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# A Black-box Identification Method for Automated Discrete Event Systems

Ana Paula Estrada-Vargas, Ernesto López-Mellado, and Jean-Jacques Lesage, *Members, IEEE*

**Abstract**— This paper deals with the identification of discrete event manufacturing systems that are automated using a programmable logic controller (PLC). The behavior of the closed loop system (PLC and Plant) is observed during its operation and is represented by a single long sequence of observed input/output (I/O) signals vectors. The proposed method follows a black-box and passive identification approach that allows addressing large and complex industrial DES and yields compact and expressive interpreted Petri net (IPN) models. It consists of two complementary stages; the first one obtains, from the I/O sequence, the reactive part of the model composed by observable places and transitions. The I/O sequence is also mapped into a sequence of the created transitions, from which the second stage builds the non observable part of the model including places that ensure the reproduction of the observed input output sequence. This method, based on polynomial-time algorithms on the size of the input data, has been implemented as a software tool that generates and draws the IPN model; it has been tested with input/output sequences obtained from real systems in operation. The tool is described and its application is illustrated through a case study.

**Note to practitioners**— Automated modeling of controlled discrete manufacturing systems can be achieved by efficient identification algorithms that cope with large and complex plants performing concurrent and repetitive tasks a priori unknown. The black-box identification procedure processes an input/output sequence recorded during the system functioning for a long period of time, and then yields a comprehensive model of the closed-loop controlled system; this model approximates closely the actual behavior of the compound system controller-plant. A tool based on identification algorithms constitutes an excellent resource for computer-aided reverse engineering of controlled manufacturing systems. The method proposed herein allows processing sequences composed by thousands of I/O vectors in few seconds.

**Index Terms**— Discrete Event Systems, Black-box Identification, Interpreted Petri Nets.

## I. INTRODUCTION

IDENTIFICATION of discrete event systems (DES) allows building systematically a mathematical model (Petri nets, automata) that describes the behavior of an unknown or ill-known system based on the observation of its evolution. Observations consist of data revealing the system activity:

sequences of operations, events, messages, signals etc., and the models allow reproducing the observed behavior.

### A. Related works

DES identification has been first addressed as a problem of grammatical inference [1], [2] for obtaining finite automata (FA) that represents a given language. Afterwards, Petri net (PN) models have been proposed for coping with more complex systems exhibiting concurrent behavior; in [3] an algorithm for constructing PN models is presented.

Several approaches for identification of DES have been proposed in literature in various formulations and from diverse approaches. Below there is an overview of the main approaches; other works can be found in detailed surveys on identification methods [4] and [5].

In [6], [7] methods based on Integer Linear Programming (ILP) are proposed; they allow obtaining accurate Petri nets from a set of transition sequences that can be fired from the initial marking. These methods require the *a priori* knowledge of the set of transitions and of the number of places, what makes difficult their application to identify real DES as black boxes, since the only available information after observation is the evolution of input and outputs signals exchanged between the control system and the plant.

In [8], [9] it is described an efficient method to incrementally construct an IPN model from a single output vectors sequence. The considered DESs to identify must be event-detectable by the outputs. Applying this method to an I/O sequence would lead to models in which same output changes caused by different input evolutions would not be distinguished, and then incorrect behavior could be introduced.

The method presented in [10] is dedicated to fault detection and isolation (FDI). It allows obtaining a finite automaton representing precisely a set of cyclic I/O sequences. An extension to distributed identification and distributed FDI has been presented in [11]. However, due to the usage of finite automata, structural information such as parallelism cannot be explicitly expressed into the models, what makes this approach inefficient for applications such reverse engineering. Other proposals related with FDI in the PN framework are presented in [12] and [13].

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In [14], [15] an event sequence is observed, as well as the corresponding output symbols of a DES to produce an IPN model, in which the sequence and the observed output vectors are reproducible. This method requires the definition of an event list, which is not available *a priori* in the context of black-box identification problem addressed in this work. An alternative to this lack of events list could be the consideration of all the observed input changes. In this case, models with several paths describing input changes would be constructed, in which some input/output relations would not be explicitly observed. This work has been extended in [16] towards the determination of stochastic transitions for FDI purposes.

In [17] a technique for constructing a Petri net-like model that describes the relationship between tasks from a sequence of workflow events is presented. This technique allows the discovering of events belonging to certain threads and synchronization points (forks and joins of tasks) through a probabilistic analysis of metrics such as the entropy, number and regularity of task occurrences. It is assumed that all the workflow operations are observable.

In [18] the modeling of a workflow is also considered. The input of the algorithm is a workflow log of several workflow instances composed by several tasks. Workflow instances are recorded sequentially, even if tasks may be executed in parallel. Based on the information in the workflow log and by making some assumptions about completeness of the log, a process model in the form of a workflow net is deduced.

### B. Black-box approach

Beyond the theoretical interest of defining model synthesis methods from event sequences, the challenges of applying identification methods to actual industrial automated systems are related to the scalability of the algorithms and technological issues: the techniques must be efficient to cope with large and complex systems that handle actual signals.

In our approach we deal with Programmable Logic Controller (PLC) based automated systems. The aim is to discover, from observations of the system behavior expressed as a single sequence of PLC input and output signals how components of the system are interrelated, and to construct a concise model which can explicitly show the discovered behavior, in particular, concurrency, synchronization, resource sharing, etc. Identification of systems in operation involves two important aspects to consider: the system operation and the observation process. Technological issues of both aspects must be considered in the proposed algorithms to construct suitable abstractions.

In previous works [19], [20] an I/O sequence is considered to compute an IPN including cyclic behavior. Although the proposed methodology is scalable due to the algorithms efficiency, the obtained models are close to finite automata and can be huge, due to the explicit representation of observed input changes that could not be relevant to define the output evolution.

### C. Contribution

In this paper we address these problems by analyzing the observed sequence to establish a clearer relation between inputs and outputs of the controller. The proposed method allows building a reduced representation of the observable part of the model which yields consequently, a reduced complete IPN. It consists of two complementary stages; the first one obtains, from the I/O sequence, the reactive part of the model composed by observable places and transitions. A first version of this stage has been presented in [21]. The I/O sequence is mapped into a sequence of the created transitions, from which the second stage builds the non-observable part of the model including places that ensure the reproduction of the observed input output sequence. This method, based on polynomial-time algorithms on the size of the input data, has been implemented as a software tool that generates and draws the IPN model [22]. None of the black-box identification approaches in related works allows obtaining such well structured models. The present article gathers both stages of the method; it includes a detailed presentation of the revised results, additional illustrative examples, all the proofs omitted in the conference papers, and a case study regarding the identification of a real process.

### D. Contents

The paper is organized as follows. In Section II IPN basic notions are overviewed. Section III states the problem of industrial automated systems identification and overviews the two steps method. Such steps are explained in Section IV and Section V. Finally, the implementation details and a case study are presented in Section VI.

## II. INTERPRETED PETRI NETS

This section contains the basic concepts and notation of PN and IPN used in this paper.

**Definition 1:** An ordinary Petri Net structure  $G$  is a bipartite digraph represented by the 4-tuple  $G = (P, T, Pre, Post)$  where:  $P = \{p_1, p_2, \dots, p_{|P|}\}$  and  $T = \{t_1, t_2, \dots, t_{|T|}\}$  are finite sets of vertices named places and transitions respectively;  $Pre(Post) : P \times T \rightarrow \{0,1\}$  is a function representing the arcs going from places to transitions (from transitions to places).

The incidence matrix of  $G$  is  $W = W^+ - W^-$ , where  $W^- = [w_{ij}^-]$ ;  $w_{ij}^- = Pre(p_i, t_j)$ ; and  $W^+ = [w_{ij}^+]$ ;  $w_{ij}^+ = Post(p_i, t_j)$  are the pre-incidence and post-incidence matrices respectively.

A marking function  $M : P \rightarrow Z^+$  represents the number of tokens residing inside each place; it is usually expressed as a  $|P|$ -entry vector.  $Z^+$  is the set of nonnegative integers. In particular, in this paper  $M : P \rightarrow \{0,1\}$ ; the PN is referred as 1-bounded or *safe*.

**Definition 2:** A Petri Net system or Petri Net (PN) is the pair  $N = (G, M_0)$ , where  $G$  is a PN structure and  $M_0$  is an initial marking.

In a PN system, a transition  $t_j$  is *enabled* at marking  $M_k$  if  $\forall p_i \in P, M_k(p_i) \geq Pre(p_i, t_j)$ ; an enabled transition  $t_j$  can be fired reaching a new marking  $M_{k+1}$ . This behavior is

represented as  $M_{k \rightarrow j} \rightarrow M_{k+1}$ . The new marking can be computed as  $M_{k+1} = M_k + Wu_k$ , where  $u_k(i) = 0, i \neq j, u_k(j) = 1$ ; this equation is called the PN state equation. The reachability set of a PN is the set of all possible reachable markings from  $M_0$  firing only enabled transitions; this set is denoted by  $R(G, M_0)$ .

**Definition 3.** A Petri net *circuit* is a path of vertices linked by arcs starting and ending in the same node. A circuit is said to be *simple* if it does not use the same transition more than once, and *elementary* if it does not use the same place more than once.

Now it is defined IPN, an extension to PN that allows associating input and output signals to PN models. This definition is adapted from [23].

**Definition 4 :** An interpreted Petri net (IPN)  $(Q, M_0)$  is a labeled net structure  $Q = (G, \Sigma, \Phi, \lambda, \varphi)$  with an initial marking  $M_0$  where:

- $G$  is a PN structure,
- $\Sigma = \{I_1, I_2, \dots, I_r\}$  is the input alphabet,
- $\Phi = \{O_1, O_2, \dots, O_q\}$  is the output alphabet,
- $\lambda : T \rightarrow C \times E$  is a labeling function of transitions, where
- $C = \{C_1, C_2, \dots\}$  is the set of input conditions in which every  $C_i$  is a Boolean function on  $\Sigma$ ; when a  $C_i$  is always true it is denoted as “=1”, and
- $E = \{E_1, E_2, \dots\}$  is the set of input events conditions; every  $E_i$  is a Boolean function of input events, build on  $\Sigma$ ; events are denoted as  $I_{i\_0}$  and  $I_{i\_1}$  for representing that the input value changes from 1 to 0, or from 0 to 1 respectively. A condition  $E_i$  may not exist; this is denoted as “ $\varepsilon$ ”.

In an IPN, a transition  $t_j$  will be fired if a)  $t_j$  is enabled, **and** b) condition  $C(t_j)$  is true, **and** c) the event in  $E(t_j)$  occurs.

- $\varphi : R(Q, M_0) \rightarrow (Z^+)^q$  is an output function, that associates with each marking in  $R(G, M_0)$  a  $q$ -entry output vector, where  $q = |\Phi|$  is the number of outputs.  $\varphi$  is represented by a  $q \times |P|$  matrix, such that if the output symbol  $O_i$  is present (turned on) every time that  $M(p_i) \geq 1$ , then  $\varphi(i, j) = 1$ , otherwise  $\varphi(i, j) = 0$ .

The state equation of PN is completed with the marking projection  $Y_k = \varphi M_k$ , where  $Y_k \in (Z^+)^q$  is the  $k$ -th output vector of the IPN.

**Definition 5:** A place  $p_i \in P$  is said to be *observable* if the  $i$ -th column vector of  $\varphi$  (denoted as  $\varphi(\bullet, i)$ ) is not null. Otherwise it is *non-observable*.  $P = P^{obs} \cup P^{nobs}$ , and  $P^{obs} \cap P^{nobs} = \emptyset$ ; where  $P^{obs}$  is the set of observable places and  $P^{nobs}$  the set of non-observable places.

### III. IDENTIFICATION OF INDUSTRIAL AUTOMATED SYSTEMS

#### A. The process PLC+Plant

In this work we consider systems composed by a Controller (a PLC) and a Plant denoted as {PLC + Plant} working on a closed loop. The input signals of the PLC (outputs of the Plant) are generated by the sensors of the Plant. The output signals of the PLC (inputs of the Plant) control the actuators of

the Plant.

The identification is made with respect to the inputs-outputs of the PLC (Fig. 1). A PLC cyclically performs three main steps: i) Input reading, where signals are read from the sensors; ii) Program execution, to determine the new outputs values for the actuators; and iii) Output writing, where the control signals to the actuators are set. At each end of the Program execution phase, the current value of all  $r$  inputs and  $q$  outputs, called I/O vector, is captured and recorded in a data base.

Regarding the implementation of the data link between PLC and identification data base, we use the UDP (User Datagram Protocol) connection presented in [24]. Tests performed using a Siemens PLC (CPU 315-2 DP) equipped with a program leading to a PLC-cycle time of 25 to 30ms have shown that this connection is reliable and efficient: no data packets got lost during the transmission and the execution of the PLC program is not delayed by the capture of data.

The only available data for the identification procedure is therefore a single sequence of I/O vectors whose length depends on the observation duration:

$$w = \begin{bmatrix} I(1) \\ O(1) \end{bmatrix}, \begin{bmatrix} I(2) \\ O(2) \end{bmatrix}, \begin{bmatrix} I(3) \\ O(3) \end{bmatrix}, \dots, \begin{bmatrix} I(k) \\ O(k) \end{bmatrix}, \dots \quad (1)$$

$I(k)$  and  $O(k)$  are vectors whose entries are respectively the values of the  $r$  inputs  $I_1, I_2, \dots, I_r$  and  $q$  outputs  $O_1, O_2, \dots, O_q$  at the  $k$ -th PLC cycle. Furthermore we denote  $I_i(k)$  and  $O_i(k)$  the values of input  $I_i$  and output  $O_i$  respectively at the  $k$ -th cycle.

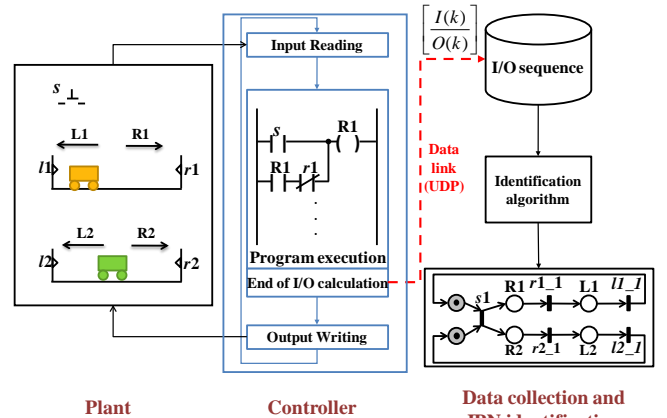


Fig. 1. {PLC + Plant} compound and identification procedure.

#### B. Event types

In order to analyze signals evolution, we compute *event vectors*, i.e., the difference between two consecutive I/O vectors. Each event vector can be decomposed into input and output event vectors:

$$E(k) = \begin{bmatrix} IE(k) \\ OE(k) \end{bmatrix} = \begin{bmatrix} I(k+1) \\ O(k+1) \end{bmatrix} - \begin{bmatrix} I(k) \\ O(k) \end{bmatrix},$$

where

$$IE(k) = \begin{bmatrix} IE_1(k) \\ IE_2(k) \\ \vdots \\ IE_r(k) \end{bmatrix} \quad \text{and} \quad OE(k) = \begin{bmatrix} OE_1(k) \\ OE_1(k) \\ \vdots \\ OE_q(k) \end{bmatrix} \quad (2)$$

Regarding input and output event vectors and the PLC cycle described in the previous subsection, there only exist four

situations (behavior types) between consecutive I/O vectors that could be observed, which are explained by different occurring phenomena:

*Type 1.*  $IE(k) \neq 0$  and  $OE(k) \neq 0$

An input change has provoked directly an output change, and consequently, a state evolution. This I/O reactive causality is observed at the same PLC cycle.

*Type 2.*  $IE(k) = 0$  and  $OE(k) \neq 0$

The controller has arrived at step  $k-1$  to a state in which, given the input values, an output (and state) evolution is allowed at step  $k$ .

*Type 3.*  $IE(k) \neq 0$  and  $OE(k) = 0$

Let  $X(k)$  be the internal current state of the controller,

a)  $X(k-1) \neq X(k)$  An input evolution has provoked a non-observable state evolution of the controller.

b)  $X(k-1) = X(k)$  It has occurred an input evolution to which the controller is not sensitive.

*Type 4.*  $IE(k) = 0$  and  $OE(k) = 0$

a)  $X(k-1) \neq X(k)$  It has occurred a non-observable state evolution of the controller which is not exhibited by any input nor output change.

b)  $X(k-1) = X(k)$  The controller remains in a stable state, i.e., no state evolution condition is satisfied.

All of these situations should be taken into account to represent the system dynamics. Our aim in this work is to express the system's behavior extractible from the I/O vector sequence as an IPN.

### C. Input-Output identification approach

#### C.1 Overview of the method

The purpose in this research is not only to compute an IPN model in which the observed sequence is reproducible, but also to achieve expressivity and compactness in the identified model allowing representing causal relationship and concurrency of the involved operations.

The method processes off-line the I/O-sequence  $w$  captured during the process operation and delivers an IPN model that reproduces the observed behavior ( $w$ ).

The method is outlined here with the help of a simple example. It regards a controller handling 3 inputs ( $s$ ,  $x$ ,  $y$ ) and 3 outputs (A, B, C), from which the following I/O sequence is obtained:

$$w = \begin{matrix} s \\ x \\ y \\ A \\ B \\ C \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The method consists of two main steps which are outlined below.

**Step1.** Discovering the reactive input/output behavior. In this step is determined the observable part of the IPN consisting of subnets, named fragments, composed by observable places labeled with output symbols, and transitions labeled with algebraic expressions of input symbols (Fig. 2).

From the sequence  $w$ , a corresponding sequence of transitions  $S = t_1 t_2 t_3 t_4 t_1 t_2 t_5 t_6 t_1 t_2 t_3 t_4 t_1 t_2 t_5$  is obtained.

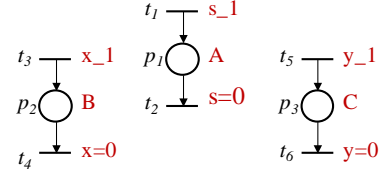


Fig 2. First step of the identification method: IPN fragments.

**Step2.** Determining the non-observable part of the IPN and the initial marking  $M_0$ . The sequence  $S$  is processed for obtaining causal and concurrency relationships useful for determining the non-observable places that relate the fragments such that  $S$  (thus  $w$ ) can be executed from  $M_0$  (Fig. 3).

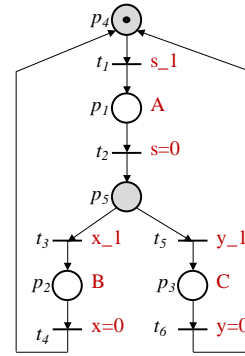


Fig 3. Second step: assembled IPN fragments

#### C.2 Dealing with event types

Since situations *Type 1* and *Type 2* (cf. section III.B) are directly observable by an output change, they can be straightforwardly modeled in an IPN. Such a modeling is performed by the first step of our method.

The *Type 1* situation represents a direct input/output reactive behavior, and thus the modeling is quite easy: the input change is associated with the label of a transition and the output change is represented as arcs relating such a transition with the observable places representing outputs involved.

In the *Type 2* situation the input values which lead to the output evolution are not observed at the same PLC cycle (i.e. at the same event vector). In order to represent such a behavior, the context (the values of the inputs) in which the output changes occur is analyzed; in this case, the output change is modeled such as in the *Type 1* situation, but the label of the corresponding transition contains only a condition on inputs levels (the input change is  $\epsilon$ ).

The *Type 3* situation is divided in two, depending on whether or not there is an internal state evolution of the controller. Situation *Type 3.a* is the case of the input events which provoke internal state evolutions and eventually lead to an output event of *Type 2*. Such internal evolutions cannot be directly computed, but can be inferred. By looking in the sequence built in Step 1, the order in which transitions appear can be determined. Such internal state inference will be performed by the second step of our method and will be

modeled by the addition of non observable places assuring the order of the transition firings, such as in Fig. 3.

In the situation *Type 3.b* there is no internal state evolution, and thus there is nothing to be inferred, as well as the situation *Type 4*, where there are neither input nor output events occurring in a PLC cycle. Consequently, the sequence stored in the database will be built by adding a new I/O vector only when it is different to the last one. Notice that in this work we can only infer internal state evolutions by means of transition firing order. Other type of internal evolutions, such as timers or counters, is out of the scope of this work. We can now make the description of the two identification steps.

#### IV. IDENTIFICATION OF THE OBSERVABLE BEHAVIOR

In this section the first step of the method is presented. The introduced concepts and algorithms are illustrated through a simple case study inspired from a manufacturing example.

##### A. Overview and case study description

Algorithm 1 summarizes the steps of the procedure to identify the {PLC + Plant} observable behavior; the steps will be described in detail in the next sub-sections.

##### Algorithm 1. Computing the observable IPN components

**Input:** I/O sequence  $w$

**Output:** Observable incidence matrix  $\varphi W$  and labeling transition function  $\lambda$

1) Analyze sequence  $w$  in order to

- Compute events vector sequence
- Compute elementary events
- Compute Direct and Indirect Causality Matrices
- Construct Output Event Firing Functions
- Find Input events with differed influence

2) Use computed data in the previous step to

- Compute transitions of the IPN and their labeling  $\lambda$
- Compute observable incidence matrix  $\varphi W$

**Example 1.** The purpose of this system (Fig. 4) is to sort parcels according to their size. It has 9 signal sensors from the system:  $a_0, a_1, a_2, b_0, b_1, c_0, c_1, k_1, k_2$ , and 4 signals to the actuators:  $A+, A-, B, C$ . This example has been used in other publications [19], [20] and we describe it to confront this work against previous results.

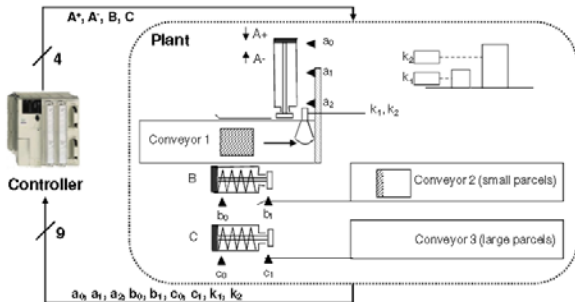


Fig. 4. Layout of the system case study.

##### B. Events vector sequence

In Fig. 5 the beginning of an I/O vector sequence is shown for illustrative purposes; however, recall that treated sequences are usually very much longer (thousands of vectors). We have included in the sequence the result of first sub-step of the algorithm, i.e., the computed event vectors (below the arrows) between each two consecutive I/O vectors.

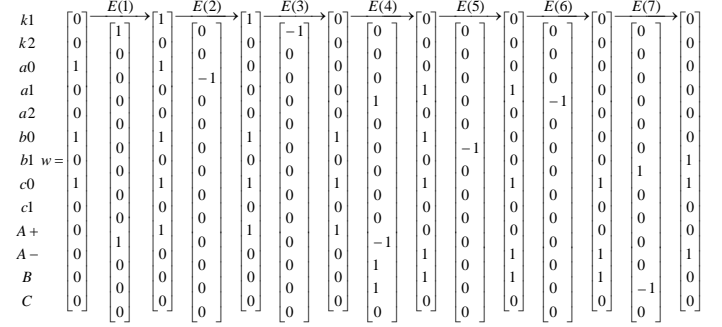


Fig. 5. Beginning of I/O vector sequence.

##### C. Elementary events

In order to analyze the system behavior in a deeper way, event vectors can be decomposed into a set of elementary events (simply called events):

$$IE(k) = \{IE_{k_1}, IE_{k_2}, \dots, IE_{k_r}\} = \bigcup_{i=1}^{i=r} IE_{k_i} \text{ s.t. } I_i(k+1) - I_i(k) \neq 0 \quad (3)$$

$$OE(k) = \{OE_{k_1}, OE_{k_2}, \dots, OE_{k_q}\} = \bigcup_{i=1}^{i=q} OE_{k_i} \text{ s.t. } O_i(k+1) - O_i(k) \neq 0 \quad (4)$$

If no elementary input (output) event occurs in  $E(k)$ , we denote it as  $IE(j) = \{\varepsilon\}$  ( $OE(j) = \{\varepsilon\}$ ). The rising edge event of input  $I_i$  (output  $O_i$ ) is denoted as  $I_{i\_1}$  ( $O_{i\_1}$ ). The falling edge event of input  $I_i$  (output  $O_i$ ) is denoted as  $I_{i\_0}$  ( $O_{i\_0}$ ).

Table 1 shows the elementary events computed for the example sequence.

TABLE I  
ELEMENTARY EVENTS LIST FOR EXAMPLE 1

Event vector	Elementary input events	Elementary output events
$E(1)$	$IE(1) = \{k1\_1\}$	$OE(1) = \{A+_1\}$
$E(2)$	$IE(2) = \{a0\_0\}$	$OE(2) = \{\varepsilon\}$
$E(3)$	$IE(3) = \{k1\_0\}$	$OE(3) = \{\varepsilon\}$
$E(4)$	$IE(4) = \{a1\_1\}$	$OE(4) = \{A+_0, A-_1, B\_1\}$
$E(5)$	$IE(5) = \{b0\_0\}$	$OE(5) = \{\varepsilon\}$
$E(6)$	$IE(6) = \{a1\_0\}$	$OE(6) = \{\varepsilon\}$
$E(7)$	$IE(7) = \{b1\_1\}$	$OE(7) = \{B\_0\}$

##### D. Direct and Indirect Causality Matrices

As stated in Section III, the influence of some input signals over the outputs setting is observed at the same PLC cycle. In order to discover such an input/output direct relationship, we analyze the relative frequency of the occurrence of both input events  $IE_i$  and output events  $OE_k$ , with respect to the occurrence of  $OE_k$  along the whole sequence of events. This relationship can be naturally expressed as the conditional probability of the occurrence of an output event  $OE_k$ , given



that a certain input event  $IE_i$  has occurred at the same PLC cycle:

$$Prob(OE_k | IE_i) = \frac{N_{Obs}(OE_k, IE_i)}{N_{Obs}(OE_k)} \quad (5)$$

where  $N_{Obs}(\cdot)$  denotes the number of observed occurrences. Using all values  $Prob(OE_k|IE_i)$ , a matrix can be filled. We call such a matrix the *Direct Causality Matrix (DM)*, in which every  $DM_{ik} = Prob(OE_k|IE_i)$ . Fig. 6 presents the computed DM matrix for the *Example 1*, considering a sequence much longer than the presented one.

	A+ 1	A+ 0	A- 1	A- 0	B 1	B 0	C 1	C 0
k1_1	0.111	0.111	0.111	0.111	0.000	0.200	0.000	0.000
k1_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
k2_1	0.222	0.000	0.000	0.000	0.000	0.000	0.000	0.000
k2_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a0_1	0.222	0.000	0.000	1.000	0.000	0.000	0.000	0.000
a0_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a1_1	0.000	0.444	0.444	0.000	1.000	0.000	0.000	0.000
a1_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
a2_1	0.000	0.556	0.556	0.000	0.000	0.000	1.000	0.000
a2_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
b0_1	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000
b0_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
b1_1	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
b1_0	0.000	0.000	0.000	0.111	0.000	0.000	0.000	0.000
c0_1	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000
c0_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
c1_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
c1_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Fig. 6. Direct Causality Matrix for the Example 1

Similarly, conditional probability has been used in [17] for determining the relationship between workflow operations.

With the DM matrix, we can find Evolution *Type 1* simply by looking at each column the values that add up to 1, since this represents the total number of occurrences of event  $OE_k$ . For example, from Fig. 6 we can discover that the output event A+\_0 is *always* provoked by event a1\_1 (in 44.4% of the observed cases) or by event a2\_1 (in 55.6% of the observed cases). The general case where several input events can provoke an output event is formalized on the next section.

Similarly, to discover input/output non direct relationship, we look at the input values when a certain output event occurs. We compute the occurrence probability of an output event  $OE_k$ , given that certain input has a given value  $IL_i$  at the same PLC cycle:

$$Prob(OE_k | IL_i) = \frac{N_{Obs}(OE_k, IL_i)}{N_{Obs}(OE_k)} \quad (6)$$

We construct the *Indirect Context Matrix (IM)* in which every  $IM_{ik} = Prob(OE_k|IL_i)$ . The IM matrix for *Example 1* is shown in Fig. 7.

Using the IM matrix we can discover evolution *Type 2* by inspecting in every column the values that add up to 1 which are not zero in the DM matrix. In the *Example 1*, k1=1 and k2=1 are input values which *can* provoke A+\_1 output event, even if they were *not always* observed at the same PLC cycle.

Now we will present how these relations can be automatically discovered from the DM and IM matrices.

	A+ 1	A+ 0	A- 1	A- 0	B 1	B 0	C 1	C 0
k1=1	0.444	0.111	0.111	0.333	0.000	0.250	0.200	0.200
k1=0	0.556	0.889	0.889	0.667	1.000	0.750	0.800	0.800
k2=1	0.556	0.000	0.000	0.333	0.000	0.250	0.000	0.200
k2=0	0.444	1.000	1.000	0.667	1.000	0.750	1.000	0.800
a0=1	1.000	0.000	0.000	1.000	0.000	0.500	0.000	0.000
a0=0	0.000	1.000	1.000	0.000	1.000	0.500	1.000	1.000
a1=1	0.000	0.444	0.444	0.000	1.000	0.000	0.000	0.000
a1=0	1.000	0.556	0.556	1.000	0.000	1.000	1.000	1.000
a2=1	0.000	0.556	0.556	0.000	0.000	0.000	1.000	0.000
a2=0	1.000	0.444	0.444	1.000	1.000	1.000	0.000	1.000
b0=1	1.000	1.000	1.000	0.556	0.000	0.000	1.000	1.000
b0=0	0.000	0.000	0.000	0.444	1.000	1.000	0.000	0.000
b1=1	0.000	0.000	0.000	0.111	1.000	1.000	0.000	0.000
b1=0	1.000	1.000	1.000	0.889	0.000	0.000	1.000	1.000
c0=1	1.000	1.000	1.000	0.889	0.000	1.000	1.000	0.000
c0=0	0.000	0.000	0.000	0.111	1.000	0.000	0.000	1.000
c1=1	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000
c1=0	1.000	1.000	1.000	1.000	0.000	1.000	1.000	0.000

Fig. 7. Indirect Context Matrix of the Example 1.

### E. Computing Firing Functions of Output Events

It can be noticed that the occurrence of every output event  $OE_k$  is caused by one or several input events occurring at the same PLC cycle and by a condition on the input values. In order to represent such conditions, a firing function  $\chi(OE_k)$  has to be defined for every  $OE_k$ . It is called the Output Event Firing Function (OEFF):

$$\chi(OE_k) = G(OE_k) \bullet F(OE_k)$$

where  $G(OE_k)$  is a function of input events and  $F(OE_k)$  is a function of inputs levels which allow the triggering of the output event  $OE_k$ .

We compute  $G(OE_k)$  as a conjunction of disjunctions of input events  $E_j$ :

$$G(OE_k) = \Pi Disj E_j \quad (8)$$

where each disjunction  $Disj E_j = (IE_x \oplus \dots \oplus IE_z)$  involves those variables corresponding to non-zero column values of the DM matrix, which add up to 1, i.e. those satisfying conditions:

$$DM_{xj} \neq 0, DM_{yj} \neq 0, \dots, DM_{zj} \neq 0 \quad (9)$$

$$DM_{xj} + DM_{yj} + \dots + DM_{zj} = 1 \quad (10)$$

Similarly,  $F(OE_k)$  is computed as a conjunction of disjunctions of input levels  $L_j$ :

$$F(OE_k) = \Pi Disj L_j \quad (11)$$

with  $Disj L_j = (IL_x \oplus \dots \oplus IL_z)$  such that

$$IM_{xj} \neq 0, IM_{yj} \neq 0, \dots, IM_{zj} \neq 0 \quad (12)$$

$$IM_{xj} + IM_{yj} + \dots + IM_{zj} = 1 \quad (13)$$

$$DM_{xj} \neq 0, DM_{yj} \neq 0, \dots, DM_{zj} \neq 0 \quad (14)$$

At the end of the computing, for every output signal  $O_i$ , we will have the input events and input conditions to produce its rising and falling edges  $O_{i-1}$  and  $O_{i-0}$  respectively. This can be easily translated into IPN fragments, as shown in Fig. 8.

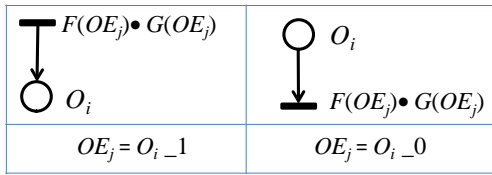


Fig. 8. Rising and falling edges of output  $O_i$

#### F. Input events with differed influence on the outputs

Notice that condition  $DM_{xj} \neq 0$ ,  $DM_{yj} \neq 0, \dots, DM_{zj} \neq 0$  requires that the inputs related to the output change were observed at least once changing its value at the same PLC cycle that the considered output. This condition may be restrictive if the input-output reaction is not observed in the same event vector. For example, in order to avoid component damages, in the absence of an input sensor to indicate that a pusher has been retracted, there may be some security temporizations which do not allow another actuator reacting at the moment an input condition has been satisfied.

In such cases, the input-output reaction would not be found and thus there may be an output event with empty conditions on its firing function. In order to find the correct  $OEFF$ , we can relax the condition to consider input events which have been observed in previous event vectors instead of the same event vector. Formally, we can compute:

$$Prob(OE_k | IE_i) = \frac{N_{Obs}(OE_k, IE_{i(\text{previous vector})})}{N_{Obs}(OE_k)} \quad (15)$$

But this time, the computation is done by considering  $IE_i$  occurred at the previous event vector than  $OE_k$ . A new  $OEFF$  can be computed using new values instead of those of the  $DM$  matrix. If the computed  $OEFF$  has still empty conditions, we can take the previous to the previous event vector and successively while empty conditions are computed. In the *Example 1*, such relaxing condition is not necessary, since, as it can be noticed, no empty conditions have been computed. However, in the experimental case study of Section VI.B, such a technique is applied.

For the *Example 1*,  $D = \emptyset$ ; the computed PN fragments are shown in Fig. 9.

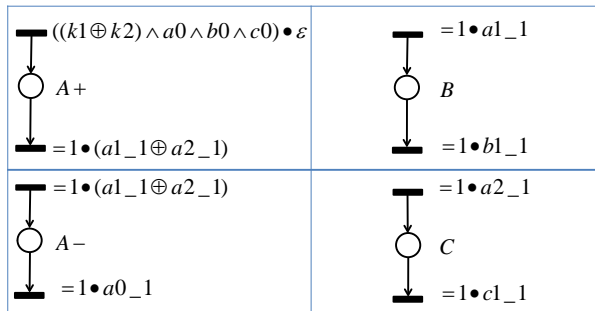


Fig. 9. IPN fragments for Example 1

#### G. Fusion of IPN fragments

As stated below, at each PLC cycle, several input and state conditions could lead to the simultaneous occurrence of several output events. This behavior is reproduced by merging

such conditions into a unique transition, which is labeled by a firing function computed from individual firing functions of each output event. This is captured in the model as a fusion of IPN fragments as shown in Fig. 10.

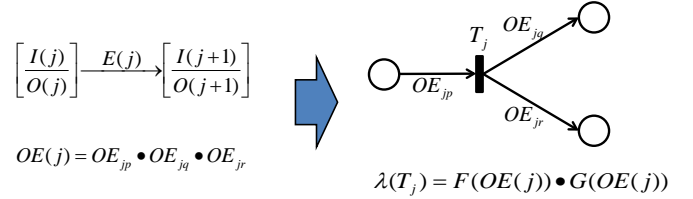


Fig. 10. IPN representation of several output events at the same cycle

The construction of the observable IPN can be systematically done with the next procedure:

#### Algorithm 2.

**Input:** I/O sequence  $w$ , I/O events sequence  $E$ , Matrices  $DM$  and  $IM$ , Differed input set  $D$

**Output:** Observable incidence matrix  $\varphi C$ , labeling transition function  $\lambda$ , and sequence of transitions  $S$

1.  $P \leftarrow \{p_1, p_2, \dots, p_q\}$  //Create  $q$  observable places, one for every output of the system
2.  $S \leftarrow \varepsilon$  //Initialize the sequence  $S$
3.  $\forall E(j), j=1, \dots, |E|$  **do** //Consider all the computed I/O events in  $E$ 
  - 3.1. **if**  $OE(j) = 0$  and  $\exists IE_s, \dots, IE_u \in IE(j) \cap D$  //There is not an output change in  $E(j)$ , but  $IE(j)$  contains elementary input events  $IE_s, \dots, IE_u$  belonging to  $D$ 

**then**

    - 3.1.1.  $T \leftarrow T \cup \{t_j\}$ ;  $\lambda(t_j) \leftarrow IE_s \bullet \dots \bullet IE_u$ ;  
 $W(t_j, p_i) \leftarrow 0, \forall p_i \in P$  //If it has not been created before, create a new zero transition  $t_j$  (a zero column in the incidence matrix) representing input changes  $IE_s, \dots, IE_u$
    - 3.1.2.  $S \leftarrow S \cdot t_j$  //Concatenate  $t_j$  to  $S$
  - 3.2. **else if**  $OE(j) \neq 0$  //There is an output change in  $E(j)$ 

**then**

    - 3.2.1.  $\forall OE_{jk} \in OE(j)$  //Consider all the elementary output events in  $OE(j)$  in order to compute  $G(OE(j))$  and  $F(OE(j))$ 
      - 3.2.1.1.  $\forall DisjE_i \in G(OE_{jk})$ , do  $DisjE_i' \leftarrow DisjE_i \cap IE_{jk}$  // Look into  $IE(j)$  the input event  $IE_{jk}$  which has satisfied  $DisjE_i$  and assign it to  $DisjE_i'$
      - 3.2.1.2.  $G'(OE_{jk}) \leftarrow \Pi DisjE_i'$  //Combine into  $G'(OE_{jk})$  all the conditions  $DisjE_i'$  which have satisfied  $G(OE_{jk})$
      - 3.2.1.3.  $G(OE(j)) \leftarrow \Pi G'(OE_{jk})$  //Combine into  $G(OE(j))$  all the input event conditions  $G'(OE_{jk})$  which have satisfied all the events  $OE_{jk}$
      - 3.2.1.4.  $\forall DisjL_i \in F(OE_{jk})$ , do  $DisjL_i' \leftarrow DisjL_i \cap I(j+1)$  // Looking the  $I(j+1)$  vector as a set of Boolean variables, save into  $DisjL_i'$  the input value  $IL_{ik}$  which has satisfied  $DisjL_i$
      - 3.2.1.5.  $F'(OE_{jk}) \leftarrow \Pi DisjL_i'$  //Combine into  $F'(OE_{jk})$  all the conditions which have made true  $F(OE_{jk})$
      - 3.2.1.6.  $F(OE(j)) \leftarrow \Pi F'(OE_{jk})$  //Combine into  $F(OE(j))$  all the conditions which have produced all the  $OE_{jk}$



3.2.2.  $T \leftarrow T \cup \{t_j\}$ ,  $\lambda(t_j) = F(OE(j)) \bullet G(OE(j))$  //If it has not been created before, create a new transition  $t_j$  and label it with the computed  $F(OE(j))$  and  $G(OE(j))$

3.2.3.  $\forall p_i \in P$ , **do**

**If**  $O_{q-1} \in OE(j)$  **then**  $W(t_j, p_q) \leftarrow -1$ ,

**else if**  $O_{q-0} \in OE(j)$  **then**  $W(t_j, p_q) \leftarrow -1$ ,

**else**  $W(t_j, p_{jq}) \leftarrow 0$  //for all elementary output events in  $OE(j) = OE_{jp} \bullet OE_{jq} \dots \bullet OE_{jr}$ , put a 1 into the line corresponding to  $OE_{jk}$  if it is a rising event, and a -1 if it is a falling event; for the rest of the lines, assign a 0.

3.2.4.  $S \leftarrow S \cdot t_j$  //Concatenate  $t_j$  to  $S$

**Complexity of Algorithm 2.** Let  $r' < r$  and  $q' < q$  be respectively the maximum number of input and output elementary events appearing simultaneously in an event vector, and  $|E|$  be the length of the events sequence. The Algorithm 2 processes each one of the events in  $E$ . When a transition should be added to represent one of such events, an appropriate firing function should be computed. If only inputs changed, it is only necessary to include in the firing function the elementary input events with differed influence. This is achieved in  $O(r')$ . If there is at least an output change, for each one of the output elementary events which have occurred, we need search for each individual firing function the input events and input conditions that produced the evolution. This is performed in  $O(q'(r' \log r'))$ . Thus, the complexity of the procedure for building the transition sequence and fragments is  $O((q'r' \log r')/|E|)$ . Consequently, the Algorithm 2 can be executed in polynomial time on the size of the input data.

**Property 1.** The transitions sequence  $S$  is a translation of the I/O sequence  $w$  into transition firings of the PN-fragments built by Algorithm 2.

**Proof.** It is easy to see that at the steps 3.1.2 and 3.2.4 of Algorithm 2,  $S$  is formed by concatenating the computed transitions from the event sequence produced by  $w$ . This allows that the reactive behavior can be reproduced in the created IPN model. ■

Fig. 11 shows how events  $E(1)$  and  $E(4)$  are treated by the algorithm. For  $E(1)$  the elementary output event  $A+_{-1}$  in  $OE(1)$  is analyzed and function  $\lambda(t_1) = (k1 \bullet a0 \bullet b0 \bullet c0) \bullet (\varepsilon)$  is extracted considering that  $k1=1$ ,  $a0=1$ ,  $b0=1$ , and  $c0=1$  are the input values which have satisfied  $\chi(A+_{-1})$ . For  $E(4)$  all elementary output events  $A+_{-0}$ ,  $A-_{-1}$  and  $B_{-1}$  in  $OE(4)$  are considered and their Firing Functions  $\chi(A+_{-0}) = (=1) \bullet (a1_{-1} \oplus a2_{-1})$ ,  $\chi(A-_{-1}) = (=1) \bullet (a1_{-1} \oplus a2_{-1})$  and  $\chi(B_{-1}) = (=1) \bullet (a1_{-1})$  are combined into  $\lambda(t_2) = (=1) \bullet (a1_{-1})$ .

Notice that interesting labeling functions have been computed. For example, the output event  $A+_{-1}$  is provoked by the presence of a piece ( $k1=1$  or  $k2=1$ ) and it occurs only when the three components corresponding to outputs  $A+$ ,  $B$ , and  $C$  are on its initial position ( $a0=1$ ,  $b0=1$  and  $c0=1$ ).

At the end of the procedure, the following observable incidence matrix  $\varphi W$  and labeling functions are obtained, as well as the transition sequence  $S$ , which is the projection of  $w$  over  $T$ :  $S = t_1 t_2 t_3 t_4 t_1 t_2 t_4 t_3 t_5 t_6 t_7 t_4 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_4 t_1 t_2 t_3 t_4 t_1$ .

$t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_4 t_5 t_6 t_7 t_4 t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_4 t_5 t_6 t_7 t_4 t_5 t_6 t_7 t_4 t_1 t_2 t_3 t_4 t_1 t_2 t_3 t_4 t_1 t_2 t_4 t_3 t_1 t_2 t_3 t_4 t_1 t_2 t_3 t_4 t_1$ .

	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$		$\lambda(t_1) = (k1 \wedge a0 \wedge b0 \wedge c0) \bullet (\varepsilon)$
$A+$	1	-1	0	0	1	-1	0		$\lambda(t_2) = (=1) \bullet (a1_{-1})$
$A-$	0	1	0	-1	0	1	0		$\lambda(t_3) = (=1) \bullet (b1_{-1})$
$B$	0	1	-1	0	0	0	0		$\lambda(t_4) = (=1) \bullet (a0_{-1})$
$C$	0	0	0	0	0	1	-1		$\lambda(t_5) = (k2 \wedge a0 \wedge b0 \wedge c0) \bullet (\varepsilon)$
									$\lambda(t_6) = (=1) \bullet (a2_{-1})$
									$\lambda(t_7) = (=1) \bullet (c1_{-1})$

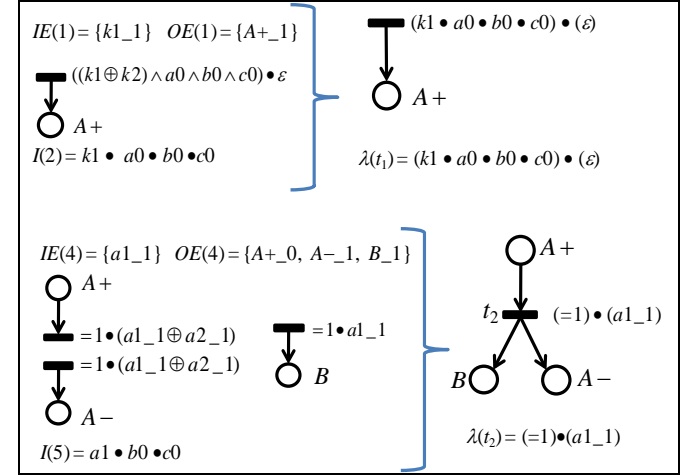


Fig. 11. Treatment of  $E(1)$  and  $E(4)$  by the Algorithm 2.

The corresponding partial model is shown in Fig. 12. The inferring procedure which allows discovering the non-observable behavior is described in next section.

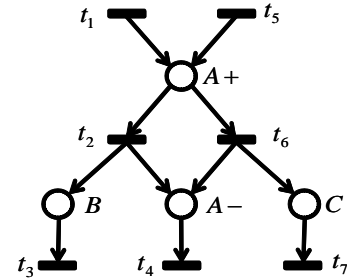


Fig. 12. Observable IPN model

## V. IDENTIFICATION OF THE NON-OBSERVABLE BEHAVIOR

### A. Problem (re)statement

The previously described procedures allow obtaining an observable structure which represents the reactive behavior of the system. Given that events and transitions of the net are completely defined, we need to add non-observable places to translate an aggregation of the non-observable dynamics of the process in such a way that the global PN will reproduce the whole behavior of the system. By adding non-observable places (depicted as grey circles), we make the inference of situation *Type 3.a* described in Section III.B, which is the case of input events provoking internal state evolutions.



In order to find which is the situation occurring between every pair of transitions in  $Seq$ , some other definitions are now introduced. The following notion is the *systematical precedence* of a transition  $t_j$  with respect to another transition  $t_k$ ; it establishes a necessary condition for  $t_j$  to occur repeatedly.

**Definition 8.** A transition  $t_j$  is *preceded systematically* by  $t_k$ , denoted as  $t_k \angle t_j$  iff  $t_k$  is always observed between two apparitions of  $t_j$  in  $S$ . By convention, we say that  $t_j \angle t_j$  if  $t_j$  was observed at least twice in  $S$ . Then the *Systematical Precedence Set* of a transition  $t_j$  is given by the function  $SP: T \rightarrow 2^T$ , that indicates which transitions *must* be fired to re-enable the firing of  $t_j$ , i.e.  $SP(t_j) = \{t_k \mid t_k \angle t_j\}$ . If  $t_j$  was observed only once in  $S$ , then  $SP(t_j) = \emptyset$ .

In the sequence  $S$  from *Example 2*, one may compute that  $t_1 \angle t_1$ ,  $t_2 \angle t_1$ ,  $t_3 \angle t_1$ , and  $t_4 \angle t_1$ , thus  $SP(t_1) = \{t_1, t_2, t_3, t_4\}$ . Notice that  $SP(t_j)$  is the set of transitions that must invariantly occur to fire  $t_j$  repeatedly. The rest of the  $SP$  sets are:  $SP(t_2) = \{t_1, t_2, t_3, t_4\}$ ,  $SP(t_3) = \{t_1, t_2, t_3\}$ ,  $SP(t_4) = \{t_4\}$ ,  $SP(t_5) = \{t_4, t_5, t_6, t_7\}$ ,  $SP(t_6) = \{t_4, t_5, t_6, t_7\}$ ,  $SP(t_7) = \{t_4, t_5, t_6, t_7\}$ .

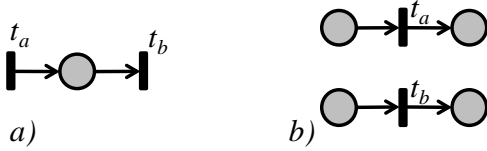


Fig. 14. Structures that represent  $t_a < t_b$ . a) shows a causal relationship from  $t_a$  to  $t_b$ , whereas b) shows a concurrent relationship between  $t_a$  and  $t_b$ .

**Definition 9.** Two transitions  $t_a, t_b$  are called transitions in a two length cycle relationship (named two-cycle transitions) if  $S$  contains the subsequence  $t_a t_b t_a$  or the subsequence  $t_b t_a t_b$ . The two-cycle transitions set  $TC$  of  $S$  is given by  $TC = \{(t_a, t_b) \mid t_a, t_b \text{ are in a two-cycle}\}$ .

From the sequence  $S$  in *Example 2*, we observe that the set of transitions in a two-cycle is  $TC = \emptyset$ .

**Remark.** Computing  $Seq$ ,  $SP$  and  $TC$  can be executed in polynomial time on the size of  $S$ .

We will now extract some structural properties regarding  $N$  from  $S$ . The previously defined terms will be used to determine which situation between causality and concurrency is the most appropriated for every pair of consecutively observed transitions in  $S$ .

### C. Causal and concurrency relationships

#### 1) Causal relationship

In order to determine that two transitions are causally related as shown in Figure 14.a, several conditions stated below must be fulfilled.

**Proposition 1.** If  $t_a \angle t_b$  ( $t_a \in SP(t_b)$ ) then, there must exist in  $N$  a simple elementary circuit (SE circuit) to which both  $t_a$  and  $t_b$  belong.

**Proof.** Suppose that there is not a SE circuit containing  $t_a$  and  $t_b$ . Thus, right after the firing of  $t_b$ , all the tokens in  $t_b^*$  (the output places of  $t_b$ ) could be displaced by transition firings

through some path to  $t_b^*$  (the input places of  $t_b$ ), enabling  $t_b$  without needing to fire  $t_a$ , which implies that  $t_a \notin SP(t_b)$ . ■

**Proposition 2.** If  $t_a < t_b$  and  $t_a \angle t_b$ , then there must exist in  $N$  a place from  $t_a$  to  $t_b$ .

**Proof.** Suppose that there is not a place from  $t_a$  to  $t_b$ . In order to allow the observation  $t_a < t_b$ , both  $t_a$  and  $t_b$  should be enabled simultaneously. By Proposition 1, there is at least one SE circuit containing  $t_a$  and  $t_b$  and thus, at least one path from  $t_a$  to  $t_b$ . Thus, if  $t_a$  and  $t_b$  are enabled simultaneously and  $t_a$  is fired, all paths from  $t_a$  to  $t_b$  contain two tokens. If all transitions in a path from  $t_a$  to  $t_b$  are fired, then there will be two tokens in one of the input places of  $t_b$ , resulting in a non-safe net. Then, at least one of the transitions  $t_i$  in each path from  $t_a$  to  $t_b$  must be conditioned to the previous firing of  $t_b$ . But if  $t_b$  is fired, all the transitions in paths from  $t_a$  to  $t_b$  can be fired and all the transitions in paths from  $t_b$  to  $t_b$  which do not include  $t_a$  can be fired; thus  $t_b$  will be enabled before  $t_a$  fires and as a consequence  $t_a \notin SP(t_b)$ . ■

**Proposition 3.** If  $t_a < t_b$  and  $t_b \angle t_a$ , then there must exist in  $N$  a place from  $t_a$  to  $t_b$ .

**Proof.** Suppose that there is not a place from  $t_a$  to  $t_b$ . Then, before the observation of  $t_a < t_b$ , both  $t_a$  and  $t_b$  must be enabled, and thus the occurrence of  $t_b < t_a$  is possible. Furthermore, together with  $t_b \angle t_a$  and by Proposition 2 implies that there should be a place from  $t_b$  to  $t_a$ . However, at the firing of  $t_b$  there are two tokens in such a place, and thus the net is not safe. ■

**Proposition 4.** If  $(t_a, t_b) \in TC$ , then there must exist in  $N$  a place from  $t_a$  to  $t_b$  and a place from  $t_b$  to  $t_a$ .

**Proof.** The sequence  $t_a t_b t_a$  must be reproducible in  $N$ . Right after the firing of  $t_a$  there is a token on its output places, and thus  $t_b$  must be at the output of such places; otherwise, there would be two tokens in such places after the second firing of  $t_a$ . Similarly, right after the first firing of  $t_a$ , there are no tokens on its input places, and thus  $t_b$  must be at the input of such places; otherwise,  $t_a$  could not be fired again. The same reasoning can be applied to reproduce the sequence  $t_b t_a t_b$ . ■

Notice that when two transitions are observed consecutively and one is systematically preceded by the other, a causal relationship is found. Also, when two transitions are involved in a two-cycle relation, they are in a causal relationship each other. Observe that all of these relationships are structural, and thus they do not depend of the initial marking of the net.

**Definition 10.** The causal relationship set *CausalR* keeps track of all the causal relationships in  $S$ .  $CausalR = \{(t_a, t_b) \mid (t_a < t_b) \text{ and } (t_a \angle t_b \text{ or } t_b \angle t_a \text{ or } (t_a, t_b) \in TC)\}$ .

From the  $Seq$  set in *Example 2* (see Definition 6), the  $SP$  sets (see Definition 8) and the  $TC$  set (see Definition 9) we compute  $CausalR = \{(t_1, t_2), (t_2, t_3), (t_4, t_1), (t_2, t_4), (t_5, t_6), (t_6, t_7), (t_7, t_4), (t_4, t_5), (t_3, t_1)\}$ .

If a couple of transitions  $(t_a, t_b)$  in the  $Seq$  set, belongs also to *CausalR*, then there must be a place from  $t_a$  to  $t_b$  in order to

constrain the observed firing order. For the rest of the transition couples in  $Seq$ , we must decide if a place should exist to relate them. Next, we will discuss some cases where the existence of a place can be discarded.

## 2) Concurrency relationship

If two transitions  $t_a$  and  $t_b$  are concurrent, there must not exist a place neither from  $t_a$  to  $t_b$  nor from  $t_b$  to  $t_a$ ; otherwise, the firing of one would constrain the firing of the other one.

**Definition 11.** The set of all pairs of concurrent transitions is called  $ConcR = \{(t_a, t_b) \mid t_a \parallel t_b\}$ .

If the sequence  $w$  is complete, (consequently,  $S$ ) i.e., if it exhibits all of the possible behavior of the observed system, we can find concurrence between transitions that are not in a causal relation, as shown in the next proposition.

**Proposition 5.** Let  $t_a, t_b$  be two transitions which have been observed consecutively in a complete sequence  $S$  in both orders, i.e.  $(t_a, t_b) \in Seq$ ,  $(t_b, t_a) \in Seq$ . Then  $(t_a, t_b) \notin CausalR$  and  $(t_b, t_a) \notin CausalR$  if and only if  $t_a \parallel t_b$ .

**Proof.** Suppose that  $(t_a, t_b) \notin ConcR$ . Without loss of generality, we suppose there is a place  $p_{ab}$  from  $t_a$  to  $t_b$ . Since  $(t_b, t_a) \in Seq$ , there must also be a place  $p_{ba}$  from  $t_b$  to  $t_a$ ; otherwise,  $t_a$  could be enabled simultaneously with  $t_b$  to allow  $t_b < t_a$  and  $t_a$  may be fired, yielding to the presence of two tokens in the place  $p_{ab}$  and breaking the safeness condition. Since  $(t_a, t_b) \notin CausalR$ ,  $t_b \notin SP(t_a)$  and thus there must be at least one path from  $p_{ab}$  to  $p_{ba}$  which does not contain  $t_b$ . Similarly, there must be at least one path from  $p_{ba}$  to  $p_{ab}$  which does not contain  $t_a$ . Since  $(t_a, t_b) \notin TC$ ,  $t_a t_b t_a$  should not be enabled and thus, there must be at least one SE circuit to which  $t_a$  belongs, but  $t_b$  does not belong. The resulting net violates the free-choice conditions (observe Fig. 15).

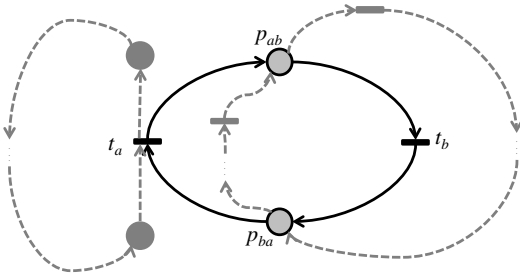


Fig. 15. Structure where  $(t_a, t_b) \in Seq$  and  $(t_b, t_a) \in Seq$  but  $(t_a, t_b) \notin ConcR$

Suppose now that  $(t_a, t_b) \in ConcR$ . That means that they can be both enabled simultaneously and one can be fired without needing the firing of the other one, and thus  $t_a \notin SP(t_b)$  and  $t_b \notin SP(t_a)$ . Also, since there cannot be any place from  $t_a$  to  $t_b$  nor from  $t_b$  to  $t_a$ , neither the subsequence  $t_a t_b t_a$ , nor the subsequence  $t_b t_a t_b$  can be enabled, and thus  $(t_a, t_b) \notin CausalR$  and  $(t_b, t_a) \notin CausalR$ . ■

Notice that our methodology allows computing also non free-choice nets. Only in the case where the system includes a behavior like the one shown in Fig. 15, the transitions  $t_a$  and  $t_b$  would be wrongly considered as concurrent and the existence of links from  $t_a$  to  $p_{ab}$  and from  $t_b$  to  $p_{ba}$  would be missed.

However, the obtained model would be still capable to reproduce the sequence  $S$ .

It is well known that in practice, the sequence  $w$  is not complete, since in the general case, the observed systems do not show all their possible behavior during a finite time of data collection. In fact, it is not possible to assure that the whole behavior of a system has been observed. The consideration of Proposition 5 is then very restrictive, since it demands the observation of all possible behavior; it could lead to the construction of incorrect models in case of incomplete sequences. Then, some less constraining rules to find concurrence must be considered. Next, we present several properties which allow us to identify couples of transitions which must be concurrent in the identified net  $N$ .

First, we will introduce the notion of *Sequential Independence*, which is a characteristic of concurrent transitions. Later, the propositions to find concurrency will be introduced.

**Definition 12.** Two transitions  $t_a$  and  $t_b$  are *Sequentially Independent* if  $t_a \notin SP(t_b)$  and  $t_b \notin SP(t_a)$ .

From the  $SP$  sets of Example 2 (see Definition 8) we compute the set of Sequentially Independent transitions:  $\{(t_1, t_5), (t_1, t_6), (t_1, t_7), (t_2, t_5), (t_2, t_6), (t_2, t_7), (t_3, t_4), (t_3, t_5), (t_3, t_6)\}$ .

Observe the net in Fig. 16 which is composed by two independent t-components  $X_1$  and  $X_2$  with supports  $\langle X_1 \rangle = \{t_a, t_i\}$  and  $\langle X_2 \rangle = \{t_b, t_k\}$  respectively. In a sequence belonging to the language of such a net, transitions belonging to different t-components are sequentially independent. In fact,  $SP$  sets of this net correspond exactly to t-components of the net.

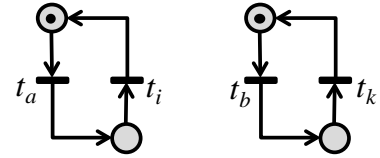


Fig. 16. A net with two t-components

**Proposition 6.** Let  $t_a$  and  $t_b$  be two transitions in  $S$  which have been observed consecutively in both orders  $(t_a < t_b$  and  $t_b < t_a)$ . If:

- $(t_a, t_b) \notin CausalR$  and  $(t_b, t_a) \notin CausalR$ ,
  - and  $|SP(t_a)| > 1$  and  $|SP(t_b)| > 1$ ,
- then  $t_a \parallel t_b$ .

**Proof.** Suppose that  $t_a$  and  $t_b$  are not concurrent. Without loss of generality, we suppose there is a place  $p_{ab}$  from  $t_a$  to  $t_b$ . Since  $t_b < t_a$  has been observed, there must be also a place  $p_{ba}$  from  $t_b$  to  $t_a$  (and as consequence  $N$  contains a two-transition cycle); otherwise,  $t_a$  could be enabled simultaneously with  $t_b$  to allow  $t_b < t_a$  and  $t_a$  may be fired, yielding to the presence of two tokens in the place  $p_{ab}$  and breaking the safeness condition. Since  $t_b \notin SP(t_a)$ , there must be at least one path leading from  $p_{ab}$  to  $p_{ba}$  not including  $t_b$ . Since  $|SP(t_a)| > 1$ , there must be at least one circuit including  $t_a$  and not including  $p_{ab}, p_{ba}$  nor  $t_b$ . Since  $t_a \notin SP(t_b)$ , there must be at least one path leading from

$p_{ba}$  to  $p_{ab}$  not including  $t_a$ . Consider the first transition  $t_x$  of this path. The free-choice conditions are not satisfied, since  $t_x$  and  $t_a$  share  $p_{ba}$  as input place, but  $t_a$  has at least one different input place. ■

We may observe that for *Example 2*,  $(t_3, t_4)$  are sequentially independent (see Definition 12), however,  $|SP(t_4)| = 1$  and thus we cannot infer any concurrence.

When  $SP(t_j)$  is a singleton, it means that it belongs to several elementary circuits and then Proposition 6 does not allow to find concurrent transitions to  $t_j$ . But if  $t_j$  is included in the  $SP$  of other transitions, we may find some concurrence relations, as shown in the next proposition.

**Proposition 7.** Let  $t_a$  and  $t_b$  be two transitions in  $S$  that have been observed consecutively in both orders ( $t_a < t_b$  and  $t_b < t_a$ ). If  $t_a$  and  $t_b$

- are *Sequentially Independent* and
  - there exists a transition  $t_k$  such that  $t_a \angle t_k$  ( $t_a \in SP(t_k)$ ) and  $t_b \angle t_k$  ( $t_b \in SP(t_k)$ )
- then  $t_a \parallel t_b$ .

**Proof.** Suppose that it does not hold that  $t_a \parallel t_b$ . Without loss of generality, we suppose that there is a place from  $t_a$  to  $t_b$ . Since  $t_a \in SP(t_k)$  and  $t_b \in SP(t_k)$ , after the firing of  $t_k$ , both  $t_a$  and  $t_b$  must be fired before the next firing of  $t_k$ . Since  $t_b < t_a$  may happen, the place from  $t_a$  to  $t_b$  must be marked. However  $t_a < t_b$  may occur too, leading to the presence of two tokens in the same place after the firing of  $t_a$ , and making the net not safe. ■

Fig. 17 shows an example of the case characterized by Proposition 7. It is the general case of transitions belonging to concurrent threads ( $t_a, t_c$  and  $t_b, t_d, t_e, t_f$  respectively), which are eventually synchronized by one transition ( $t_k$ ). If we make several firings to build a transition sequence, eventually the  $SP$  sets would become:  $SP(t_k) = \{t_k, t_a, t_c, t_b, t_d, t_f\}$ ,  $SP(t_a) = SP(t_c) = \{t_k, t_a, t_c\}$ ,  $SP(t_b) = SP(t_f) = \{t_k, t_b, t_d, t_f\}$ ,  $SP(t_e) = \{t_e, t_d\}$ ,  $SP(t_d) = \{t_d\}$ . Even if  $SP(t_d)$  is a singleton, the synchronization point  $t_k$  help us to find by Proposition 7 several concurrent relationships:  $t_a \parallel t_b$ ,  $t_a \parallel t_d$ ,  $t_a \parallel t_f$ ,  $t_c \parallel t_b$ ,  $t_c \parallel t_d$ , and  $t_c \parallel t_f$ .

In *Example 2*,  $(t_3, t_4)$  are *Sequentially Independent* (see Definition 12), and we have determined that  $t_3 \angle t_1$  and  $t_4 \angle t_1$  (see Definition 8), thus we can conclude that  $t_3 \parallel t_4$ .

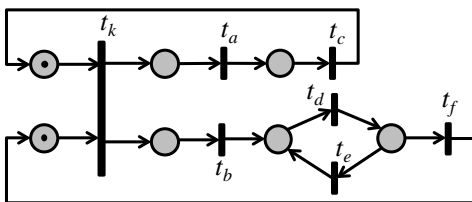


Fig. 17. Concurrent threads synchronized by a transition

If concurrent transitions do not belong to synchronized threads, conditions of the next propositions help us to find a subset of concurrent transitions which do not depend from another transition  $t_k$ .

**Proposition 8.** Let be two transitions  $t_a$  and  $t_b$  which have

been observed consecutively in both orders ( $t_a < t_b$  and  $t_b < t_a$ ). If  $t_a$  and  $t_b$  are:

- Sequentially Independent*, and
  - $\exists t_k$  such that  $t_k \in SP(t_b)$ ,  $t_k \notin SP(t_a)$ , and
  - $(t_a, t_k) \in Seq$
- then  $t_a \parallel t_b$ .

**Proof.** Suppose there is a place  $p_{ab}$  from  $t_a$  to  $t_b$ . Since  $t_b < t_a$  has also been observed, there must be also a place  $p_{ba}$  from  $t_b$  to  $t_a$ ; otherwise,  $t_a$  should be enabled simultaneously with  $t_b$  to allow  $t_b < t_a$  and  $t_a$  may be fired, yielding to the presence of two tokens in  $p_{ab}$ . Since there exists  $t_k$  such that  $t_k \in SP(t_b)$ , then there must be a SE circuit containing both  $t_b$  and  $t_k$ . If such a circuit contains places  $p_{ba}$  or  $p_{ba}$ , it is not possible to fire  $t_a < t_k$  and thus such a circuit must contain another input place  $p_{kb}$  of  $t_b$  and another output place  $p_{bk}$  of  $t_b$ . Now, to accomplish that  $t_b \notin SP(t_a)$ , there must be at least one path leading from  $p_{ab}$  to some input place of  $t_a$  not including  $t_b$ . Consider the first transition  $t_x$  of this path. In order to respect the free-choice conditions,  $p_{kb}$  should be an input place of  $t_x$ , making the occurrence of  $t_a < t_k$  impossible. ■

**Definition 13.** The *Inverse Systematical Precedence* set of a transition  $SP^{-1}: T \rightarrow 2^T$  contains the transitions which are dependent of a common transition to re-enable their firing:

$$PS^{-1}(t_j) = \{t_k \mid t_k \neq t_j \text{ and } t_j \in PS(t_k)\} \quad (13)$$

**Proposition 9.** Let  $t_a$  and  $t_b$  be two transitions which have been observed consecutively in both orders ( $t_a < t_b$  and  $t_b < t_a$ ). If  $t_a$  and  $t_b$  are: *Sequentially Independent*, and  $SP^{-1}(t_a) \neq \emptyset$ ,  $\forall t_j \in SP^{-1}(t_a)$ ,  $t_j \parallel t_b$ , then  $t_a \parallel t_b$ .

**Proof.** Suppose there is a place  $p_{ab}$  from  $t_a$  to  $t_b$ . Since  $t_b < t_a$  has also been observed, there must be also a place  $p_{ba}$  from  $t_b$  to  $t_a$ ; otherwise,  $t_a$  should be enabled simultaneously with  $t_b$  to allow  $t_b < t_a$  and thus  $t_a$  may be fired, yielding to the presence of two tokens in the place from  $t_a$  to  $t_b$ . Since  $t_b \notin SP(t_a)$ , there must be at least one path leading from  $p_{ab}$  to  $p_{ba}$  not including  $t_b$ . Similarly, there must be at least one path leading from  $p_{ba}$  to  $p_{ab}$  not including  $t_a$ . Since  $SP^{-1}(t_a) \neq \emptyset$ , there is at least one transition  $t_j$  concurrent to  $t_b$  such that  $t_j \angle t_a$  and there must be a SE circuit including  $t_a$  and  $t_j$ . Such a circuit cannot contain  $p_{ab}$  nor  $p_{ba}$  otherwise  $t_j$  may be able to fire without need of firing  $t_a$ . Consider the input place  $p_x$  of  $t_a$  in this path. The free-choice conditions are not satisfied between  $p_x$  and  $p_{ba}$ : they share  $t_a$  as output transition, but  $p_{ba}$  has at least another output transition. ■

An example where Proposition 9 can be used is shown in Fig. 18.  $SP^{-1}(t_a) = \{t_{j1}, t_{j2}\}$  and  $t_{j1} \parallel t_b$ ,  $t_{j2} \parallel t_b$  are determined by Proposition 6. Consequently,  $t_a \parallel t_b$ .

**Remark.** Computing *CasualR* and *ConcR* can be executed in polynomial time on the size of  $S$ .



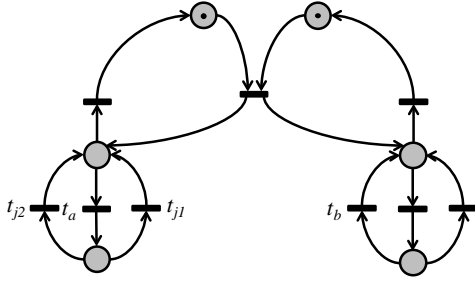


Fig. 18. Concurrency between transitions whose  $SP$  is a singleton

#### D. Building the non-observable PN

We will use now the computed data from sequence  $S$  to infer internal evolutions of the system. We will make an analysis of causal and concurrency relations that have been found between consecutive transitions in order to compute non-observable places of the net.

**Definition 14.** The set  $Seq' = (Seq - CausalR) - ConcR$  contains the set of transition pairs  $(t_a, t_b)$  which have been observed consecutively, but are not in a causal relation or in a concurrency relation.

Until now, we have computed for *Example 2* that  $Seq = \{(t_1, t_2), (t_2, t_3), (t_3, t_4), (t_4, t_1), (t_2, t_4), (t_4, t_3), (t_3, t_5), (t_5, t_6), (t_6, t_7), (t_7, t_4), (t_4, t_5), (t_3, t_1)\}$ ,  $CausalR = \{(t_1, t_2), (t_2, t_3), (t_4, t_1), (t_4, t_5), (t_5, t_6), (t_6, t_7), (t_7, t_4), (t_2, t_4), (t_3, t_1)\}$  and  $ConcR = \{(t_3, t_4), (t_4, t_3)\}$ . Thus,  $Seq' = \{(t_3, t_5)\}$ . This means that there is a relationship which has not been explained.

If  $Seq' \neq \emptyset$ , then there are two possibilities for the remaining transition pairs  $(t_a, t_b)$  in  $Seq'$ :

- They are both input and output transitions of a place with several input and output transitions
- They are concurrent, but  $w$  (thus,  $S$ ) is not complete enough to find such a relationship

Since our goal is to approximate as much as possible the language generated by identified IPN, to the observed sequence  $S$ , we assume that if we have observed two transitions consecutively ( $t_a < t_b$ ) but by none of the previous propositions we have determined that they are concurrent, thus the firing of  $t_a$  has enabled  $t_b$ . This is made in order to preserve in the PN the firing order observed in  $S$ . Then, a place will be added from  $t_a$  to  $t_b$ ; this denoted by  $[t_a, t_b]$ .

When it is found that  $[t_a, t_c]$  and  $[t_b, t_c]$ , and the involved transitions are related by a single place, this is represented as  $[t_a t_b, t_c]$ . In general, a place  $p$  can be denoted as  $[t_{a1} t_{a2} \dots t_{al}, t_{b1} t_{b2} \dots t_{bh}]$ , where  $t_{ai}$  are the input transitions of  $p$  and  $t_{bi}$  are the output transitions of  $p$ , and  $l=|p|$ ,  $h=|p^*|$ , as illustrated in Fig. 19.

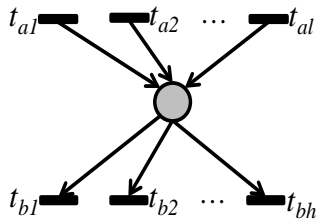


Fig. 19. A PN place  $p = [t_{a1} t_{a2} \dots t_{al}, t_{b1} t_{b2} \dots t_{bh}]$

The same place could be used to relate several consecutive transitions. If a transition  $t_k$  has been observed followed by two transitions  $t_{ai}, t_{aj}$  in  $S$  ( $t_k < t_{ai}$  and  $t_k < t_{aj}$ ), there are two cases to represent such observations into the PN model: the case of selection, where they are represented with the same place  $[t_k, t_{ai} t_{aj}]$  (Fig. 20a) or the case of concurrence, where they are represented with different places  $[t_k, t_{ai}] [t_k, t_{aj}]$  (Fig. 20b).

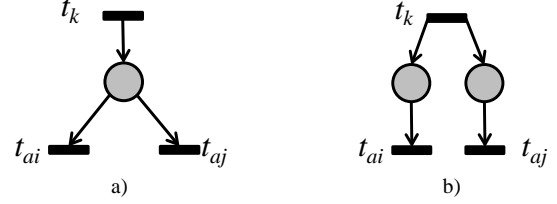


Fig. 20. Selection and parallelism representation. a) Shows the case where  $t_{ai}, t_{aj}$  are not concurrent and have not been observed consecutively whereas b) shows the case where  $t_{ai}, t_{aj}$  are concurrent or have been observed consecutively.

In a generalized form, for every set  $t_k < t_{a1}, \dots, t_k < t_{aw}$  of non-concurrent consecutive transition pairs with the same first transition  $t_k$ , we can thus merge all  $t_k < t_{a1}, \dots, t_k < t_{ax}$  whose second transitions  $t_{a1} \dots t_{aw}$  are non-concurrent nor consecutive and represent them into a single place  $[t_k, t_{a1} \dots t_{aw}]$ , as illustrated in Fig. 21.

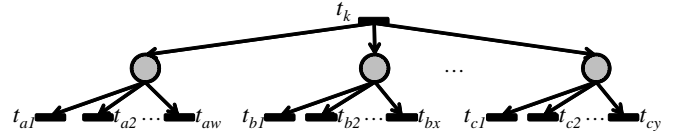


Fig. 21. A PN place  $p = [t_{a1} t_{a2} \dots t_{al}, t_{b1} t_{b2} \dots t_{bh}]$

Once we have made the first merging, all places  $[t_{k1}, t_{a1} \dots t_{aw}]$ ,  $[t_{k2}, t_{a1} \dots t_{aw}]$ ,  $\dots$ ,  $[t_{kz}, t_{a1} \dots t_{aw}]$  whose input transitions are non-concurrent nor consecutive and whose output transitions are the same, can be merged into a single place as illustrated in Fig. 22.

**Remark.** Building the non-observable PN can be executed in polynomial time on the size of  $Seq$ .

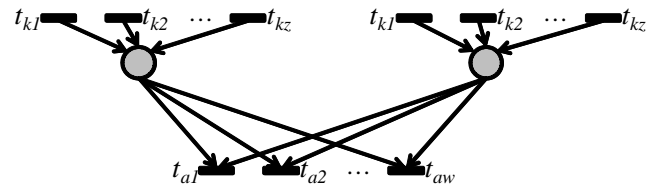


Fig. 22. Selection and concurrence between pre-transitions

#### E. Initial marking

Once the structure of the net is built, the initial marking can be computed by allowing the firing of  $S$ . All transitions are processed, from the last transition till the first one. The processing of a transition is as follows:

- If its output places are unmarked, the tokens in such places are retired,
- Tokens are added to its unmarked input places.

**Example 2 (Cont.).** By considering the couples of consecutive non-concurrent transitions in  $Seq'$  (which in this example is only  $(t_3, t_5)$  – see Definition 14), the places:  $[t_1, t_2]$

$[t_2, t_3]$   $[t_3, t_1t_5]$   $[t_4, t_1t_5]$   $[t_5, t_6]$   $[t_6, t_7]$  and  $[t_2t_7, t_4]$  are computed. The PN structure and the computed initial marking is shown in Fig. 23.

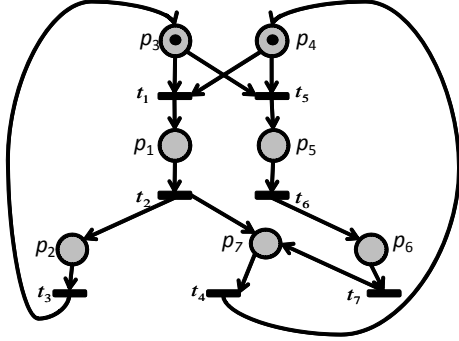


Fig. 23.  $(G^{nobs}, M_0)$  the non-observable IPN of Example 1

#### F. Token flow verification

As stated before, with the proposed mechanisms in last section, the sequence  $w$  may not have shown enough combinations which allow us to determine concurrence. If the sequence  $w$  were complete, all the concurrent and sequential behavior could be found and represented, according to Proposition 6. However, since we know that  $w$  could not be complete, in order to approximate the language of the identified IPN to  $S$  as much as we can, we have considered that if two transitions have not been declared as concurrent, they must be in a sequential relationship. But if the transitions are actually concurrent, the sequential consideration could lead us to links or places in the built model which restrict too much the behavior of the system and don't allow the firing of  $S$ . Now, we present some notions that will help us to verify if added places until now do not interfere in the correct reproduction of  $S$ .

**Proposition 10.** If the IPN model has been correctly build, every computed non-observable place  $p$  in  $N$  must fulfill the place input/output flow equation:

$$\sum_{t_i \in \bullet p} Occ(t_i) = \sum_{t_i \in p \bullet} Occ(t_i) \pm 1$$

where  $Occ(t_k)$  is the number of occurrences of  $t_k$  in  $S$ .

**Proof.** Equation follows straightforward from the IPN transition enabling and firing conditions and from the fact that  $(G^{nobs}, M_0)$  must be safe. ■

**Proposition 11.** If there exists a place  $p$  such that  $|\bullet p|=1$ , then  $\forall t_j \in p \bullet, t_k \in SP(t_j), SP(t_j) \neq \emptyset$  where  $t_k$  is the input transition of  $p$ . Also, if there exists a place  $p$  such that  $|p \bullet|=1$ , then  $\forall t_j \in \bullet p, t_k \in SP(t_j), SP(t_j) \neq \emptyset$  where  $t_k$  is the output transition of  $p$ .

**Proof.** If  $|\bullet p|=1$ , for the re-enabling of  $t_j$ ,  $p$  must be marked and the only way to do so is the firing of  $t_k$ , and thus  $t_k \in SP(t_j)$ . Similarly, if  $|p \bullet|=1$ , for the re-enabling of  $t_j$ ,  $p$  must be unmarked and the only way to do so is the firing of  $t_k$ , thus  $t_k \in SP(t_j)$ . ■

**Correction rule.** If the input/output flow equation or the conditions in Proposition 11 are not satisfied by some place,

the arcs relating transitions which are not in *CausalR* are removed. If there are not *CausalR* represented in such a place, it is deleted.

**Example 2 (Cont.).** In the model of Figure 23, we verify the input/output flow equation for each place. From Example 2, we can compute  $Occ(t_1)=12$ ,  $Occ(t_2)=11$ ,  $Occ(t_3)=11$ ,  $Occ(t_4)=20$ ,  $Occ(t_5)=9$ ,  $Occ(t_6)=9$ , and  $Occ(t_7)=9$ . We check also the condition of Proposition 11.

- $p_1: Occ(t_1) = Occ(t_2) (\pm 1), t_1 \in SP(t_2), t_2 \in SP(t_1)$
- $p_2: Occ(t_2) = Occ(t_3) (\pm 1), t_2 \in SP(t_3), t_3 \in SP(t_2)$
- $p_3: Occ(t_3) \neq Occ(t_1) + Occ(t_5) (\pm 1), t_3 \in SP(t_1), t_3 \in SP(t_5)$
- $p_4: Occ(t_4) = Occ(t_1) + Occ(t_5) (\pm 1), t_4 \in SP(t_1), t_4 \in SP(t_5)$
- $p_5: Occ(t_5) = Occ(t_6) (\pm 1), t_5 \in SP(t_6), t_6 \in SP(t_5)$
- $p_6: Occ(t_6) = Occ(t_7) (\pm 1), t_6 \in SP(t_7), t_7 \in SP(t_6)$
- $p_7: Occ(t_2) + Occ(t_7) = Occ(t_4) (\pm 1), t_4 \in SP(t_2), t_4 \in SP(t_7)$

As can be observed,  $p_3$  is a wrong place, since  $Occ(t_3) \neq Occ(t_1) + Occ(t_5) \pm 1$ . Since  $(t_3, t_5) \in Seq'$ ; this means that the sequence is not complete, and thus the causal relationship we assumed between  $t_3$  and  $t_5$  is wrong. In order to fix this, we can delete the arc going from place  $p_3$  to transition  $t_5$ . After this correction, all of the conditions from Proposition 10 and Proposition 11 are satisfied.

Finally, the identified IPN of the sorting system described in Example 1 is obtained by merging the observable model in Fig. 12 and the non-observable model from Fig. 23 after applying the places correction. We can also delete non-observable implicit places. Then the IPN shown in Fig. 24, which reproduces  $w$ , is the final result of the model merging.

In the supplementary file [26] several additional examples regarding the method for identifying a non observable model from a sequence  $S$  are included.

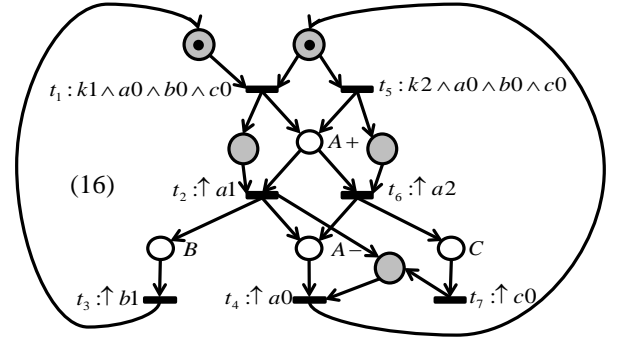


Fig. 24. Final IPN model for the process in Example 1

#### G. Features of the method

##### G.1 Reproducibility of $S$

**Proposition 12.** The PN model  $(G^{nobs}, M_0)$  built with the previous procedures summarized in Algorithm 3 reproduces the sequence  $S$ .

**Proof.** Regard that we have computed the following sets:

- *Seq* containing all the consecutive transition couples in  $S$ . If we represent into a net all couples in *Seq*, the net will be able to reproduce  $S$ ,
- *CausalR* containing transition couples  $(t_a, t_b) \in Seq$  that must be related by a place,

- *ConcR* containing transition couples  $(t_a, t_b) \in Seq$ , that must not be related by any place.

If the set  $Seq' = (Seq - CausalR) - ConcR = \emptyset$ , it means that all transition couples  $(t_a, t_b) \in Seq$  are correctly represented in  $N$  and thus the sequence  $S$  is reproducible. If  $Seq' \neq \emptyset$ , it means that there are still transition couples that cannot be distinguished as concurrent or sequential. Thus, by merging several couples in  $Seq$ , all couples in  $Seq'$  are considered as sequential by creating places with several input and output transitions. If they are actually sequential, all the verification rules are satisfied. Otherwise, they are actually concurrent and they are corrected using the described procedure. Once they are corrected, it only remains places relating sequential transitions and thus the sequence  $S$  is reproducible. ■

### G.2 Performance

Given that all of the procedures of *Algorithm 3* are executed in polynomial time on  $|S|$ , the construction of  $(G^{nobs}, M_0)$  is efficiently performed.

Note also that the application of *Algorithm 3* to a sequence  $S$  yields always the same PN model, due to that all the constructive steps in the procedures are deterministically performed, i.e. there are not random selections on the input and intermediate data.

## VI. METHOD IMPLEMENTATION AND APPLICATION

Based on the presented algorithms, a software tool has been developed to automate the IPN model synthesis. The architecture of the tool is shown in Fig. 25.

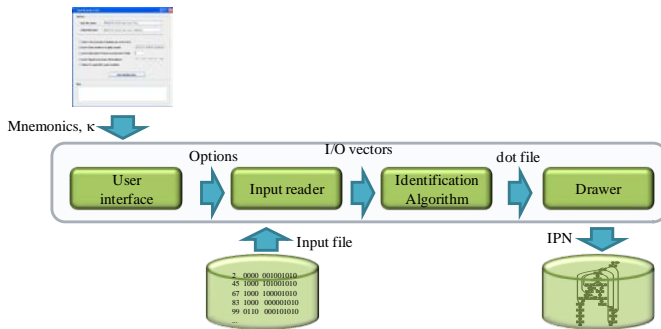


Fig. 25. Software architecture

The user interface allows capturing the input/output sequence and shows the obtained model graphically. Following input data is provided to the tool: the name of a text file containing the I/O sequence (with one line per I/O vector), the names of the input and output signals, and the desired name for the output file. Additionally it is specified the order in which inputs and outputs appear in the txt file (since depending on data collection procedure, order could change) and the index numbers of the signals to take into account if a mask is going to be applied (some inputs or outputs could be ignored like indicator lights or push-buttons).

Later, an input reader component processes the input file and transforms the input/output sequence into a vector

sequence. These vectors are delivered to a component called Algorithm in which the identification procedure is implemented. The output of this component is an XML file that can be opened with the Platform Independent Petri net Editor (PIPE [25]), which is an editor for visualization and analysis of Petri nets.

The presented identification tool has been tested on several examples of diverse size and complexity. A small size case study regarding an actual manufacturing system is described in the supplementary files to this article [26] in which the use of such a software tool is illustrated.

## VII. CONCLUSION

The proposed identification method discovers the actual input-output relation of PLC controlled discrete event systems. The technique allows building a concise IPN model in which the transitions are labeled with sufficient conditions on the inputs which represent both the input changed and the inputs execution context. The obtained structure is remarkably more clear and expressive than that synthesized with a previous method.

The technique copes with complex automated DES because it takes into account technological characteristics of actual controlled systems, and because it is based on efficient algorithms. This feature is not still addressed in current literature on the matter, in which several features considered in the current stated problem have not been dealt.

The algorithms issued from the present method have been implemented as a software tool and tested on experimental case studies which are very close to actual industrial discrete event processes. The performed tests reveal the efficiency of the methods when data including thousands of input-output vectors are processed in few seconds.

Due to this is a black-box approach, the obtained models represent the observed behavior; consequently, when the observation has been made for a long time, the identified IPN approximates closely the actual behavior. Afterwards this model can be completed using available knowledge on the process.

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## Biographies



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