

**This document must be cited according  
to its final version which is published in a conference as:**

**V.Grelet, P. Dufour, M. Nadri, V.Lemort, T. Reiche,  
"Explicit multi model predictive control  
of a waste heat Rankine based system for heavy duty trucks",  
54rd IEEE Conference on Decision and Control (CDC),  
Osaka, Japan, pp. 179-184, december 15-18, 2015.**

**You downloaded this document from the  
CNRS open archives server, on the webpages of Pascal Dufour:  
<http://hal.archives-ouvertes.fr/DUFOUR-PASCAL-C-3926-2008>**

# EXPLICIT MULTI-MODEL PREDICTIVE CONTROL OF A WASTE HEAT RANKINE BASED SYSTEM FOR HEAVY DUTY TRUCKS

Vincent GRELET<sup>1,2,3</sup>, Pascal DUFOUR<sup>2</sup>, Madiha NADRI<sup>2</sup>, Vincent LEMORT<sup>3</sup>  
and Thomas REICHE<sup>1</sup>

<sup>1</sup>Volvo Group Trucks Technology Advanced Technology and Research, 1 avenue Henri Germain, 69800 Saint Priest, France

<sup>2</sup>Université de Lyon, Lyon F-69003, Université Lyon 1, CNRS UMR 5007, Laboratory of Process Control and Chemical Engineering (LAGEP), Villeurbanne 69100, France

<sup>3</sup>LABOTHAP, University of Liege, Campus du Sart Tilman Bat. B49 B4000 Liege, Belgium

54<sup>th</sup> IEEE Conference on Decision and Control (CDC 2015)  
15-18 December, Osaka, Japan

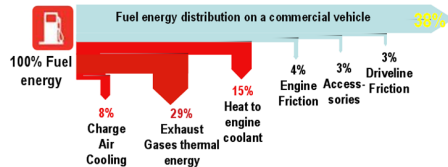


## Table of contents

- 1 Context and motivations
- 2 Rankine cycle based heat recovery system
  - Rankine process
  - Studied system and controller objective
- 3 Nonlinear evaporator detailed model
- 4 Controller development
  - Identification
  - Piecewise linear approach
  - MMPC strategy
- 5 Simulation results
- 6 Conclusion and next steps

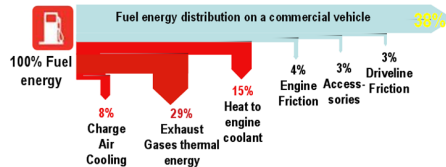
## Context and motivations

In nowadays heavy duty engines, a major part of the chemical energy contained in the fuel is released to the ambient through heat.



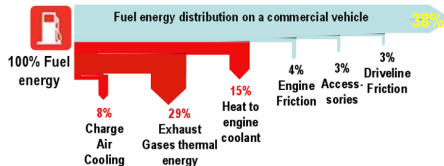
## Context and motivations

- 👎 In nowadays heavy duty engines, a major part of the chemical energy contained in the fuel is released to the ambient through heat.
- 👍 Waste heat recovery based on the Rankine cycle is a promising technique to increase fuel efficiency.



## Context and motivations

- 🚫 In nowadays heavy duty engines, a major part of the chemical energy contained in the fuel is released to the ambient through heat.
- 👍 Waste heat recovery based on the Rankine cycle is a promising technique to increase fuel efficiency.
- 🚫 Dynamic models needed for concept optimization, fuel economy evaluation and control algorithm development.



## Rankine process

- Liquid compression ( $1 \rightarrow 2$ ) from condensing to evaporating pressure by means of the pump power  $\dot{W}_{in}$ .
- Preheating ( $2 \rightarrow 3a$ ), vaporization ( $3a \rightarrow 3b$ ) and superheating ( $3b \rightarrow 3c$ ) by means of the input heat power  $\dot{Q}_{in}$ .
- Vapor expansion ( $3c \rightarrow 4$ ) from evaporating to condensing pressure creating power  $\dot{W}_{out}$  on the expander shaft.
- Condensation ( $4 \rightarrow 1$ ) releasing heat  $\dot{Q}_{out}$  in the heat sink.

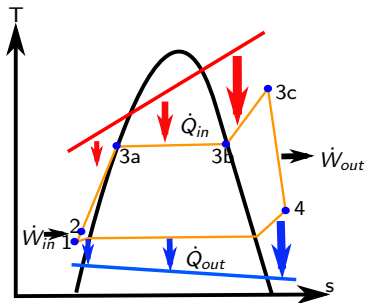
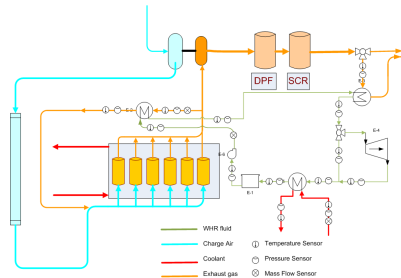


Figure: Temperature-entropy diagram of the Rankine cycle

## Studied system and controller objective

- Recover heat from both EGR and exhaust in a serial configuration.
- Working fluid: water ethanol mixture.
- Focus on the control of the working fluid superheat at the expansion machine inlet.
- Even more critical when using a kinetic expander.
- Control issue: Reduce the deviation of the superheat around its set point to have safe and efficient operation.





## Nonlinear evaporator detailed model

### Model representation

$$\dot{x}_i = f_i(x_i, u), \quad (1)$$

$$u^T = [\dot{m}_{f_0} \quad P_{f_0} \quad h_{f_0} \quad \dot{m}_{g_L} \quad T_{g_L}], \quad x_i^T = [\dot{m}_{f_i} \quad h_{f_i} \quad T_{w_{int_i}} \quad T_{g_i} \quad T_{w_{ext_i}}] \quad (2)$$

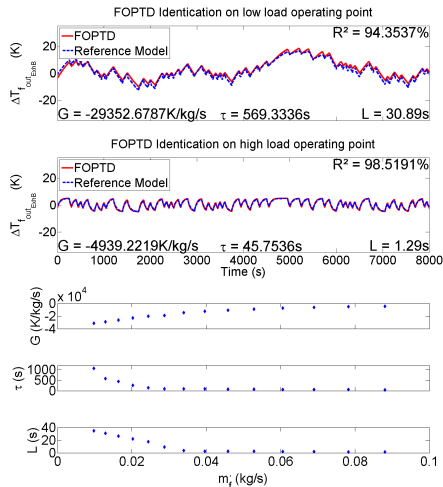
$$f_i(x_i, u) = \left[ \begin{array}{l} \frac{\dot{m}_{f_{i-1}} \frac{h_{f_{i-1}}}{\rho_{f_{i-1}}} \frac{\partial \rho_{f_{i-1}}}{\partial h_{f_{i-1}}} + \frac{1}{\rho_{f_{i-1}}} \frac{\partial \rho_{f_{i-1}}}{\partial h_{f_{i-1}}} \alpha_{f_i} A_{exch_{int_f}} (T_{f_i} - T_{w_{int_i}})}{1 - \frac{h_{f_i}}{\rho_{f_i}} \frac{\partial \rho_{f_i}}{\partial h_{f_i}}} - \dot{m}_{f_i} \\ \frac{(\dot{m}_{f_{i-1}} h_{f_{i-1}} - \dot{m}_{f_i} h_{f_i}) - \alpha_{f_i} A_{exch_{int_f}} (T_{f_i} - T_{w_{int_i}})}{\rho_{f_i} V_f} \\ \frac{\alpha_{f_i} A_{exch_{int_f}} (T_{f_i} - T_{w_{int_i}}) + \alpha_{g_i} A_{exch_{int_g}} (T_{g_i} - T_{w_{int_i}})}{\rho_{w_{int}} V_{w_{int}}} \\ \frac{\dot{m}_g c_{p_g} (T_{g_i}) (T_{g_{i-1}} - T_{g_i}) - \alpha_g \left[ A_{exch_{int_g}} (T_{g_i}^* - T_{w_{int_i}}) - A_{exch_{ext_g}} (T_{g_i} - T_{w_{ext_i}}) \right]}{\rho_{g_i} V_g c_{p_g} (T_{g_i})} \\ \frac{\alpha_{amb} A_{exch_{ext_{amb}}} (T_{amb} - T_{w_{ext_i}}) + \alpha_g A_{exch_{ext_g}} (T_{g_i} - T_{w_{ext_i}})}{\rho_{w_{ext}} V_{w_{ext}}} \end{array} \right]$$

## Identification

- Dynamic relation between  $u$  (working fluid mass flow) and  $y$  (working fluid superheat) can be described around an operating point by a first order plus time delay (FOPTD) model:

$$\frac{y(s)}{u(s)} = \frac{G}{1 + \tau s} e^{-Ls}, \quad (3)$$

- High variation in FOPTD parameters shows high nonlinearity.
- Linear time invariant controller will hardly achieve the control objective with good performance under transient driving cycle.



## Piecewise linear approach

- Multi linear model approach consists into identifying a bank of  $N$  linear models and combine them by means of a weighting scheme.
- Global model output is (at time  $t_k$ ):

$$y_k = \sum_{i=1}^N y_{i,k} W_{i,k} \quad (4)$$

- Key design issues are : 1/ the selection of the good model(s) in the bank. 2/ linear models mixing.
- Modeling error of the  $i^{\text{th}}$  model at the current time  $t_k$  is defined by:

$$\epsilon_{i,k} = y_{p,k} - y_{i,k}. \quad (5)$$

## Weighting scheme

### Bayesian recursive scheme

$$p_{i,k} = \frac{\exp(-\frac{1}{2}\epsilon_{i,k} K \epsilon_{i,k}^T p_{i,k-1})}{\sum_{m=1}^N (\exp(-\frac{1}{2}\epsilon_{m,k} K \epsilon_{m,k}^T p_{m,k-1}))} \quad (6)$$

$$W_{i,k} = \begin{cases} \frac{p_{i,k}}{\sum_{m=1}^N p_{m,k}} & \text{for } p_{i,k} > \delta \\ 0 & \text{for } p_{i,k} < \delta \end{cases} \quad (7)$$

where  $K$  is a vector and  $\delta$  a scalar.

### New proposed scheme

$$\tilde{\epsilon}_{i,k} = \frac{\epsilon_{i,k}^2}{\sum_{m=1}^N \epsilon_{m,k}^2} \quad (8)$$

$$X_{i,k} = (1 - \tilde{\epsilon}_{i,k}) \prod_{j \neq i, j=1}^{j=N} \tilde{\epsilon}_{j,k} \quad (9)$$

$$\tilde{X}_{i,k} = \frac{X_{i,k}}{\sum_{m=1}^N X_{m,k}} \quad (10)$$

$$W_{i,k} = \frac{1}{1 + T_s} \tilde{X}_{i,k} \quad (11)$$

where  $T$  is a scalar.

## Optimization problem

### Model Predictive Control cost function for set-point tracking

$$\left\{ \begin{array}{l} \min_{u_{inf} \leq u_k \leq u_{sup}} J(u_k) = \int_{t_k}^{t_k+t_p} (y_p(t) - y^{sp})^2 + w_u \Delta u_k^2 dt, \\ \Delta u_k = u_k - u_{k-1}, \end{array} \right. \quad (12)$$

where  $w_u$  is a scaling factor and a penalty weight.

### Modeling error

$$e_k = y_{p,k} - y_k. \quad (13)$$

Output prediction  $y_p(t)$  in (12) can be written based on the  $N$  models and feedback:

$$y_p(t) = y(t) + e_k. \quad (14)$$

### Output response of a FOPTD model

$$y_i(t) = y_{p,k} e^{-\frac{(t-t_k)}{\tau_i}} + \int_{t_k}^t \left( e^{-\frac{(t-s)}{\tau_i}} \frac{G_i}{\tau_i} u(s - L_i) \right) ds. \quad (15)$$

## MMPC framework

### Integration intervals

A model response (15) can be developed as:

$$y_i(t) = y_{p,k} e^{-\frac{(t-t_k)}{\tau_i}} + \frac{G_i}{\tau_i} e^{-\frac{t}{\tau_i}} \int_{t_k}^t (e^{\frac{s}{\tau_i}} u(s - L_i)) ds, \quad (16)$$

Based on the time delay  $L_i$ , let us define:

$$\begin{cases} \lambda_i &= \max\{a_i \in \mathbb{N} | a_i \leq \frac{L_i}{T_s}\} \\ \Delta L_i &= L_i - \lambda_i T_s, \in \mathbb{R}^+. \end{cases} \quad (17)$$

Integration in (16) is done by parts, where the  $\lambda_i + 2$  time intervals are:

$s$	$s - L_i$	$u(s - L_i)$
$t_k \rightarrow t_k + \Delta L_i$	$t_k - L_i \rightarrow t_k - \lambda_i$	$u(t_k - \lambda_i - 1)$
$t_k + \Delta L_i \rightarrow t_k + \Delta L_i + T_s$	$t_k - \lambda_i \rightarrow t_k - \lambda_i + 1$	$u(t_k - \lambda_i)$
...	...	...
$t_k + \Delta L_i + (\lambda_i - j) T_s \rightarrow t_k + \Delta L_i + (\lambda_i - j + 1) T_s$	$t_k - j \rightarrow t_k - j + 1$	$u(t_k - j)$
...	...	...
$t_k + \Delta L_i + (\lambda_i - 1) T_s \rightarrow t_k + L_i$	$t_{k-1} \rightarrow t_k$	$u(t_{k-1})$
$t_k + L_i \rightarrow t$	$t_k \rightarrow t - L_i$	$u(t_k) = u_k$

## MMPC framework

### Explicit MMPC formulation

Once integrated, (16) is a linear expression in the optimization argument  $u_k$ :

$$y_i(t) = y_{p,k} f_{1i}(\tau_i, t_k, t) + f_{2i}(T_s, G_i, \tau_i, \Delta L_i, \lambda_i, t_k, t, u(\text{past})) + u_k f_{3i}(G_i, \tau_i, L_i, t_k, t), \quad (18)$$

where the  $f_i$  may be explicitly defined offline and updated online at each time  $t_k$ . Hence the initial cost function is:

$$J(u_k) = \int_{t_k}^{t_k+t_p} \left( \left( \sum_{i=1}^N (w_{i,k} y_i(t)) + e_k - y_k^{sp} \right)^2 + w_u \Delta u_k^2 \right) dt \quad (19)$$

where the prediction horizon  $t_p = \max(t_{pi}) \forall i$  may be tuned as:

$$\begin{cases} t_{pi} = \gamma_p * \tau_i + L_i; \gamma_p \in \mathbb{R}^+, \\ \text{e.g.: } \gamma_p = 1 \text{ (63\% of the dynamics is predicted)} \\ \text{or } \gamma_p = 3 \text{ (95\% of the dynamics is predicted)}. \end{cases} \quad (20)$$

## MMPC framework

### Explicit MMPC formulation

Based on the step response series (16) of the  $N$  linear FOPTD models:

$$\begin{aligned}
 J(u_k) = & \beta_{2,k}(N, G_i, \tau_i, L_i, t_p, w_u, w_{i,k})u_k^2 \\
 & + \beta_{1,k}(N, T_s, G_i, \tau_i, L_i, t_p, \Delta L_i, \lambda_i, w_u, y_{p,k}, y_k^{sp}, e_k, u(\text{past}), w_{i,k})u_k \\
 & + \beta_{0,k}(N, T_s, G_i, \tau_i, L_i, t_p, \Delta L_i, \lambda_i, w_u, y_{p,k}, y_k^{sp}, e_k, u(\text{past}), w_{i,k}),
 \end{aligned} \tag{21}$$

Minimization of (21) obtained with the first order optimality at each  $t_k$ :

$$\frac{\partial J}{\partial u_k} = 0 \text{ at } u_k = u_k^{\min}. \tag{22}$$

Calculation of  $u_k^{\min}$  is then straightforward:

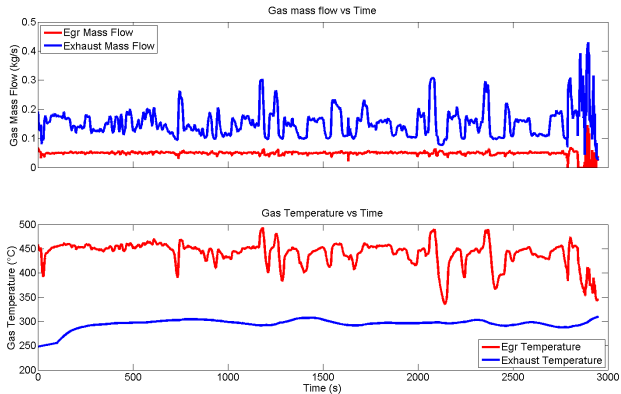
$$u_k^{\min} = \frac{-\beta_{1,k}}{2\beta_{2,k}} \tag{23}$$

which leads to the explicit formulation of the solution  $u_k^*$ :

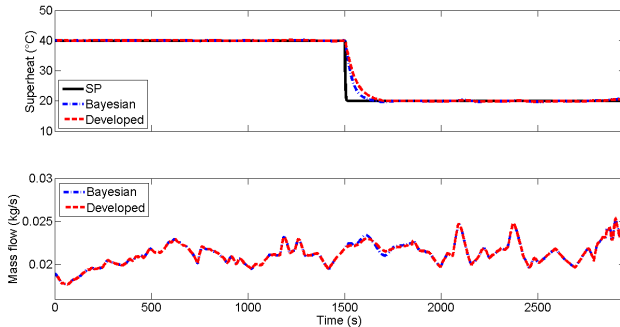
$$\begin{cases}
 \text{if } u_{\text{inf}} \leq u_k^{\min} \leq u_{\text{sup}} : u_k^* = u_k^{\min} \\
 \text{if } u_k^{\min} \leq u_{\text{inf}} : u_k^* = u_{\text{inf}} \\
 \text{if } u_{\text{sup}} \leq u_k^{\min} : u_k^* = u_{\text{sup}}.
 \end{cases} \tag{24}$$



## Input disturbances



## Tracking error and manipulated variable



## Conclusion and next steps

### Conclusion

- New modeling weighting scheme based on a piecewise linear approach has been developed and validated.
- New scheme has less tuning parameters than the Bayesian scheme.
- Explicit MMPC strategy for Rankine cycle based heat recovery system is presented.
- MMPC is compliant with classical automotive integration constraints (i.e., basic CPU and fast sampling time).

### Next steps

- Experimental validation.
- Robustness study.

## Contacts and discussion

### Authors

- Vincent GRELET: greletv@live.com
- Pascal DUFOUR: dufour@lagep.univ-lyon1.fr
- Madiha NADRI: nadri@lagep.univ-lyon1.fr
- Vincent LEMORT: vincent.lemort@ulg.ac.be
- Thomas REICHE: thomas.reiche@volvo.com

### Acknowledgement

This PhD thesis is collaboration between UCBL1, ULg and Volvo Trucks which is gratefully acknowledged for the funding. The French ministry of higher education and research for the financial support of the CIFRE PhD thesis 2012/549 is also acknowledged.

## $\beta_i$ expressions

$$\beta_{1,k} = \sum_{i=1}^N \left[ \beta'_{1,k} + \beta''_{1,k} + \beta'''_{1,k} + \beta''''_{1,k} \right] - 2w_u u_{k-1} t_p, \quad (25)$$

$$\beta'_{1,k} = -G_i \tau_i w_{i,k}^2 y_{p,k} \left( e^{\frac{L_i}{\tau_i}} + 2e^{-\frac{t_p}{\tau_i}} - e^{\frac{L_i - 2t_p}{\tau_i}} - 2 \right) \quad (26)$$

$$\begin{aligned} \beta''_{1,k} = & -G_i^2 \tau_i u(t_{k-\lambda_i-1}) w_{i,k}^2 e^{-\frac{2t_p}{\tau_i}} \left( e^{\frac{\Delta L_i}{\tau_i}} - 1 \right) \left( e^{\frac{t_p}{\tau_i}} - 1 \right) \dots \\ & \left( e^{\frac{L_i}{\tau_i}} - 2e^{\frac{t_p}{\tau_i}} + e^{\frac{L_i + t_p}{\tau_i}} \right) \end{aligned} \quad (27)$$

## $\beta_i$ expressions

$$\beta_{1,k}''' = \sum_{j=1}^{\lambda_i} G_i^2 \tau_i u(t_{k-j}) w_{i,k}^2 e^{\frac{\Delta L_i - t_p - j T_s + \lambda_i T_s}{\tau_i}} \left( e^{\frac{T_s}{\tau_i}} - 1 \right) \left( e^{\frac{L_i - t_p}{\tau_i}} - 2 \right) \dots$$

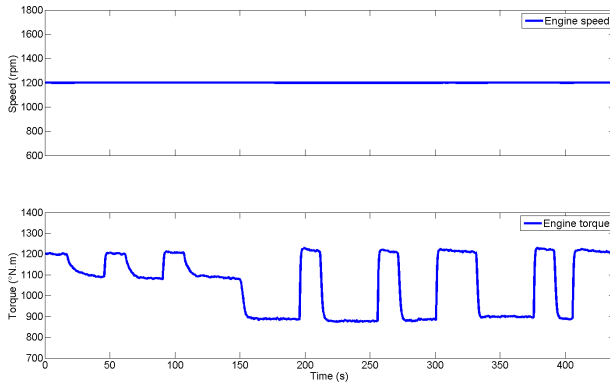
$$- G_i^2 \tau_i u(t_{k-j}) w_{i,k}^2 e^{\frac{\Delta L_i - j T_s + \lambda_i T_s}{\tau_i}} \left( e^{\frac{L_j}{\tau_i}} - 2 \right) \left( e^{\frac{T_s}{\tau_i}} - 1 \right) \quad (28)$$

$$\beta_{1,k}'''' = 2 G_i w_{i,k} \left( e_k - y^{SP} \right) \left( t_p + \tau_i e^{\frac{L_i - t_p}{\tau_i}} - \tau_i e^{\frac{L_j}{\tau_i}} \right) \quad (29)$$

$$\beta_{2,k} = \frac{G_i^2 w_{i,k}^2 \left( 2 t_p + 4 \tau_i e^{\frac{L_i - t_p}{\tau_i}} - \tau_i e^{\frac{2 L_i - 2 t_p}{\tau_i}} - 4 \tau_i e^{\frac{L_j}{\tau_i}} + \tau_i e^{\frac{2 L_j}{\tau_i}} \right)}{2} + w_u t_p \quad (30)$$

## Input disturbances

### Engine speed and torque



## Tracking error and manipulated variable

### Developed weighting scheme

