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# **A Particle Filtering and Kernel Smoothing-based Approach for New Design Component Prognostics**

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## **ABSTRACT**

This work addresses the problem of predicting the Remaining Useful Life (RUL) of components for which a mathematical model describing the component degradation is available, but the values of the model parameters are not known and the observations of degradation trajectories in similar components are unavailable. The proposed approach solves this problem by using a Particle Filtering (PF) technique combined with a Kernel Smoothing (KS) method. This PF-KS method can simultaneously estimate the degradation state and the unknown parameters in the degradation model, while significantly overcoming the problem of particle impoverishment. Based on the updated degradation model (where the unknown parameters are replaced by the estimated ones), the RUL prediction is then performed by simulating future particles evolutions. A numerical application regarding prognostics for Lithium-ion batteries is considered. Various performance indicators measuring precision, accuracy, steadiness and risk of the obtained RUL predictions are computed. The obtained results show that the proposed PF-KS method can provide more satisfactory results than the traditional PF methods.

Key words: Prognostics; Remaining Useful Life; Parameter Estimation; Particle Filtering; Kernel Smoothing; Battery

## **1. INTRODUCTION**

Prognostics is an engineering discipline which is mainly focused on the estimation of the future degradation path, remaining useful life and potential risk associated to an equipment [1]. Technical approaches of prognostics can be broadly categorized into data-driven and model-based [1-5]. Model-based prognostic methods use mathematical representations of the equipment degradation evolution in order to predict the equipment Remaining Useful Life (RUL) [6-9]. They usually require the knowledge of the values of the model parameters, which are typically estimated considering the results of experimental tests or by observing the degradation behaviors of similar components. In practice, the whole model-based prognostics process is divided into an off-line and an on-line phase. During the off-line phase, a mathematical model of the equipment degradation is built using the available physical knowledge on the involved degradation mechanisms and historical degradation data. Some statistical or artificial intelligence methods are, then, applied to the available historical data in order to estimate model parameters. For example, in [10] the authors used an adaptive-network-based fuzzy inference system and robust relevance vector machine to build the steelmaking process model which reduced the

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effect of noise and outliers in the historical data; in [11], a combined relevance vector machine and exponential regression method was used to estimate the ball bearings degradation and predict its RUL based on real world vibration-based degradation data; in [12, 13], the authors used artificial neural networks combined with optimization algorithms to estimate the State-of-Charge of batteries. During the on-line phase, the acquired degradation measurements or condition monitoring data are used to adapt the degradation model to the current degradation situation and to predict the equipment RUL. For example, vibration data were used in [14] to predict the RUL distribution of bearings and the RUL distribution of Gyros were estimated using data collected during the testing process in [15].

The novelty of the present work is that we consider, within the framework of model-based prognostics, a case in which a mathematical model of the degradation process is available but the true model parameter values are unknown, and there are no data available for estimating them related to the degradation of similar components. These situations are typically encountered for safety-critical and high-value components which are characterized by very high reliability, unique or new design material composition. For these kinds of components, performing run-to-fail tests is too expensive or not feasible.

In such cases, the model parameters can be estimated by resorting to: i) expert knowledge, with uncertainty possibly expressed in the form of interval of values and ii) a sequence of data collected during the component operating life until the present time. In this setting of very limited available information, the objective is to jointly estimate i) the parameters of the degradation model ii) the remaining useful life of the component. For doing this, one has to handle the following sources of uncertainty i) measurement uncertainty, ii) intrinsic randomness of the degradation process, iii) uncertainty in the model parameters and iv) uncertainty on the future operational and environmental conditions.

In model-based prognostics, the problem of predicting the component RUL is typically tackled in two sequential steps: 1) estimation of the component degradation state at the present time and 2) prediction of the future evolution of the component degradation.

Step 1) is typically handled by resorting to filter methods such as Kalman Filter [16-18], used in cases of linear degradation models and Gaussian noise, and PF, used in cases of non-linear degradation models and/or non-Gaussian noise. Examples of applications of the PF methods to non-linear degradation problems can be found in [19-22]. Notice that all these works assumed that the model parameter exact values were known. In principle, also the uncertainty on the model parameters values can be treated by adopting Bayesian filtering methods where the dimension of the state vector is extended by including also the elements for the unknown parameters [23-25]. The problem of joint estimation of system state and model parameters had been addressed in [25, 26] considering Bayesian filtering approaches. For the application of fault detection, an approach to estimating the model parameters and the system state was discussed in [26]. In the context of prognostics, with few model parameters to be estimated, Bayesian approaches had been presented in [27-29]; however, these prognostic approaches had not been developed for cases with several model parameters and very poor knowledge on their prior distributions. In particular, it has been shown that in those cases in which the true parameter values are located in the tail of the prior distributions or the prior distributions are characterized by large variances with respect to the parameter typical ranges, most particles weights tend to very small values after the Bayesian updating. Under such situations, traditional PF methods will suffer a serious particle impoverishment [30]; thus, in order to get satisfactory estimates, one would need to use a very large number of particles, with the consequence of increasing computational efforts, and rendering this solution unfeasible in practical cases.

Alternatively, the problem of particle impoverishment has been addressed by adding an artificial evolution to the particles in order to maintain their diversity [23, 31]. However, it has been shown that adding an artificial evolution causes an increase of the variance of the particles, which may obstacle the convergence of the population towards the true model parameter values [25]. Furthermore, the variance of the artificial noise is a further hyper-parameter to which the estimation results are very sensible, and which may be difficult to set in the context of the information available in the present work.

In this work, in order to overcome the problem of particle impoverishment in a case in which few available degradation measurements are available, we consider a method for parameter estimation based on a Kernel Smoothing technique. The method was proposed in [25] in a completely different problem context and is here extended to a prognostics problem. The main advantage of the method is that it can solve the problem of impoverishment without the side effects of variance increase (i.e. without adding extra artificial noise on particles).

Once the equipment degradation state at the present time and the model parameters have been estimated, it is necessary to predict the future evolution of the equipment degradation trajectory (step 2). Notice that this requires going beyond the traditional use of filtering methods, since it involves future time horizons in which no measurements are available for the Bayesian updating. This issue is addressed by adopting a procedure proposed in [32] based on the simulation of the future evolution of the particles describing the component degradation state at the present time.

The main contribution of this work consists in the proposal of a systematic method for on-line RUL prognostics of degrading equipment in the cases that 1) the true values of the parameters in the degradation model are unknown and/or affected by large uncertainties; 2) historical operation data of degradation of similar equipments are unavailable. From the methodological point of view, the novelty consists in the application of a kernel smoothing procedure to a prognostic problem which requires the prediction of the future evolution of the component degradation. The developed method for model parameters estimation and RUL prediction is applied to a numerical case study regarding the degradation of a Lithium-ion battery. Various performance indicators measuring precision, accuracy, steadiness and risk of the obtained RUL predictions are considered.

The remainder parts of the paper are organized as follows. Section 2 illustrates the problem statement. In Section 3, the PF approach to prognostics is briefly recalled, whereas in Section 4, we introduce the PF-KS method for the joint estimations of the degradation state and the model parameters. Sections 5 and 6 describe the prediction of the future degradation evolution and the component RUL. Section 7 shows a numerical application of the proposed method to the prognostic of the RUL of a Lithium-ion battery. In Section 8, we draw some conclusions and suggest potential future work.

## 2. PROBLEM STATEMENT

The objective of the work is to develop a prognostic method for RUL prediction of a degrading component, and related uncertainty. The following sources of information are considered available:

- A degradation model for the mathematical representation of the evolution of the equipment degradation. The mathematical model is typically obtained from a physical understanding of the degradation mechanism. In this work, we assume that the physical model can be formulated as a first order Markov Process:

$$x_t = g(x_{t-1}, \mathbf{p}_{t-1}, \gamma) \quad (1)$$

where  $g(x, \mathbf{p}, \gamma)$  is the recursive transition function,  $x_t$  is an indicator of the equipment degradation state at time  $t$ ,  $\mathbf{p}$  is the vector of the model parameters, whose true values are unknown,  $\gamma$  is the process noise which represents the degradation process uncertainty. Model parameters and process noise are assumed to be constant during the life of the component.

- The measurement equation which links the degradation state  $x$  and its measurements. It is typically represented by a possibly non-linear function  $h$ :

$$z_t = h(x_t, \sigma_m) \quad (2)$$

where  $z_t$  is the measurement of  $x_t$  at time  $t$  and  $\sigma_m$  is the measurement noise.

- Degradation measurements  $z_t (t=1, 2, \dots, T)$ , collected during the operating life of the component, until the present time  $T$ . For simplicity, we assume that measurements have been performed at regular intervals from  $t=1$  (in arbitrary units).
- A failure threshold defining the maximum acceptable degradation state: the equipment is considered failed when its degradation exceeds the failure threshold. In this work, we assume that the failure threshold value is known [33].

Notice that degradation measurements performed on identical or similar components are considered to be not available. On the contrary, we assume the availability of a prior estimate of the probability density functions (PDF) of the initial degradation state  $p(x_0)$  and the model parameters  $p(\mathbf{p}_0)$  based on expert judgment.

### 3. THE PARTICLE FILTERING-BASED APPROACH TO PROGNOSTICS

The PF-based approach to prognostics is typically divided into three steps (Figure 1, the solid boxes are inputs and the dotted boxes are outputs):

- 1) a filtering step for the estimation of the equipment degradation state at the present time, which is based on the use of the prior PDF of the model parameters and state provided by the experts, the degradation model and the measurements performed until the present time
- 2) a prediction step for the estimation of the future degradation evolution using the posterior PDF of the model parameters and state (output of step 1), and the updated degradation model (where the unknown parameters are replaced by the estimated ones)
- 3) the prediction of the equipment RUL which is based on the degradation state prediction (output of step 2) and the knowledge of the failure threshold.

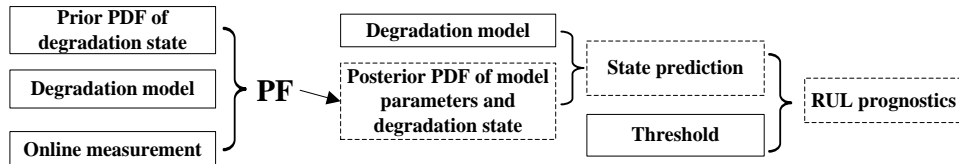


Figure 1 Sketch of the PF-based approach to RUL prognostics

Section 4 will be devoted to the illustration of the proposed method for estimating the equipment degradation state and model parameters. Section 5 and 6 will present the methods for the prediction of the future degradation evolution and the estimation of component RUL, respectively.

#### 4. ESTIMATION OF THE COMPONENT DEGRADATION STATE AT THE PRESENT TIME AND OF THE MODEL PARAMETERS

In order to facilitate the explanation of the method, in Section 4.1, we describe the basics of the PF estimation of the degradation state assuming that the parameters of the degradation model are known. Then, in Section 4.2 and 4.3, we discuss the problem of the joint estimations of the degradation state and the model parameters estimation. Section 4.4 describes the procedures of kernel smoothing.

##### 4.1. Particle filtering estimate of the degradation state

In this filtering step, we use the degradation model, the noisy measurements and the prior PDF of the degradation state to estimate the posterior PDF of the degradation state. A detailed description of the PF method for the estimation of the system state can be found in [8, 34-36]. The operative procedure is typically divided into two steps: prediction and update. In the prediction stage, one knows  $p(x_{t-1} | z_{1:t-1})$ , and, by using Eq.(1) and the Chapman-Kolmogorov equation, the prediction distribution of the degradation at the next time  $p_f(x_t | z_{1:t-1})$  can be obtained:

$$p_f(x_t | z_{1:t-1}) = \int p(x_t | x_{t-1}) p(x_{t-1} | z_{1:t-1}) dx_{t-1} \quad (3)$$

When the new measurement  $z_t$  arrives, one can update and calculate the posterior PDF  $p(x_t | z_{1:t})$  via the Bayesian rule:

$$p(x_t | z_{1:t}) = \frac{p(x_t | z_{1:t-1}) p(z_t | x_t)}{\int p(x_t | z_{1:t-1}) p(z_t | x_t) dx_t} \quad (4)$$

Usually, except for the situation of linear Gaussian state space models (Kalman filter) and hidden finite-state space Markov chains (Wohnam filter), it is not possible to evaluate analytically the PDF in Eq.(4), since this requires the calculation of complex high-dimensional integrals. The PF solution is, then, based on the Monte Carlo sampling method. By sampling a large number of samples (called particles) from a proposal PDF  $q(x_t | z_{1:t})$ , the estimated posterior PDF  $p_e(x_t | z_{1:t})$  can be calculated as:

$$p_e(x_t | z_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(x_t^i - x_t) \quad (5)$$

where  $x_t^i$  ( $i=1,2,...,N$ ) are the particles sampled from  $q(x_t | z_{1:t})$  and  $w_t^i$  is the weight associated to the particle  $x_t^i$  given by:

$$w_t^i = \frac{p(z_t | x_{0:t}^i) p(x_{0:t}^i)}{q(x_{0:t}^i | z_{1:t})}; \quad w_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i} \quad (6)$$

One of the most adopted choices is to consider the proposal PDF  $q(x_t | z_{1:t})$  as the transition function, namely  $q(x_t | z_{1:t}) = p(x_t | x_{t-1})$ . In this way, using Eq.(7), one can obtain the particle weights  $w_t^i$  at time  $t$ :

$$w_t^i = w_{t-1}^i p(z_t | x_t^i); w_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i} \quad (7)$$

where  $p(z_t | x_t^i)$  is called the likelihood of measurement  $z_t$  given the particle  $x_t^i$ , which can be derived from the measurement function in Eq.(2).

The following pseudocode reports the procedural steps for the estimate of the degradation state at the present time.

```

Sample  $x_0^i \sim p(x_0)$ , set initial weight  $w_0^i = N^{-1}, i = 1, 2, \dots, N$ 
t=1;
While (t<T_end)
    Sample  $x_t^i \sim p_f(x_t | z_{0:t-1})$ ;
    Calculate the weight  $w_t^i = w_{t-1}^i L(z_t | x_t^i) / \sum_{i=1}^n L(z_t | x_t^i)$ 
    Calculate the state estimation and posterior PDF  $\hat{x}_t = \sum_{i=1}^N w_t^i x_t^i$ ,  $p_e(x_t | z_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(x_t^i - x_t)$ 
    Calculate the criteria for resampling  $ESS_t \approx \left( \sum_{i=1}^n (w_t^i)^2 \right)^{-1}$ 
    if  $ESS_t \leq N/2$ 
        resample with probability  $P(k_t^i = k_{t-1}^i) = w_{t-1}^i$ 
    end if
    t=t+1;
end while

```

## 4.2. Combined State and Parameter estimation

Several methods such as Maximum likelihood, Expectation Maximization, Minimum Mean Square Error and other hybrid methods have been used to estimate degradation model parameters on the basis of a batch of historical data [37-41]. The main limitation of the application of these approaches to prognostics is that they consider the problem of parameter estimation distinct from that of RUL prognostics. This causes that every time a new degradation measurement becomes available, a completely new estimation of the parameters needs to be performed. Then, with the new parameter values, the prognostic model has to be newly applied, without any benefits from the previous results.

Also Bayesian approaches such as the Overall Bayesian Method (OBM) and the Recursive Bayesian Method (RBM) were applied to the problem of parameter estimation in prognostic problems [42-44]. Although OBM provides satisfactory results, it cannot be used in this work since it relies on the use of historical batch data. In [44], the authors compared the performance of OBM, RBM and PF with respect

to the RUL prognostic objective; the obtained results show that PF is the most suitable method for online parameter estimation and RUL prognostics.

### 4.3. Combined State and Parameter estimation using PF

The combined estimate of the equipment degradation state and model parameters can be performed by using an augmented PF [25, 34, 44-46]. The idea is to consider the model parameters as elements of the state vector which is estimated by the PF. Thus, the generic augmented  $i$ -th particle  $k_t^i$ , is represented by:

$$k_t^i = \{x_t^i, \mathbf{p}_t^i, w_t^i\}$$

where  $x_t^i$  represents the degradation state,  $\mathbf{p}_t^i$  the model parameters at time  $t$  and  $w_t^i$  is the weight associated to the particle. Eq.(1) becomes a system of two equations, one describing the transition of the state and the other the transition of the parameters:

$$\begin{aligned} x_t^i &= g_1(x_{t-1}^i, \mathbf{p}_{t-1}^i, \gamma_1) \\ \mathbf{p}_t^i &= g_2(\mathbf{p}_{t-1}^i, \gamma_2) \end{aligned} \quad (8)$$

Different strategies have been considered to set the function  $g_2$  and, thus, to simulate the evolution of the model parameters values. In [44], the distributions of the parameters were kept unchanged during the prediction stage:

$$p(\mathbf{p}_t | z_{1:t-1}) = p(\mathbf{p}_{t-1} | z_{1:t-1}) \quad (9)$$

Thus, within a PF approach, the evolution of the values of the parameters in the particles becomes:

$$\mathbf{p}_{t+1}^i = \mathbf{p}_t^i \quad (10)$$

This approach will be here referred to as Particle Filtering with unchanged parameter distribution (PF-UC). In [47], a dual PF method was used to handle the problem: the main idea of this dual PF approach is to decouple the state and parameters estimates and perform them sequentially: one PF is used for the degradation state estimate and the other PF for the model parameters estimate.

These two options have been shown to properly work when few model parameters are to be estimated, but they tend to suffer the problems of particle degeneracy and impoverishment in the case in which several model parameters needed to be estimated [48]. Resampling methods [34, 49], such as bootstrap resampling with replacement, can be used to avoid the problem of particle degeneracy, but are typically not effective with respect to the problem of particle impoverishment, since they tend to reduce the diversity in the particle population. Advanced resample methods such as those described in [50, 51] overcome the problem of particle impoverishment in low dimension of state space, but they have been shown to be not effective in high dimensional state space.

The problem of the particle impoverishment has been addressed in several works [23, 52, 53] by adding an artificial evolution to the particles parameters evolution equation. In practice, Eq.(10) becomes:

$$\mathbf{p}_{t+1}^i = \mathbf{p}_t^i + N(0, \sigma_{AN}^2) \quad (11)$$

where  $\sigma_{AN}^2$  is the variance of the artificial noise. This approach will be referred to as Particle Filtering with Artificial Evolution (PF-AE). Although it has been shown that the method is able to provide more accurate estimates of the parameter values, it is difficult to apply it to our prognostic problem since it



requires a proper setting of the value of the variance  $\sigma_{AN}^2$ , which is difficult to achieve by trial and error attempts, due to the unavailability of complete test trajectories. In [27, 52, 54, 55], the authors showed that, if too small values of  $\sigma_{AN}^2$  are used, the particles convergence to the parameter true values is too slow and the problem of particle impoverishment was still be encountered. Whereas, if large values of  $\sigma_{AN}^2$  are used, the convergence to the parameters true values was not achieved. Furthermore, it was shown that adding an artificial noise to the parameters values cause an increase of the variance of the particle population according to:

$$V(\mathbf{p}_{t+1}^i) = V(\mathbf{p}_t^i) + \sigma_{AN}^2 \quad (12)$$

where  $V(\mathbf{p}_{t+1}^i)$  is the variance of the parameters in the population after the artificial evolution is added [25]. In practice, every time we add the artificial evolution, the variance of the parameter value  $\mathbf{p}_t^i$  will increase by  $\sigma_{AN}^2$ .

In order to overtake these difficulties, in this work we consider an alternative PF approach based on the Kernel Smoothing algorithm.

#### 4.4. The Kernel Smoothing algorithm

Kernel smoothing is based on the application of two different procedures to the population of particles: shrinkage and perturbation. Shrinkage aims at moving the particle parameters values toward their estimated values, whereas perturbation adds a controlled noise to the particle parameters in order to maintain the desired variance in the population [45, 56, 57].

##### ➤ Shrinkage

The particle shrinkage is performed by applying to the particle model parameters values the transform:

$$\tilde{\mathbf{p}}_t^i = \mathbf{p}_t^i \sqrt{1-h^2} + \hat{\mathbf{p}}_t \left(1 - \sqrt{1-h^2}\right) \quad (13)$$

where the vector  $\tilde{\mathbf{p}}_t^i$  contains the parameters values of the  $i$ -th particle after the shrinkage. The direction of shrinkage is the expected value of the parameter values  $\hat{\mathbf{p}}_t$ , at time  $t$ . The kernel parameter,  $h \in [0,1]$ , determines the degree of shrinkage: higher is its value, deeper is the shrinkage. If  $h=1$ , the model parameters completely shrink to the expected values  $\hat{\mathbf{p}}_t$ ; whereas if  $h=0$ , no shrinkage is applied to the parameters. To set  $h$ , some authors suggested using the value  $h=0.1$  [45], whereas other authors suggested optimizing the value of  $h$  using historical data [56].

After shrinkage, the parameters variance in the population of particles will decrease from  $V(\mathbf{p}_t^i)$  to  $(1-h^2)V(\mathbf{p}_t^i)$ . Then, the parameter vector  $\tilde{\mathbf{p}}_t^i$  is used in Eq.(13) to predict the degradation state value  $x_{t+1}^i$ .

##### ➤ Perturbation

Perturbation is used to find the particle parameters values  $\mathbf{p}_{t+1}^i$ , at time  $t+1$ . In practice, an artificial evolution, i.e. a noise of variance  $h^2V(\mathbf{p}_t^i)$ , is added to the particle parameters values at time  $t$ :

$$\mathbf{p}_{t+1}^i = \tilde{\mathbf{p}}_t^i + N(0, h^2 V(\mathbf{p}_t^i)) \quad (14)$$

Notice that the effect of adding the perturbation is to increase the variance of the parameters values in the population:

$$V(\mathbf{p}_{t+1}^i) = V(\tilde{\mathbf{p}}_t^i) + h^2 V(\mathbf{p}_t^i) = V(\mathbf{p}_t^i) \quad (15)$$

Thus, different from the approach of Eq.(11) based on the addition of a Gaussian noise on the parameter values, the combined effect of the shrinkage and perturbation procedures can keep unmodified the variance of the parameters values in the population.

In the following, we will call this method as PF-KS.

## 5. PREDICTION OF THE FUTURE DEGRADATION EVOLUTION

Once the posterior PDF of the equipment degradation state and the model parameter values have been estimated, it is possible to predict the future evolution of the equipment degradation trajectory. Notice that this requires going beyond the traditional use of filtering methods, since it involves future time horizons in which no measurements are available for the Bayesian updating in Eq.(6) [31]. In practice, the estimated posterior PDF,  $p_e(x_t | z_{1:t})$ , obtained at the previous step, should be propagated through Eq.(1) until the future time of interest  $t+l$ . This requires to compute [32]:

$$p_f(x_{t+l} | z_{1:t}) = \int \dots \int \prod_{j=t+1}^{t+l} p(x_j | x_{j-1}) p_e(x_t | z_{1:t}) \prod_{j=t}^{t+l-1} dx_j \quad (16)$$

where  $p_f(x_{t+l} | z_{1:t})$  is the predicted PDF of degradation state at time  $t+l$ . However, the numerical evaluation of the integral in Eq.(16) is a computationally highly demanding task. A review of methods for the computation of  $p_f(x_{t+l} | z_{1:t})$  can be found in [54]. In this work, in order to reduce the computation demand, we follow the procedure proposed in [32] based on the assumption that the error generated by considering the particle weights constant from time  $t$  to time  $t+l$  is negligible with respect to other sources of error, such as those caused by model inaccuracy or by the assumptions made for setting process and measurement noise [58]. According to this procedure, the predictive PDF of the degradation state at time  $t+l$ ,  $p_f(x_{t+l} | z_{1:t})$ , is given by:

$$p_f(x_{t+l} | z_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(x_{t+l} - x_{t+l}^i) \quad (17)$$

where the particle state  $x_{t+l}^i$  is obtained by iteratively applying Eq.(1) to the state  $x_t^i$ .

## 6. PREDICTION OF COMPONENT RUL

Once the predicted degradation state PDF,  $p(x_{t+l} | z_{1:t})$ , is available, the RUL can be found by calculating the earliest time at which the degradation state exceeds the failure threshold  $x_{th}$  [19]. In practice, the RUL associated to the  $i$ -th particle at the present time  $t$  is given by:

$$RUL_t^i = \left\{ (T_t^i - 1 - t) \mid g(x_{T_t^i-1}^i, \mathbf{p}_t^i, \gamma) < x_{th}, g(x_{T_t^i}^i, \mathbf{p}_t^i, \gamma) \geq x_{th} \right\} \quad (18)$$

where  $T_t^i$  can be found by iteratively applying Eq.(1) to simulate the particles evolution. Thus, the predicted RUL distribution is given by: [8, 21, 36]

$$p_f(RUL | z_{1:t}, x_i < x_{th}) \approx \sum_{i=1}^N w_t^i \delta(RUL - RUL_t^i) \quad (19)$$

The details and derivations of Eq.(18) and (19) can be found in [8, 21].

## 7. NUMERICAL APPLICATION

In this section, we investigate the performance of the proposed method in an application of RUL prognostics of a Lithium-ion battery.

### 7.1. Degradation Model

A detailed description of the battery degradation mechanism can be found in [23, 59-61]. The quantity typically used to represent the battery degradation state is the battery capacity  $q(t)$ . According to the model proposed in [23, 59], the battery degradation is characterized by a first phase during which the battery capacity slowly decreases, followed by a second phase characterized by a fast decreasing process. These two phases can be described by a double exponential model:

$$q(t) = p_1 \cdot \exp(p_2 \cdot t) + p_3 \cdot \exp(p_4 \cdot t) + N(0, \sigma_p^2) \quad (20)$$

where  $p_1, p_2, p_3$  and  $p_4$  are the four model parameters ( $p_1, p_3$  determine the initial state and  $p_2, p_4$  the degradation rate),  $t$  is the number of charge/discharge cycles experienced by the battery and  $\sigma_p^2$  is the process noise. Generally,  $p_1$  and  $p_4$  are positive,  $p_2$  and  $p_3$  are negative. Eq.(20) can be rewritten in a recursive form as:

$$\begin{aligned} q_1(t) &= q_1(t-1) \cdot \exp(p_2) + N(0, \sigma_1^2) \\ q_2(t) &= q_2(t-1) \cdot \exp(p_4) + N(0, \sigma_2^2) \end{aligned} \quad (21)$$

where  $q_1(t)$  and  $q_2(t)$  are two first-order independent Markov processes, with Gaussian process noises characterized by variance  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. Within this formulation, the present degradation state  $q(t)$  can be obtained from  $q_1(t)$  and  $q_2(t)$  as:

$$q(t) = p_1 \cdot q_1(t) + p_3 \cdot q_2(t) \quad (22)$$

Eq.(22) constitutes the degradation model corresponding to Eq.(21), whereas Eq.(20) is introduced only to compute the degradation state but is not used within the PF approach. The measurement equation is:

$$Q(t) = p_1 \cdot q_1(t) + p_3 \cdot q_2(t) + N(0, \sigma_m^2) \quad (23)$$

where  $Q(t)$  is the measurement at the  $t$ -th charge/discharge cycle and  $\sigma_m^2$  is the measurement noise. The failure threshold of  $q(t)$  is set according to expert knowledge.

## 7.2. Generation of the Measurement Data and of the Battery Failure Times

The performance of the proposed prognostics method has been verified considering 100 simulated battery degradation trajectories. Each degradation trajectory is characterized by different values of the parameters  $p_1, p_2, p_3$  and  $p_4$ , sampled from the statistical distributions reported in Table 1. All the trajectories have been simulated considering the process and measurement noise and the failure threshold reported in Table 2.

Table 1 Distributions from which the true values of the parameters are sampled

Parameter	$p_1$	$p_2$	$p_3$	$p_4$
Value	$U(0.88,0.92)$	$U(-0.0097,-0.0008)$	$U(-0.0004,-0.0001)$	$U(0.03,0.06)$

Table 2 Process, measurement noise and threshold value

Parameter	$\sigma_1$	$\sigma_2$	$\sigma_m$	threshold
Value	0.0001	0.001	0.001	0.7172

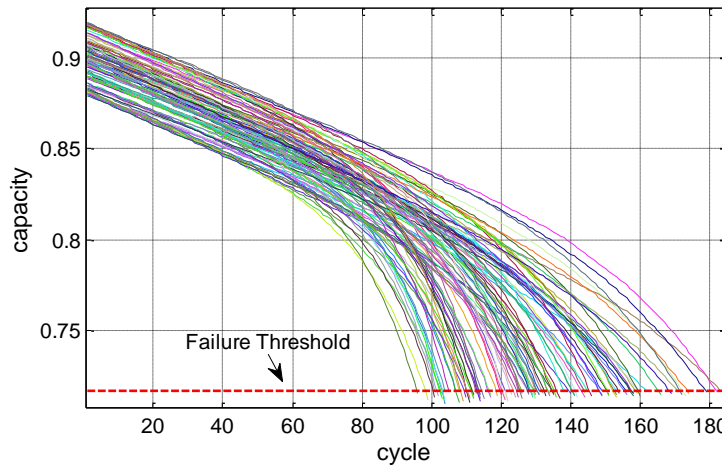


Figure 2 100 batteries degradation trajectories

Figure 2 shows the 100 simulated degradation trajectories. Due to the different values of the parameters, life durations are uncertain, ranging from 95 to 182 cycles. The sampled values of  $p_1, p_2, p_3$  and  $p_4$ , the obtained degradation state  $q$  and the failure time will be referred to as the “true” values.

## 7.3. Results

The experiment is performed assuming that the true values of  $p_1, p_2, p_3$  and  $p_4$  are not known, and the measurements performed on the test battery, i.e. the batteries whose RUL we want to predict, are available from cycle 1 to the present cycle.

Notice that the application of the proposed method for the joint estimate of the component degradation state and model parameter values has an important prerequisite known as observability. Given that in case of non-observable systems the application of filtering-based method can produce poor estimates, one should verify whether the system state vector (degradation state and model parameters) is

observable. Whereas for linear systems the global observability of the system state can be verified by computing the rank of the observability matrix, in non-linear systems the problem is more complex. According to [62], system observability is influenced by several factors such as the prior distributions of the degradation state and model parameters, and an adequate characterization of the noise sources. In the present work, the problem of discussing the observability of the system is not addressed, but, since we are considering an artificial numerical case study, we will discuss the accuracy of the obtained results considering also different possible choices of the prior distributions. Interested readers can find in [63], an example of demonstration of the observability of a non-linear system within a particle filtering problem.

### 7.3.1. Parameters and Degradation State Estimation

In this section, we analyze the parameters and degradation state estimation obtained by the PF-KS algorithm of one of the 100 simulated batteries. The prior PDFs and the true values of the parameters  $p_1, p_2, p_3$  and  $p_4$  are reported in Table 3.

Table 3 The prior PDFs used for the four parameters

Parameter	$p_1$	$p_2$	$p_3$	$p_4$
Prior distribution	$U(0.85,1.2)$	$U(-0.001,0)$	$U(-0.001,0)$	$U(0.03,0.13)$
True value	0.887	-8.86e-4	-2.32e-4	0.0458

Notice that the prior PDFs are remarkably more dispersed than those used for the simulation of the true values of these parameters (Table 1). Furthermore, the problem is complicated by the fact that the expected values of the true distributions of the four parameters are located in the tail of the prior distributions. This setting has been chosen in order to assess whether the method can work even if the parameters prior PDFs are very uncertain and their expected values are shifted with respect to the true values.

Figure 3 shows the estimates of the expected values and 90% confidence intervals of the four parameters. The continue thick horizontal lines represent the true value, the thin continuous lines the estimate of the parameters expected values and the dotted lines the 90% confidence interval of the parameters posterior distributions. Figure 4 shows the time evolution of the PDFs of parameters  $p_3$  and  $p_4$ , with the thin-dotted line representing their true values.

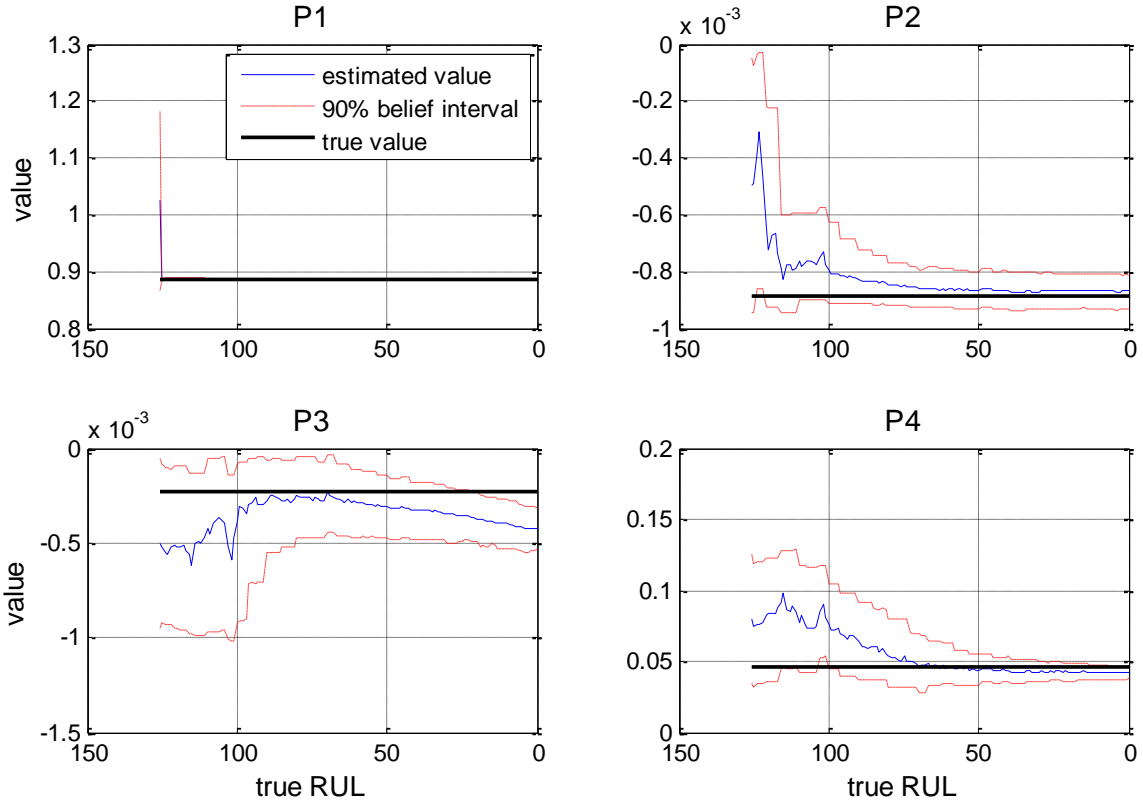


Figure 3 Parameters estimates

It can be observed that, as expected, the 90% confidence interval width tends to decrease as time passes. Furthermore, the error in the estimates of parameters  $p_1$  and  $p_2$  is very small, and on  $p_4$  is still satisfactory, whereas the algorithm seems not able to find the correct value of  $p_3$  and reduce the uncertainty on its estimate. In particular, during the last part of the battery life, the  $p_3$  expected value remarkably drifts from the true  $p_3$  value.

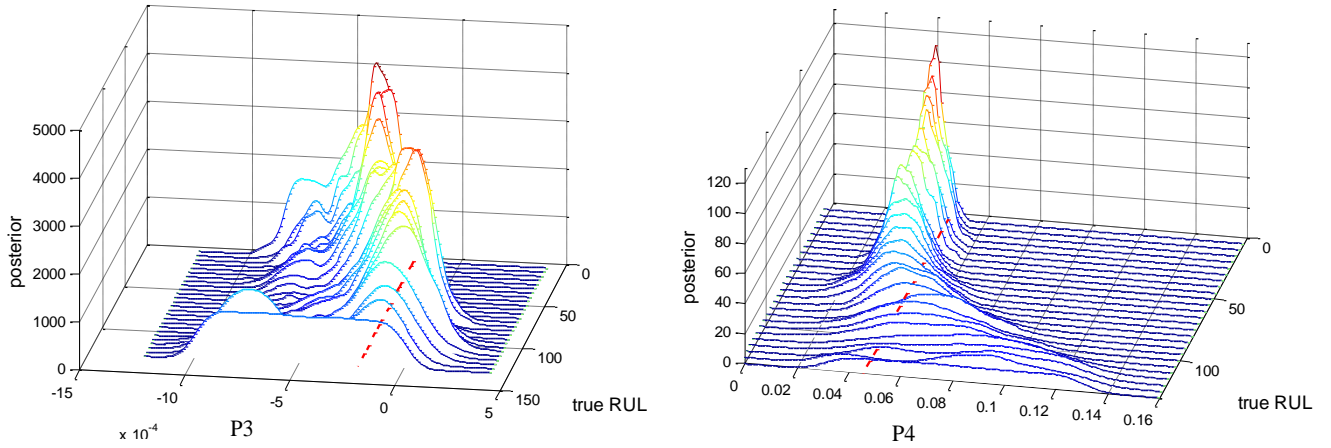


Figure 4 Posterior evolution of parameters  $p_3$  and  $p_4$

This high error on the  $p_3$  estimate can be explained by considering the interaction between the estimates of the parameters. In particular, it seems that the error done in the estimate of  $p_3$  tends to compensate the small error on the estimate on  $p_4$ . In this respect, it should be noticed that if we repeat the estimation of  $p_1, p_2, p_3$ , assuming to know the correct value of  $p_4$  (Figure 5), PF-KS is able to correctly estimate  $p_3$ . A possible explanation of this effect is that there can be different combinations of  $p_1, p_2, p_3$  and  $p_4$  values which can lead to satisfactory estimates of the current degradation state, but only one of them is satisfactory from the point of view of the model parameters.

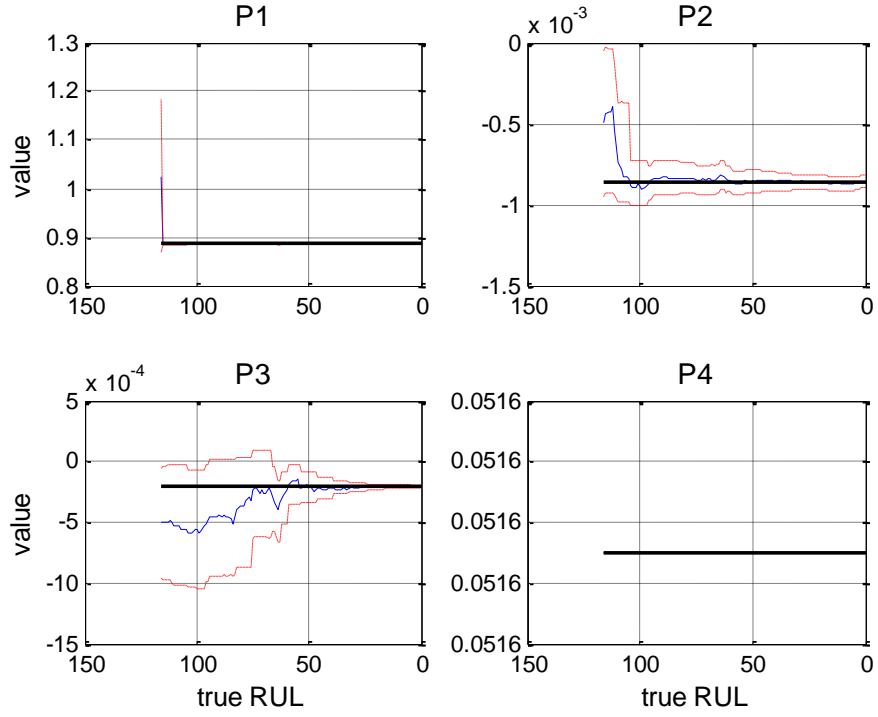


Figure 5 Parameters estimation given the true value of  $p_4$

Figure 6 shows the absolute error in the prediction of the battery capacity degradation state and its 90% confidence interval. The estimate is very satisfactory since the error range is always below 0.002.

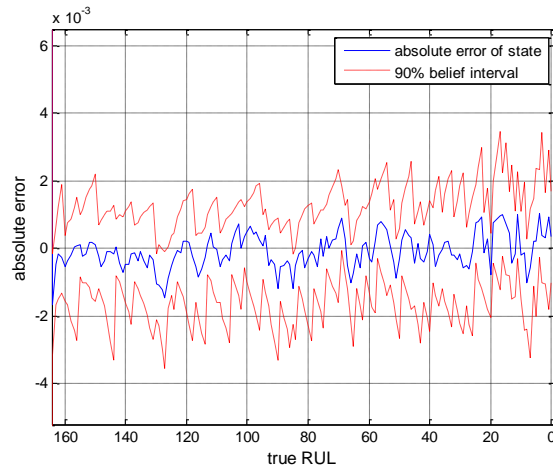


Figure 6 Absolute error on the degradation state



### 7.3.2. Prediction of the battery RUL

In this section, we show the RUL prediction of the same battery analyzed in subsection 7.3.1. Figure 7 (left) shows the RUL predictions obtained at different times, with the thin-dotted line representing the 90% confidence interval.

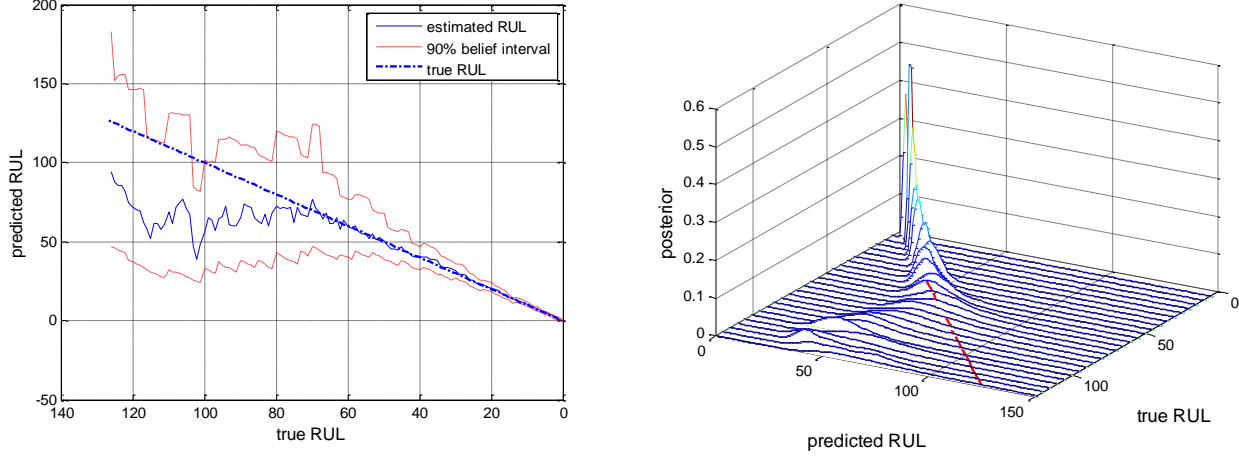


Figure 7 RUL prediction for the considered degradation trajectory

Figure 7 (right) shows the time evolution of the estimate of the RUL posterior distribution. Notice that before the true RUL reaches the value of 80 cycles, the RUL estimate tends to be very uncertain. This is due to the fact that at the beginning of the battery life, the degradation process is mainly influenced by the process  $q_1$  whereas the overall contribution of  $q_2$  is marginal (in Figure 8). Thus, the particle filtering algorithm is able to properly estimate parameters  $p_1$  and  $p_2$ , which have a remarkable effect on the battery degradation state at the beginning of the battery life, whereas not effective with respect to parameters  $p_3$  and  $p_4$ , which have negligible influence on the battery degradation state at the beginning. However, as time passes, the contribution of  $q_2$  becomes predominant and, the PF-KS algorithm is able to improve the estimates of parameters  $p_3$  and  $p_4$ , thus obtaining a good prediction of the RUL.

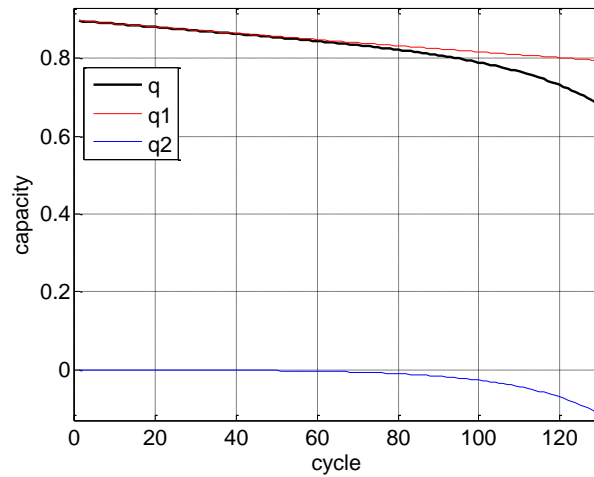


Figure 8. Effects of the terms  $q_1$  and  $q_2$  on the battery degradation  $q$

In order to obtain a more robust evaluation of the performance metrics, the prediction of the RUL has been performed for all the 100 true degradation trajectories. Notice that the application of the method to a given battery is based only on the capacity measurements simulated for that trajectory and does not employ any information coming from the measurements of the other 99 trajectories.

We measure the performance of the different PF approaches, i.e. PF-UC, PF-AE and PF-KS from the point of view of the precision, accuracy, steadiness and risk level metrics described in [31, 64] and here briefly recalled:

➤ Precision Index (PI)

PI considers the relative width of the prediction interval, which is defined by:

$$PI_t = \frac{\sup(RUL\_I_t) - \inf(RUL\_I_t)}{RUL_t} \quad (24)$$

where  $\sup(RUL\_I_t)$  and  $\inf(RUL\_I_t)$  are the upper and lower bounds of the RUL prediction 90% belief interval and  $RUL_t$  is the real RUL at time  $t$ . Small values of  $PI_t$  indicate more precise predictions.

➤ Accuracy Index (AI)

AI is defined as the relative error of RUL prediction:

$$AI_t = \frac{|RUL_t - RUL_t|}{RUL_t} \quad (25)$$

where  $RUL_t$  is the RUL estimated at time  $t$ . Small values of  $AI_t$  indicate more accurate predictions.

➤ Steadiness Index (SI)

SI measures the volatility of the RUL prediction:

$$SI_t = \sqrt{\text{var}(RUL_{(t-\Delta t):t})} \quad (26)$$

where  $\Delta t$  is the sliding time window: in this paper we take  $\Delta t = 5$ . Small values of  $SI_t$  indicate more stable predictions.

➤ Risk Index (RI)

RI is the probability of obtaining a RUL estimate smaller than the true RUL:

$$RI_t = P(RUL \leq RUL_t) = \int_{-\infty}^{RUL_t} p(RUL\_I) dRUL \quad (27)$$

where  $p(RUL\_I)$  is the estimate of the RUL PDF. Large  $RI_t$  values indicate conservative RUL prediction which are associated to lower risk for maintenance decisions.

In order to have a robust performance evaluation, we calculate the PI, AI, SI and RI indicators for the RUL predictions of the 100 batteries' degradation trajectories simulated in Section 7.2. Figure 9 reports the performance indicators averaged over the 100 trajectories, considering different values of the smoothing parameter  $h$  in PF-KS.

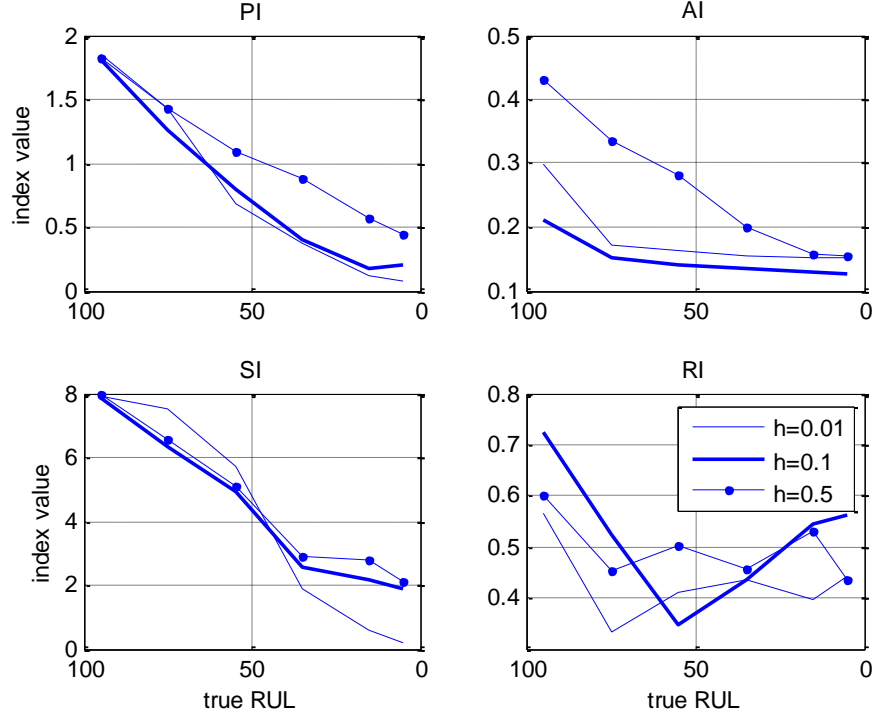


Figure 9 Performance comparison considering different values of the smoothing parameter  $h$

At the beginning of the battery life, when the true RUL value is 95, the most satisfactory performance is obtained with  $h=0.1$ . As time passes and more data become available,  $h=0.01$  leads to a faster convergence, as indicated by the lower values of the PI and SI indicators, although the RUL prediction is still less accurate (lower AI indicator) than that obtained using  $h=0.1$ . This effect is caused by the particle impoverishment problem that is faced when  $h$  is too small. On the other side, larger values of  $h$ , such as 0.5, do not allow a satisfactory convergence of the RUL and lead to unsatisfactory accuracy. Thus, our final choice has been to set the value of the parameter  $h$  to 0.1. Notice that the same value of  $h=0.1$  is suggested in [45]. Current research work is devoted to the development of an algorithm for the automatic setting of the parameter, taking into account that its optimal value can change as time passes and that the  $h$  value can be optimized each time a new measurement becomes available by considering the difference between the predicted and posterior PDFs.

Figure 10 reports the PI, AI, SI and RI indicators obtained by PF-UC, PF-AE and PF-KS. With respect to PF-AE, since there is no information or rule to set the variance of the artificial noise, we chose to use a noise variance equal to one percent of the variance of the parameter prior PDFs. This setting is supposed to provide enough variability in the particle model parameters without, at the same time, causing instabilities in the estimates. For PF-KS, the value of the smoothing parameter  $h$  is equal to 0.1. For all three methods we have used the same number of particles  $N=10,000$ , relatively large in order to not penalize the PF-UC in the comparison.

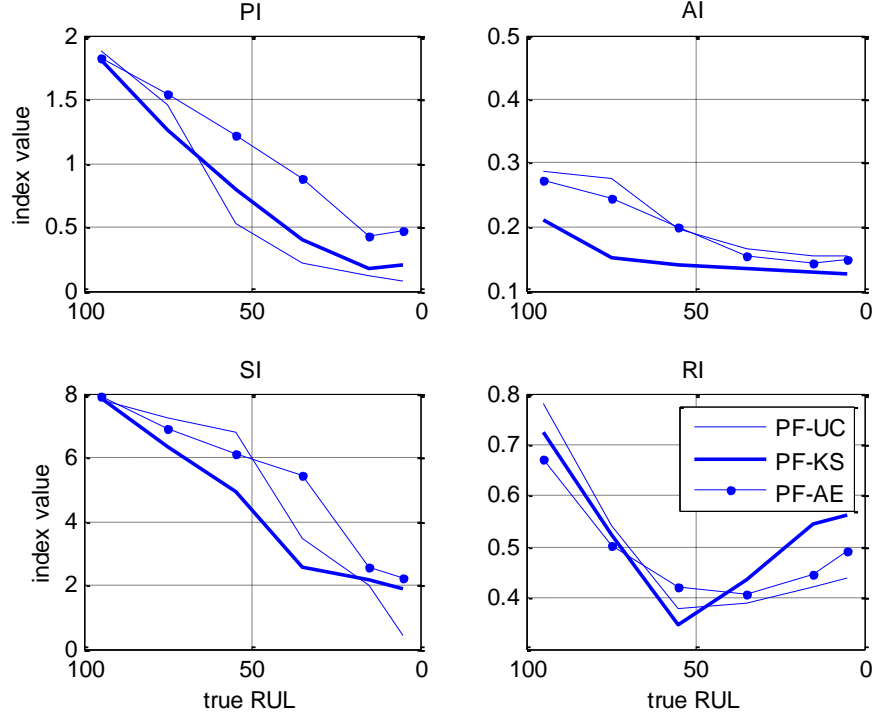


Figure 10 Performance comparison using PF-UC, PF-AE and PF-KS

Except for the precision indicator, PF-KS has the best performances. The lowest value of the PI indicator of PF-UC is due to the fact that at the end of the battery life all the particles are characterized by the same state and parameter values. Thus,  $\sup(RUL_{I_t})$  and  $\inf(RUL_{I_t})$  in Eq.(24) have very similar values, and PI becomes very small. Given the unsatisfactory accuracy of PF-UC (high value of AI), we can conclude that this method overall performance is completely unsatisfactory.

Two analyses of the sensitivity of the PF-KS method with respect to the uncertainty in the prior distributions and the number of particles are performed.

Firstly, we have considered three possible situations of prior knowledge on the parameters values. The prior PDFs setting is shown in Table 4, where the intermediate level is the same as Table 3. Figure 11 shows the obtained results. Notice that although the quality of the prior PDFs has a remarkable influence on the overall performance at the beginning of the component life, the performances in the three cases become very similar as time passes and more measurements become available. This confirms that the PF-KS method is able to provide satisfactory RUL predictions even in case of limited knowledge on the parameter values.

Table 4 Three different parameter Prior Distributions settings

very uncertain prior distributions		intermediate level of uncertainty in the prior distribution		well defined prior distributions	
$p_1$ $U(0.75,1.3)$	$p_2$ $U(-1.5,0)e-3$	$p_1$ $U(0.85,1.2)$	$p_2$ $U(-1,0)e-3$	$p_1$ $U(0.85,0.92)$	$p_2$ $U(-9.7,-8) e-4$
$p_3$ $U(-1.5,0)e-3$	$p_4$ $U(0.1,1.7)e-1$	$p_3$ $U(-1,0)e-3$	$p_4$ $U(0.3,1.3)e-1$	$p_3$ $U(-4,-1)e-4$	$p_4$ $U(0.3,0.6)e-1$

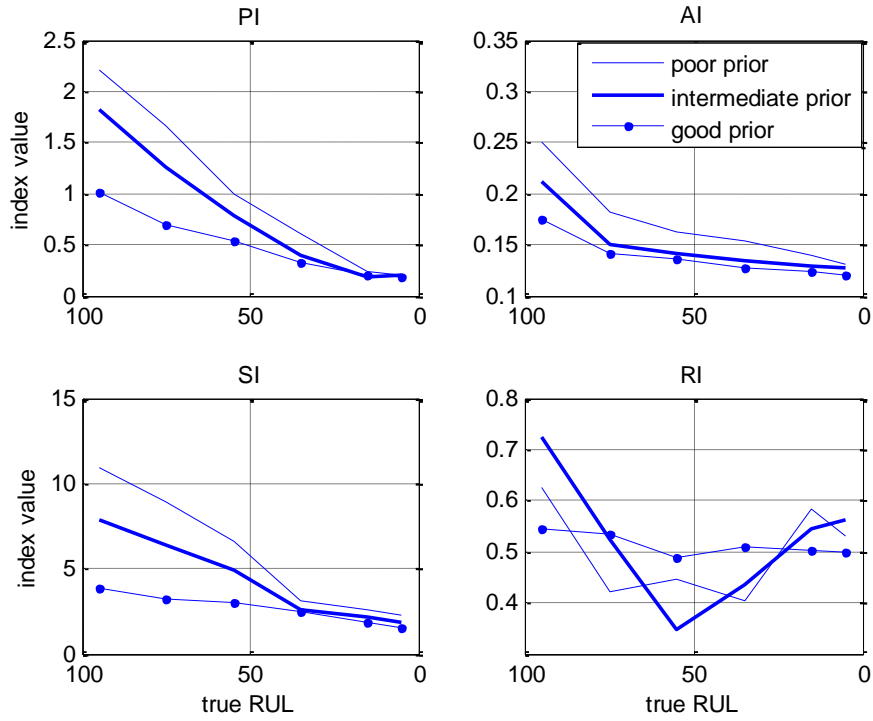


Figure 11 Comparison of the performance obtained using prior distributions of the model parameters characterized by different levels of uncertainty.

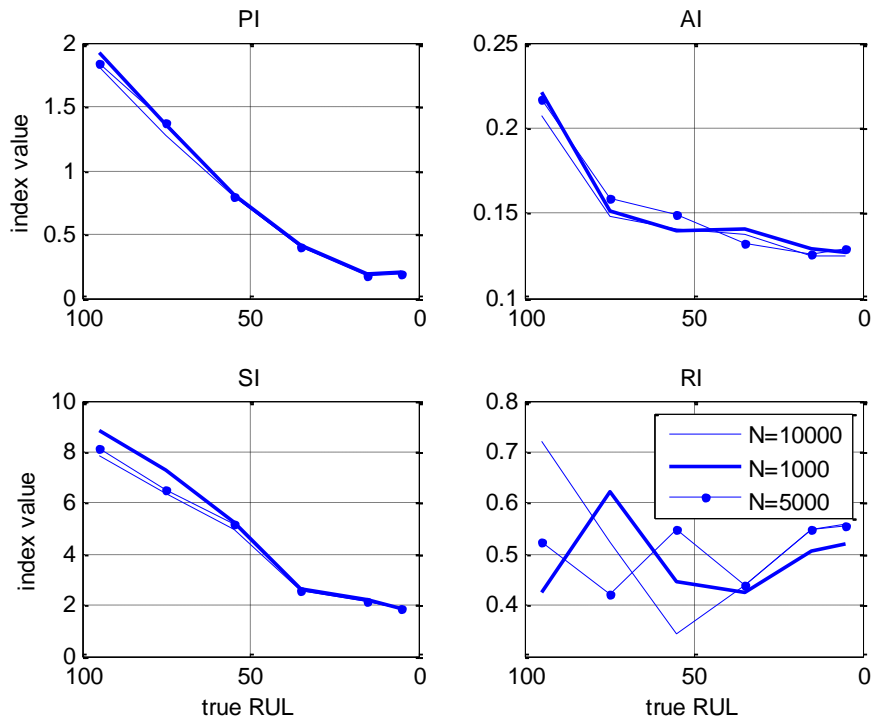


Figure 12 Performance comparison using three different numbers of particles

Finally, Figure 12 shows the performances of the methods considering different numbers of particles (1,000, 5,000 and 10,000). It can be observed that the performance indicators in the three cases are very similar. This confirms the ability of PF-KS of providing good performance without requiring high computational efforts.

## 8. CONCLUSION

In this work, we have proposed a particle filtering and kernel smoothing approach for the RUL prediction of degrading components based on a model of degradation with unknown parameters. We have assumed to know the model of degradation process and to be able to perform measurements of quantities related to the component degradation; on the other side, we have assumed that we do not know the true value of the degradation model parameters and the observations of degradation trajectories in similar components are unavailable.

The results obtained in a numerical case study regarding battery degradation have shown that the proposed method can provide estimates of the component RUL and of the model parameters which are more satisfactory than those obtained with other particle filtering approaches. From the point of view of the ability of the proposed approach to deal with the large uncertainties of the considered problem, at the beginning of the life of the component, when few observations of its degradation are available, the method provides, as expected, uncertain estimates of the model parameters and of the RUL, but as time passes and new information becomes available, the estimates become more precise and less uncertain.

The proposed approach will be further investigated in future works in order to understand if information on the sensitivity of the RUL to the different model parameters can be used to drive the search of their values more effectively. In particular, it is expected that by dynamically freezing the values of the less sensitive parameters, it is possible to improve the accuracy of the estimate of the most sensitive parameters. Furthermore, alternative methods for the prediction of the future component degradation trajectory, based on the estimate of the multidimensional integral in the Chapman-Kolmogorov equation with advanced Monte Carlo Techniques, will be explored.

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