))) \leq -\rho(R(\phi(t,j))), \tag{21}$$

as $\phi(t,j) \in C$ for all $(t,j) \in \operatorname{dom}\phi$. Thus, in view of (19), (21) and since inter-jump times are lower bounded by T in view of (10), we conclude that, by following the same lines as in the end of the proof of Theorem 1 in [14], there exists $\tilde{\beta} \in \mathcal{KL}$ such that for any solution ϕ to (5), (10) and for all $(t,j) \in \operatorname{dom}\phi$, $R(\phi(t,j)) \leq \tilde{\beta}(R(\phi(0,0)), 0.5t + 0.5Tj)$. In view of Assumption 1 and since W is continuous (since it is locally Lipschitz) and positive semi-definite, there exists $\overline{\alpha}_W \in \mathcal{K}_\infty$ such that $W(e) \leq \overline{\alpha}_W(|e|)$ for all $e \in \mathbb{R}^{n_e}$ by following similar arguments as in the proof of Lemma 4.3 in [26]. As a result, in view of Assumption 1, (18) and the definition of the function R, it holds that, for all $q \in C \cup D$, $\underline{\alpha}(|x|) \leq R(q) \leq \overline{\alpha}(|x|) + \frac{\lambda}{\theta}\overline{\alpha}_W(|e|) \leq \overline{\alpha}_R(|(x,e)|)$, where $\overline{\alpha}_R : s \mapsto \overline{\alpha}(s) + \frac{\lambda}{\theta}\overline{\alpha}_W(s) \in \mathcal{K}_\infty$. Hence, we deduce that (12) holds for any solution ϕ to (5), (10) and for all $(t,j) \in \operatorname{dom}\phi$, where $\beta : (s_1, s_2) \mapsto \underline{\alpha}^{-1}(\tilde{\beta}(\overline{\alpha}_R(s_1), \frac{1}{2}\min\{1, T\}s_2)) \in \mathcal{KL}$.

Regrading the inter-transmission times, we note that the event-triggering mechanism (10) ensures that $j - j' \leq \frac{t-t'}{T} + 1$, for any $(t', j'), (t, j) \in \operatorname{dom}\phi$ such that $t' + j' \leq t + j$. Hence, two successive transmissions are spaced by at least T units of time.

We now investigate the completeness of the maximal solutions to system (5), (10). Let ϕ be a maximal solution to (5), (10). We first show that ϕ is nontrivial, *i.e.* its domain contains at least two points (see Definition 2.5 in [24]). According to Proposition 6.10 in [24], it suffices for that purpose to prove that $\{F(q)\} \cap T_C(q) \neq \emptyset$ for any $q := (x, e, \tau) \in C \backslash D$, where $F(q) := (f(x,e), g(x,e), 1)$ and $T_C(q)$ is the tangent cone[2] to C at q. Let $q \in C \backslash D$. If q is in the interior of C, $T_C(q) = \mathbb{R}^{n_x+n_e+1}$ and the required condition holds. If q is not in the interior of C, necessarily $\tau = 0$ as $q \in C \backslash D$, in this case $T_C(q) = \mathbb{R}^{n_x+n_e} \times \mathbb{R}_{\geq 0}$ and we see that $F(q) \in T_C(q)$, in

[2]The tangent cone to a set $S \subset \mathbb{R}^n$ at a point $x \in \mathbb{R}^n$, denoted $T_S(x)$, is the set of all vectors $\omega \in \mathbb{R}^n$ for which there exist $x_i \in S, \tau_i > 0$ with $x_i \to x, \tau \to 0$ as $i \to \infty$ such that $\omega = \lim_{i\to\infty}(x_i - x)/\tau_i$ (see Definition 5.12 in [24]).

view of (5). Hence, ϕ is nontrivial according to Proposition 6.10 in [24]. In view of (5), (10) and (12), ϕ_x and ϕ_τ cannot explode in finite time. Recall that the network-induced error is $\phi_e = (\phi_{e_y}, \phi_{e_u})$ with $\phi_{e_y} = \phi_y(t_j, j) - \phi_y(t, j)$, $\phi_{e_u} = \phi_u(t_j, j) - \phi_u(t, j)$ for $j > 0$ and $(t, j) \in \operatorname{dom}\phi$ where we write $\operatorname{dom}\phi = \cup_{j\in\{0,\dots,J\}}([t_j, t_{j+1}], j)$ with $J \in \mathbb{Z}_{\geq 0} \cup \{\infty\}$. Hence, in view of (2), (3), (12) and since g_p, g_c are continuous, it holds that, for all $j > 0$ and $(t, j) \in \operatorname{dom}\phi$, $|\phi_{e_y}(t, j)| \leq |g_p(\phi_{x_p}(t_j, j))| + |g_p(\phi_{x_p}(t, j))| \leq 2\max|g_p(z)|$ with $|z| \leq \beta(|(\phi_x(0, 0), \phi_e(0, 0))|, 0)$. Similarly, we obtain, for all $j > 0$ and $(t, j) \in \operatorname{dom}\phi$, $|\phi_{e_u}(t, j)| \leq 2\max|g_c(z_1, z_2)|$ with $|z_1| \leq \beta(|(\phi_x(0, 0), \phi_e(0, 0))|, 0)$ and $|z_2| \leq \max|g_p(z_1)|$. When $j = 0$, we have that $|\phi_{e_y}(t, 0)| \leq |\phi_{e_y}(0, 0)| + |g_p(\phi_{x_p}(0, 0)) - g_p(\phi_{x_p}(t, 0))|$ and $|\phi_{e_u}(t, 0)| \leq |\phi_{e_u}(0, 0)| + |g_c(\phi_{x_c}(0, 0), \phi_y(0, 0)) - g_c(\phi_{x_c}(t, 0), \phi_y(0, 0))|$ and we can derive similar bounds on the interval $[0, t_1]$. As a result, and since ϕ_e is reset to 0 at each jump, ϕ_e cannot blow up in finite time. As a consequence, ϕ cannot explode in finite time. Let $G(x, e, \tau) := (x, 0, 0)$ denote the jump map (5). The solutions to (5), (10) cannot leave the set $C \cup D$ after a jump since $G(D) \subset C$ in view of (5), (10). Thus, we conclude that maximal solutions to (5), (10) are complete according to Proposition 6.10 in [24]. Finally, we note that if Assumption 1 holds locally, then there exists $\Delta > 0$ such that (19) and (21) hold on the invariant set $|(x, e)| \leq \Delta$ and consequently (12) holds locally. □

Proof of Proposition 1. Let $W(e) = |e|$, for all $e \in \mathbb{R}^{n_e}$. Then, in view of (15), we have that, for all $x \in \mathbb{R}^{n_x}$ and almost all $e \in \mathbb{R}^{n_e}$, $\langle \nabla W(e), \mathcal{A}_2 x + \mathcal{B}_2 e\rangle \leq |\mathcal{A}_2 x| + |\mathcal{B}_2||e|$. Hence, condition (8) holds with $L = |\mathcal{B}_2|$ and $H(x) = |\mathcal{A}_2 x|$. Let $V(x) = x^T P x$, for all $x \in \mathbb{R}^{n_x}$, where P is real positive definite and symmetric. Therefore, condition (6) is satisfied with $\underline{\alpha}(|x|) = \lambda_{\min}(P)|x|^2$ and $\overline{\alpha}(|x|) = \lambda_{\max}(P)|x|^2$. Consequently, for all $e \in \mathbb{R}^{n_e}$ and all $x \in \mathbb{R}^{n_x}$, $\langle \nabla V(x), \mathcal{A}_1 x + \mathcal{B}_1 e\rangle = x^T(\mathcal{A}_1^T P + P\mathcal{A}_1)x + x^T P\mathcal{B}_1 e + e^T\mathcal{B}_1^T P x$. By post- and pre-multiplying LMI (16) respectively by the state vector (x, e) and its transpose, we obtain $x^T(\mathcal{A}_1^T P + P\mathcal{A}_1)x + x^T P\mathcal{B}_1 e + e^T\mathcal{B}_1^T P x \leq -\varepsilon_2|x|^2 - |\mathcal{A}_2 x|^2 - \varepsilon_1|\overline{C}_p x|^2 + \mu|e|^2$. As a result, condition (7) is verified with $\alpha(|x|) = \varepsilon_2|x|^2$, $\delta(y) = \varepsilon_1|y|^2$ and $\gamma = \sqrt{\mu}$. Thus, Assumption 1 holds. □

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