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Gaussian beam launching based on frame decomposition and 3d spectral partition

I.F. Arias Lopez* C. Letrou†

Abstract — **Frame decomposition has been introduced into Gaussian Beam Shooting (GBS) algorithms to perform decompositions of fields radiated by large planar apertures into a half plane in a rigorous and stable way. This work proposes a generalization of frame-based GBS to situations when fields are radiated into all directions in 3d space. Frame decomposition is applied in six planes in the spectral domain, starting from the knowledge of the radiated far field, which is partitioned through a "partition of unity" procedure. The formulation is presented, as well as numerical results which illustrate its validity and its accuracy versus frame decomposition and beam launching parameters.**

1 INTRODUCTION

Frame theory allows for complete representations of source field distributions in phase space (geometrical and associated spectral domains), in the form of superpositions of translated and phase shifted Gaussian windows [1, 2]. These Gaussian frame windows radiate fields in the form of paraxial Gaussian beams, in as much as their spectrum is sufficiently localized. Frame decomposition can thus be used at the starting point of Gaussian beam shooting algorithms [3, 4].

Until now however, frame decomposition was applied to source distributions in one plane, radiating into one half-space. Also, the fields of beams propagating along directions close to the source plane (large spectral shift of the source frame window) are not accurately approximated by the paraxial approximation, for the spectrum of their frame window source is not fully comprised in the visible domain.

To overcome these limitations, other types of initial decompositions have been proposed, generally based on sampling theorems on a sphere [5, 6]. Our contribution in this paper consists in formulating a method based on frame decompositions in planes, yet allowing for beam launching into the whole 3d space, and reducing the inaccuracy related to highly shifted beams.

The proposed method applies to frame decomposition of fields in the spectral domain, and is therefore easily applied to source fields known by their spectrum (or far field). The 3d source spectrum is supposed to be known in six planes radiating into six half-spaces. A partition of unity is then formulated which allows to synthetize the 3d far field from the summation of fields radiated by six spectra, each multiplied by a partitioning window. Frame decomposition can be applied to these "partial spectra", in each of the six planes, and the Gaussian beams launched from all the planes are summed to obtain the 3d radiated fields.

Section 2 gives a brief outline of frame decomposition and of its application in the context of a directive source radiating into a half-space. Section 3 presents the new "spectral partitioning" formulation, and section 4 numerical results obtained in the case of a half-wavelength dipole.

2 FRAME DECOMPOSITION

Frame decomposition is briefly outlined in this section. Harmonic time dependence $e^{-i\omega t}$, with ω the angular frequency, is assumed and suppressed in equations. The Fourier transform of a function $g \in L^2(\mathbb{R})$, denoted $\widetilde{g}$, is defined as $\widetilde{g}(k_x) = \int_{-\infty}^{+\infty} g(x) e^{-ik_x x}\, dx$.

2.1 Gaussian window frames in $L^2(\mathbb{R})$

In the $L^2(\mathbb{R})$ Hilbert space, the set of Gaussian functions

$$w_{mn}(x) = w(x - m\bar{x}) e^{in\bar{k}_x(x - m\bar{x})}\,,\ (m,n) \in \mathbb{Z}^2$$

$$\text{with} \quad w(x) = \sqrt{\frac{\sqrt{2}}{L}} e^{-\pi \frac{x^2}{L^2}}$$

is a frame if and only if $\bar{x}\bar{k}_x = 2\pi\nu$ with $\nu < 1$ (oversampling factor) [1]. $\bar{x}$ and $\bar{k}_x$ are respectively the spatial and spectral domain translation step.

If the set of functions w_{mn} is a Gaussian window frame, then the set $\widetilde{w}_{nm}, (n,m) \in \mathbb{Z}^2$, obtained by translations of the Gaussian function $\widetilde{w}$ in the spectral domain:

$$\widetilde{w}_{nm}(x) = \widetilde{w}(k_x - n\bar{k}_x) e^{-imn\bar{x}k_x}$$

is also a Gaussian window frame in $L^2(\mathbb{R})$.

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Frames are complete sets hence any function $f \in L^2(\mathbb{R})$, or its Fourier transform, can be expressed as summations of weighted frame windows:

$$f = \sum_{(m,n)\in\mathbb{Z}^2} a_{mn} w_{mn} \; ; \; \widetilde{f} = \sum_{(m,n)\in\mathbb{Z}^2} a_{mn} e^{imn\bar{k}_x \bar{x}} \; \widetilde{w}_{mn}$$

with the (non unique) a_{mn} complex coefficients called "frame coefficients". One set of such coefficients can be calculated by projecting the function or its Fourier transform on a "dual frame" of functions [2, 4].

2.2 Frame based Gaussian beam shooting

Frames in $L^2(\mathbb{R}^2)$ are easily defined as product frames: $w_{mnpq} = w_{mn} w_{pq}$, $(m,n,p,q) \in \mathbb{Z}^4$. If a y-polarized field distribution radiating into the $z > 0$ half-space is decomposed on such a frame, with A_{mnpq} the frame coefficients, then the radiated field at any point M with $z > 0$ is obtained as:

$$\overrightarrow{E}(M) = \sum_{m,n,p,q} A_{mnpq} e^{i(mnl_x\kappa_x + pql_y\kappa_y)} \overrightarrow{B}_{mnpq}(M)$$

where $\overrightarrow{B}_{mnpq}$ is the field radiated by the plane wave spectrum (PWS) associated to the w_{mnpq} frame window used to discretize the $E_y(x,y)$ function. For spectrally narrow frame windows, the $\overrightarrow{B}_{mnpq}$ radiated field can be expressed in the form of a Gaussian beam (ray type expression with complex curvature), based on a paraxial approximation in the spectral domain [3, 4].

3 SPECTRAL PARTITIONING

The aim of this section is to define partial PWS in different planes, so that the field at any observation point outside of the reactive region of the radiating source, can be obtained by summation of the fields calculated by GBS from several of these planes. These partial PWS radiate partial far fields, the sum of which is equal to the antenna far field.

3.1 Notations

The far field radiated by a given "source" is assumed to be known in all (θ, ϕ) directions in the global coordinate system $(O, \hat{x}, \hat{y}, \hat{z})$. Six coordinate systems $\mathcal{S}_j = (O, \hat{x}_j, \hat{y}_j, \hat{z}_j)$, $j = 1, \ldots, 6$, are introduced, and the six planes $\mathcal{P}_j = (O, \hat{x}_j, \hat{y}_j)$ will be used as source planes for the half-spaces $\mathcal{H}_j$ defined by $z_j > 0$. The PWS of the source radiated fields in the $\mathcal{P}_j$ plane is denoted $\vec{\tilde{E}}^{(j)}$ and expressed as a function of the spectral variables (k_{x_j}, k_{y_j}), the wavevector components in the $\mathcal{P}_j$ plane. Figure 1 gives an exploded view of the $\mathcal{P}_j$ planes.

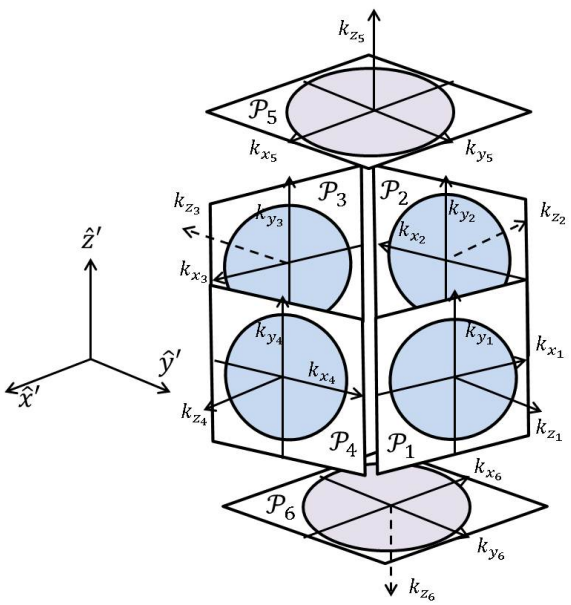

Figure 1: $\mathcal{P}_j$ planes and associated spectral variables.

Any point M in the 3d space, except for O', belongs to exactly three different $\mathcal{H}_j$ half-spaces. We shall denote J the set of indices defined by:

$$M \in \bigcap_{j\in J} \mathcal{H}_j$$

3.2 Partition of unity relation

Each $\vec{\tilde{E}}^{(j)}$ PWS is deduced from the antenna far field in the $\mathcal{H}_j$ half-space, using the classical far field asymptotic expression:

$$\vec{E}(M) \approx \frac{-i}{\lambda r_j} e^{ikr_j} cos\theta_j \vec{\tilde{E}}^{(j)}(k_{x_j}, k_{y_j}) \qquad (1)$$

with (r_j, θ_j, ϕ_j) the spherical coordinates of point M in the coordinate system $\mathcal{S}_j$, k the wavenumber, $k_{x_j} = k\, sin\theta_j\, cos\phi_j$, and $k_{y_j} = k\, sin\theta_j\, sin\phi_j$.

A partitioning function χ_j, function of (k_{x_j}, k_{y_j}), is defined in each $\mathcal{P}_j$ plane, so that at an observation point M:

$$\vec{E}(M) = \sum_{j\in J} \vec{E}^{\chi_j}(M)$$

where $\vec{E}^{\chi_j}$ is the far field radiated by the partial PWS $\vec{\tilde{E}}^{\chi_j}$ defined by:

$$\vec{\tilde{E}}^{\chi_j}(k_{x_j}, k_{y_j}) = \vec{\tilde{E}}^{(j)}(k_{x_j}, k_{y_j})\; \chi_j(k_{x_j}, k_{y_j})$$

If $\{j, j'\} \subset J$, then $\frac{k_{z_j}}{k_{z_{j'}}} \vec{\tilde{E}}^{(j)}$, expressed as a function of $(k_{x_{j'}}, k_{y_{j'}})$, is the PWS in the $\mathcal{P}_{j'}$ plane

which radiates the same far field at M as the $\vec{E}^{(j)}$ PWS defined in the $\mathcal{P}_j$ plane. With such relations, it is easily shown that the partitioning functions χ_j, $j \in J$, have to satisfy, for any set J of three intersecting half-spaces:

$$\sum_{j \in J} \chi_j(k_{x_j}, k_{y_j}) = 1$$

where the (k_{x_j}, k_{y_j}) variables are projections in the planes $\mathcal{P}_j$ of the same wavevector $\vec{k}$.

3.3 Partition of unity functions

One-variable partition of unity functions can easily be defined by translations of an even function which for positive x verifies:
- $\chi(x) = 1$ for $0 < x \leq k_L$,
- $\chi(x) = f(x - k_L)$ for $k_L \leq x \leq k_L + \delta$ (transition),
- $\chi(x) = 0$ for $x > k_L + \delta$.

We derive the function f used in the "transition" region from the Hann window, in order to minimize the effect of truncation in the transformed domain [7]. Figure 2 is an example of such a one-variable partitioning function χ.

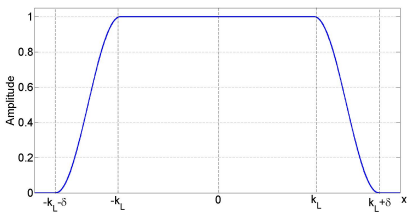

Figure 2: One-variable partitioning function with Hann function type transition.

The following constraints are imposed to the χ_j partitioning functions:

1. Functions χ_5 and χ_6, respectively used in the "upper" $\mathcal{P}_5$ and "lower" $\mathcal{P}_6$ planes (cf Fig. 1) possess circular symmetry: they are defined as functions of $k_{r_j} = \sqrt{k_{x_j}^2 + k_{y_j}^2}$, $j \in \{5, 6\}$;

2. The χ_j functions defined in the "lateral" $\mathcal{P}_j$ planes ($j \in \{1, 2, 3, 4\}$) are of the following form:

$$\chi_j(k_{x_j}, k_{y_j}) = \chi_{jx}(k_{x_j}, k_{y_j}) \chi_{jy}(k_{y_j})$$

where $\chi_{jy}(k_{y_j})$ is responsible for the partitioning with "upper" and "lower" planes.

Explicit expressions of these partitioning functions will be presented at the conference.

4 NUMERICAL RESULTS

The results presented in this section are for the case of a theoretical half-wave dipole, aligned along the z axis, with its far field given by:

$$\vec{E}(r, \theta, \phi) = -i\, 60\, I\, \frac{e^{ikr}}{r} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta}\, \widehat{\boldsymbol{\theta}}$$

with $I = 1/60$.

Figure 3 represents the dipole far field on a sphere of radius $r = 50\lambda$, as a function of the spherical angles $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$, respectively used as radial and angular polar variables in the figure. This field has been synthetized by GBS from six planes where partial spectra resulting from the "spectral partitioning" procedure described in Section 3 have been decomposed on frames of Gaussian windows.

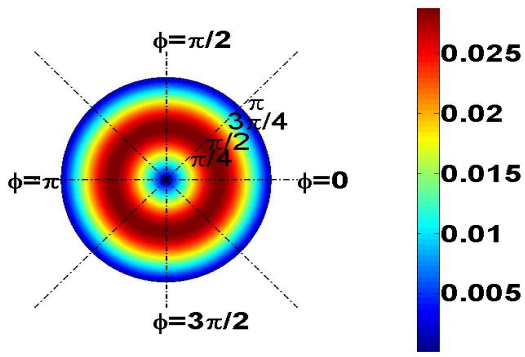

Figure 3: Half-wavelength dipole far field ($r = 50\lambda$) synthesized by GBS with "spectral partitioning".

For this computation, the same frame parameters are used in all the $\mathcal{P}_j$ planes: $\nu_\alpha = 0.16$, $L_\alpha = 10\lambda$, $\alpha = x_j, y_j$. For computational purpose, source distributions and Gaussian frame windows are truncated at a threshold value of $\epsilon = 10^{-3}$. With these parameters, the initial limits of frame indices are ± 26 for n and q (spectral domain) and ± 6 for m and p (spatial domain), in each plane. Among all the frame cofficients, only those with relative magnitude (normalized to the maximal one) larger than a "compression" parameter γ are considered non negligible. The beams weighted by "negligible" coefficients are not launched. The γ "compression" parameter was taken equal to 10^{-3}, which allowed to neglect about half of the coefficients.

Figure 4 presents a map of the absolute normalized error (normalized with respect to the maximum of the field magnitude), which is visibly coherent with the threshold and compression parameters ϵ and γ. Figure 5 gives a better view of the absolute

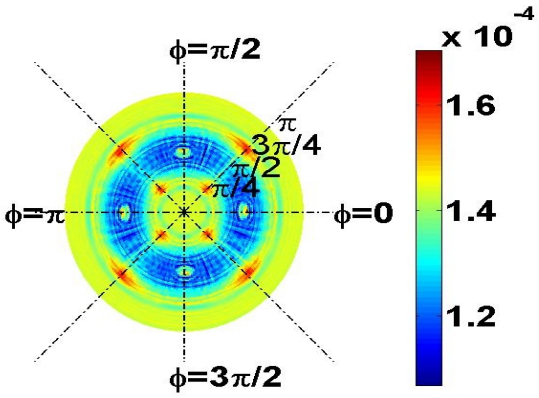

Figure 4: Absolute normalized error of the half-wavelength dipole far field ($r = 50\lambda$) synthesized by GBS with "spectral partitioning".

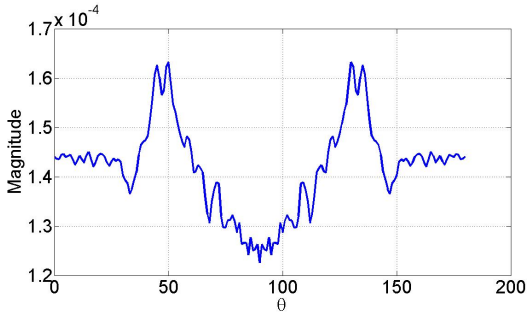

Figure 5: $\phi = \pi/4$ cut of the absolute normalized error of the half-wavelength dipole far field ($r = 50\lambda$) synthesized by GBS with "spectral partitioning".

normalized error in the $\phi = \pi/4$ half-plane. Radiated fields in the half-space $\mathcal{H}_1$ synthetized by GBS *without* partitioning, from the $\vec{\tilde{E}}^{(1)}$ PWS in the $\mathcal{P}_1$ plane, with the above frame and γ parameters, will be presented at the conference as well as the corresponding absolute normalized error map. The maximum absolute normalized error in the whole space is respectively of 1.7×10^{-4} *with* partitioning and 3.3×10^{-2} *without* partitioning. The latter is related to the lack of accuracy of highly shifted beams and of their frame coefficients.

When using the new "spectral partitioning" algorithm, the final error is clearly related to the threshold value: $\epsilon = 10^{-1}$ yields a maximum absolute normalized error of 1.7×10^{-2}, 100 times more than in the previous case, computed with $\epsilon = 10^{-3}$. The choice of the Gaussian window width parameter L_α ($\alpha = x, y$) does not affect accuracy under the following constraint: all frame windows needed to analyze the partial spectra must be centered in the "visible domain". For a given L_α value, accuracy starts degrading at distances larger than $5b$ with b the collimation distance of non tilted beams ($b = L_\alpha^2/\lambda$).

5 CONCLUSION

A "spectral partitioning" formulation, based on a partition of unity applied to the far field of a radiating source, is introduced to allow the representation of non directive radiated fields by summations of Gaussian beams. Six frame decompositions are used as the starting point of this GBS algorithm. The validity of the approach is numerically proven, as well as its accuracy. Although tested in the far field, the method is of special interest to calculate fields in the near (non reactive) zone of radiating sources.

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