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Adaptive identification of continuous-time MIMO state-space models

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CDC, Osaka, 15-18 December, 2015

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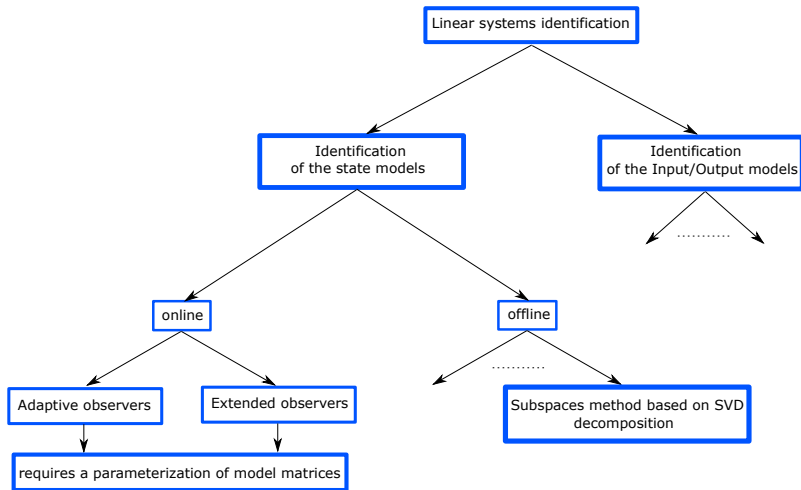
Outline

- 1 Identification problem
- 2 Estimation method
- 3 Exponential convergence
- 4 Simulation
- 5 Conclusion and future work

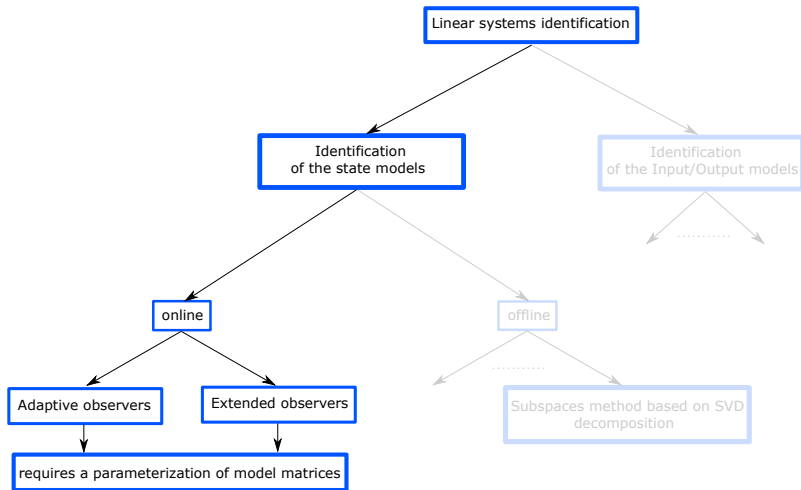
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General schema



Studied approach



MIMO (Multiple-Input Multiple-Output) systems

Consider a linear system described by a state-space model of the form

$$S: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t), \end{cases}$$

MIMO (Multiple-Input Multiple-Output) systems

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- $u \in \mathbb{R}^{n_u}$ measured inputs signals
- $y \in \mathbb{R}^{n_y}$ measured outputs signals
- $x \in \mathbb{R}^n$, unknown state
- $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$ and $D \in \mathbb{R}^{n_y \times n_u}$ are **unknown parameters**

Systems assumptions & estimation problem

System assumptions

- A1. The system S is stable $\rightarrow A$ is Hurwitz.
- A2. (A, B, C) is minimal $\rightarrow (A, B)$ is controllable and (A, C) is observable.
- A3. C is full row rank.

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Estimation problem

From the measurements of inputs u and outputs $y \rightarrow$ Estimate both the state x and the parameters (A, B, C, D) in an arbitrary state space basis.

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Main idea

- Search for a linear expression

$$M(A, B, C, D)\varphi(u, y) = f(u, y)$$

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 - $M(t) = \arg \min_M V_t(M)$ converges to M^0 the solution of the linear expression
 - $\frac{d}{dt} \nabla V_t(M) = 0 \rightarrow$ a continuous-time recursive estimator.

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 - $\frac{d}{dt} \nabla V_t(M) = 0 \rightarrow$ a continuous-time recursive estimator.
- Extract A , B , C and D from M .

Is there a linear relation $M(A, B, C, D)\varphi(y, u) = f(y, u)$?

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We can get

$$y_f = \mathcal{O}_f x + \mathcal{I}_f u_f, \quad f > n$$

with

$$y_f = \begin{bmatrix} y^\top & \dot{y}^\top & \dots & y^{(f-1)\top} \end{bmatrix}^\top \in \mathbb{R}^{n_y n}$$

$$u_f = \begin{bmatrix} u^\top & \dot{u}^\top & \dots & u^{(f-1)\top} \end{bmatrix}^\top \in \mathbb{R}^{n_u n}$$

$$\mathcal{O}_f = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{f-1} \end{bmatrix} \in \mathbb{R}^{n_y n \times n}, \quad \mathcal{I}_f = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \dots & D \end{bmatrix} \in \mathbb{R}^{n_y n \times n_u n}.$$

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Problem (noisy measurements)

The time derivatives of u and y are not measured. The only available signals are the inputs and outputs .

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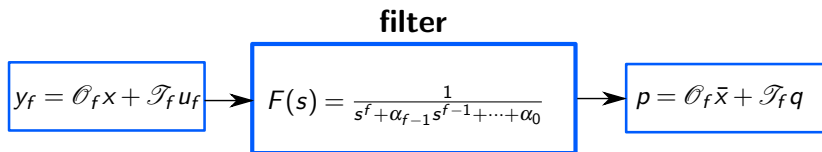
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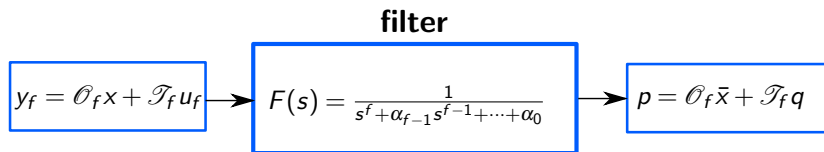
Solution (Linear stable filter)

Avoid numerical computation of derivatives \rightarrow work with filtered data.

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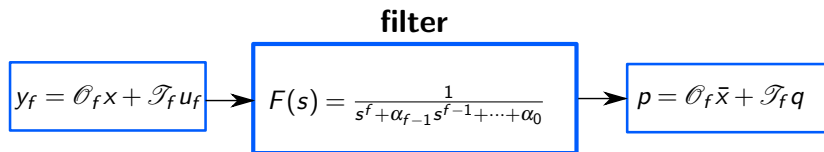


with

$$\begin{aligned} \dot{p} &= (\Lambda_c \otimes I_{n_y}) p + (I \otimes I_{n_y}) y \\ \dot{q} &= (\Lambda_c \otimes I_{n_u}) q + (I \otimes I_{n_u}) u \end{aligned}, \quad \Lambda_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & \cdots & \cdots & -\alpha_{f-1} \end{bmatrix}, \quad I = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

and \bar{x} is the filtered state of x .

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and \bar{x} is the filtered state of x .

Problem

We are still facing the difficulty that \bar{x} is unknown.

Is there a linear relation $M(A, B, C, D)\varphi(y, u) = f(y, u)$?

Solution

By a change of state coordinates \rightarrow We define a new state \tilde{x} as

$$\tilde{x} = T\bar{x}, \text{ with } T = H\theta_f \in \mathbb{R}^{n \times n} \text{ is nonsingular}$$

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Remark (choice of H)

- (A, C) is observable $\Rightarrow \theta_f$ is of full rank column \Rightarrow the probability to get a nonsingular T is one if H is randomly generated.

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- For multiple inputs single output systems $H = [I_n \quad 0_{n \times (f-n)}]$.
- For MIMO systems, if a set of O_f rows indexed by $[i_1, \dots, i_n]$ is linearly independent, then $H = [e_{i_1} \quad \dots \quad e_{i_n}]^T$ with $e_i \in \mathbb{R}^{f_y}$ the i -th canonical basis vector.

There is a linear relation $M(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})\varphi(y, u) = f(y, u)$

The system realization becomes

$$\tilde{\Sigma} : \begin{cases} \dot{z}(t) = \tilde{A}z(t) + \tilde{B}u(t) \\ y(t) = \tilde{C}z(t) + \tilde{D}u(t), \end{cases}$$

with

$$\tilde{A} = TAT^{-1}, \quad \tilde{B} = TB, \quad \tilde{C} = CT^{-1}, \quad \tilde{D} = D, \quad z = Tx$$

and \tilde{x} is the filtered state of z .

There is a linear relation $M(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})\varphi(y, u) = f(y, u)$

The system realization becomes

$$\tilde{\mathcal{S}} : \begin{cases} \dot{z}(t) = \tilde{A}z(t) + \tilde{B}u(t) \\ y(t) = \tilde{C}z(t) + \tilde{D}u(t), \end{cases}$$

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Result

This change of coordinates leads to

$$\tilde{\mathcal{O}}_f \tilde{x} = p - \tilde{\mathcal{I}}_f q \quad \text{and} \quad \tilde{x} = Hp - H\tilde{\mathcal{I}}_f q \Rightarrow M\varphi = p$$

$$M = \begin{bmatrix} \tilde{\mathcal{O}}_f & \tilde{\mathcal{I}}_f - \tilde{\mathcal{O}}_f H\tilde{\mathcal{I}}_f \end{bmatrix} \in \mathbb{R}^{fn_y \times (n+fn_u)}, \quad \varphi = \begin{bmatrix} Hp \\ q \end{bmatrix} \in \mathbb{R}^{n+fn_u}.$$

RLS method

V_t Criterion

$$V_t(M) = \frac{1}{2} e^{-\alpha t} \text{tr}[(M - M_0) P_0^{-1} (M - M_0)^T] \\ + \frac{1}{2} \int_0^t \frac{e^{-\alpha(t-\tau)}}{r(\tau)^2} \|p(\tau) - M\varphi(\tau)\|_2^2$$

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$M(t) = \underset{M}{\text{arg min}} V_t(M) \Rightarrow M(t)$ is the solution of $\nabla V_t(M) = 0$

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$M(t) = \underset{M}{\text{argmin}} V_t(M) \Rightarrow M(t)$ is the solution of $\nabla V_t(M) = 0$

- $P_0 > 0 \rightarrow P_0$ is a symmetric positive-definite matrix.
- $\alpha > 0$ is a design parameter.
- $r(\tau)^2 \geq 1$ is a normalizing upper-bounded factor.
- $M_0 = M(0)$ is the initial value of $M(t)$.

Parameters \tilde{A} and \tilde{C} adaptive identifier

Result

Setting $\frac{d}{dt} \nabla V_t(M) = 0$ yields the continuous-time RLS estimator:

$$\dot{M} = \frac{(p - M\varphi)\varphi^\top P}{r(t)^2}, \quad M(0) = M_0$$

$$\dot{P} = \alpha P - \frac{P\varphi\varphi^\top P}{r(t)^2}, \quad P(0) = P_0 \succ 0$$

$$r(t)^2 = 1 + \varphi^\top P \varphi$$

with

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \\ M_{31} & M_{32} \end{bmatrix} \Rightarrow \tilde{C} = M_{11}, \quad \tilde{A} = \begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix}^\dagger \begin{bmatrix} M_{21} \\ M_{31} \end{bmatrix}.$$

Parameters \tilde{B} and \tilde{D} adaptive identifier

The system output can be expressed on a linear regression form

$$y(t) = \tilde{C}e^{\tilde{A}t}\tilde{x}_0 + \Phi(t)\theta,$$

with

$$\theta = [\text{vec}(\tilde{B})^\top \quad \text{vec}(\tilde{D})^\top]^\top ; \Phi(t) = [\Psi(t) \quad u(t)^\top \otimes I_{n_y}],$$

and $\Psi(t)$ obeys the following differential equation

$$\dot{\Psi}(t) = \Psi(t)[I_{n_u} \otimes \tilde{A}] + u(t)^\top \otimes \tilde{C}, \Phi(0) = 0$$

Parameters \tilde{B} and \tilde{D} adaptive identifier

Result

we can derive the RLS estimator of θ

$$\dot{\theta} = Q\Phi^\top \frac{(y - \Phi\theta)}{\rho(t)^2}, \quad \theta(0) = \theta_0$$

$$\dot{Q} = \beta Q - \frac{Q\Phi^\top\Phi Q}{\rho(t)^2}, \quad Q(0) = Q_0 \succ 0$$

where

$$\rho(t)^2 = 1 + \text{tr} \left(\Phi(t)Q(t)\Phi(t)^\top \right).$$

Estimation of the state $z = Tx$

z can be generated by constructing a Luenberger observer based on the identified realization $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$.

$$\dot{\hat{z}} = \tilde{A}\hat{z} + \tilde{B}u + L(y - (\tilde{C}\hat{z} + \tilde{D}u))$$

such that $(\tilde{A} - L\tilde{C})$ is a Hurwitz matrix.

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such that $(\tilde{A} - L\tilde{C})$ is a Hurwitz matrix.

Remarks

- The state \tilde{x} is a filtered version of z
- If the filter $F(s)$ is properly chosen, then \tilde{x} contains less noise than z

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Convergence of the (\tilde{A}, \tilde{C}) -estimates

Assume that M^0 is the real value of M , then we get the error $e_M = (M - M^0)$ dynamic equation as

$$\dot{e}_M = -e_M \frac{\varphi\varphi^\top}{r^2} P$$

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Definition (persistence of excitation)

A locally integrable signal $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}^d$ is said to be Persistently Exciting (PE) if there exist constant numbers $\alpha_1, \alpha_2, T > 0$ and $t_0 \geq 0$ such that

$$\alpha_1 I \preceq \int_t^{t+T} \varphi(\tau) \varphi(\tau)^\top d\tau \preceq \alpha_2 I \quad \forall t \geq t_0.$$

Here the notation $S \preceq R$ is used to mean that $R - S$ is positive semi-definite.

Convergence of the (\tilde{A}, \tilde{C}) -estimates

Definition (sufficiency of richness)

A smooth signal $u : \mathbb{R}_+ \rightarrow \mathbb{R}^{n_u}$ is said to be Sufficiently Rich (SR) of order k if

- $u^{(i)}(0) = 0$ for $i = 0, \dots, k-1$ and
- for any $\gamma > 0$, the signal $\bar{u}_k \in \mathbb{R}^{kn_u}$ defined by

$$\bar{u}_k = \frac{1}{(s + \gamma)^k} \begin{bmatrix} u^\top & \dot{u}^\top & \dots & (u^{(k-1)})^\top \end{bmatrix}^\top$$

is PE.

Convergence of the (\tilde{A}, \tilde{C}) -estimates

Proposition

Assume that

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Proposition

Assume that

- The matrices \tilde{A} and Λ_c (the state matrix of filter $F(s)$) are Hurwitz
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- The input u is SR of order $f + m$, where $m = \deg(m_{\tilde{A}}(\lambda))$

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then the signal φ defined as

$$\varphi(t) = \begin{bmatrix} Hp \\ q \end{bmatrix}.$$

is PE.

Convergence of the (\tilde{A}, \tilde{C}) -estimates

Theorem

Assume that

- The system \tilde{S} is stable $\rightarrow \tilde{A}$ is Hurwitz.
- $(\tilde{A}, \tilde{B}, \tilde{C})$ is minimal $\rightarrow (\tilde{A}, \tilde{B})$ is controllable and (\tilde{A}, \tilde{C}) is observable.
- The input u is SR of order $f + m$

then M converges exponentially to M^o .

Convergence of the (\tilde{B}, \tilde{D}) -estimates

Assume that θ^0 is the real value of θ , then we get the error $e_\theta = (\theta - \theta^0)$ dynamic equation as

$$\dot{e}_\theta = -\frac{Q\Phi^\top\Phi}{\rho^2}e_\theta + v$$

with

$$v = \frac{Q\Phi^\top}{\rho^2} \left[\tilde{C}^o e^{\tilde{A}^o t} \tilde{x}_0 + (\Phi^o - \Phi) \theta^o \right].$$

Convergence of the (\tilde{B}, \tilde{D}) -estimates

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with

$$v = \frac{Q\Phi^\top}{\rho^2} \left[\tilde{C}^\circ e^{\tilde{A}^\circ t} \tilde{x}_0 + (\Phi^\circ - \Phi)\theta^\circ \right].$$

Remark

If we can prove that

- $v(t)$ vanishes as $t \rightarrow \infty$.
- the homogeneous part is exponentially stable.

then, we can conclude on the convergence of the estimation error $e_\theta \rightarrow 0$.

Convergence of the (\tilde{B}, \tilde{D}) -estimates

Question:

$v(t)$ vanishes as $t \rightarrow \infty$?

Convergence of the (\tilde{B}, \tilde{D}) -estimates

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Answer:

$$v(t) = \frac{Q\Phi^\top}{\rho^2} \left[\tilde{C}^\circ e^{\tilde{A}^\circ t} \tilde{x}_0 + (\Phi^\circ - \Phi) \theta^\circ \right].$$

When (\tilde{A}, \tilde{C}) converge to the true values $(\tilde{A}^\circ, \tilde{C}^\circ) \Rightarrow$
 $\lim_{t \rightarrow \infty} \tilde{C}^\circ e^{\tilde{A}^\circ t} \tilde{x}_0 = 0$ and $\lim_{t \rightarrow \infty} \Phi = \Phi^\circ \Rightarrow \lim_{t \rightarrow \infty} v(t) = 0.$

Convergence of the (\tilde{B}, \tilde{D}) -estimates

Lemma

Assume that

- The system \tilde{S} is stable $\rightarrow \tilde{A}$ is Hurwitz
- $(\tilde{A}, \tilde{B}, \tilde{C})$ is minimal
- u is SR of order at least $f + m$
- the rate of convergence defined as the largest $c > 0$ such that

$$\sup_{k \in \mathbb{N}} \left\| \tilde{C}(t) \tilde{A}(t)^k - \tilde{C}^{\circ} (\tilde{A}^{\circ})^k \right\|_2 \leq \lambda e^{-ct},$$

then $\Phi = \Phi(t, \tilde{A}(t), \tilde{C}(t))$ is PE.

If Φ is PE, then the homogeneous part of $\dot{e}_{\theta} = -\frac{Q\Phi^{\top}\Phi}{\rho^2}e_{\theta} + v$ is exponentially stable.

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Fourth order MIMO system

Consider the MIMO linear system with matrices:

$$A = \begin{bmatrix} -1.4 & 0.3 & -0.6 & -0.3 \\ 0.3 & -0.9 & -0.9 & 0.1 \\ -0.6 & -0.9 & -2.6 & -1.0 \\ -0.3 & 0.1 & -1.0 & -1.8 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -0.4 & 1.3 \\ 0.6 & 0.2 \\ 0 & 0.6 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.9 & 0 & 0 & 1.3 \\ -1.1 & -0.8 & +0.9 & +0.4 \end{bmatrix}, D = \begin{bmatrix} 0.8 & -1.5 \\ 0 & 0 \end{bmatrix}.$$

We estimate a system $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ similar to the system (A, B, C, D) .

Time performances

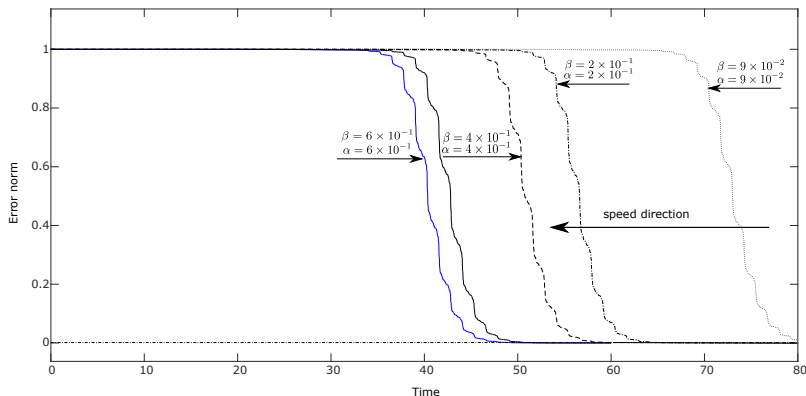


Figure: The relative error $\varepsilon_r = \frac{\|\tilde{\mathcal{O}}_f^0 \tilde{\mathcal{B}}^0 - \tilde{\mathcal{O}}_f \tilde{\mathcal{B}}\|_F}{\|\tilde{\mathcal{O}}_f^0 \tilde{\mathcal{B}}^0\|_F}$ time evolution

Frequency performances

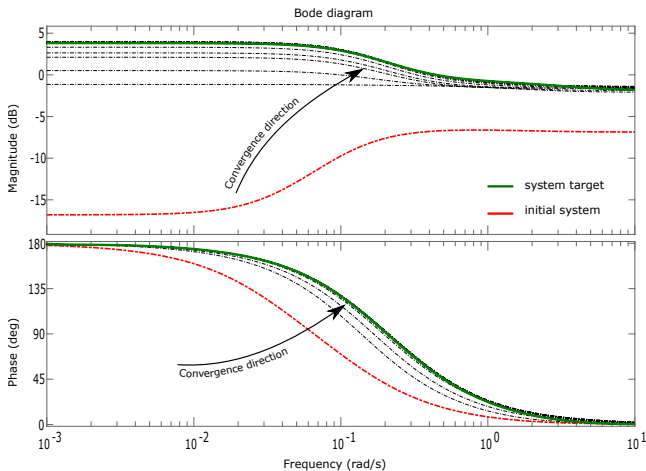


Figure: Bode diagram of output1 / input 1 transfer function convergence

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Conclusion

The main challenges with such a problem are

- Estimate time derivatives of input-output signals.
- Provide a cheap update process for the system matrices while the state is unknown.
- Analyse convergence of the whole algorithm.

Conclusion

The main challenges with such a problem are

- Estimate time derivatives of input-output signals.
- Provide a cheap update process for the system matrices while the state is unknown.
- Analyse convergence of the whole algorithm.

Future work

- Study of the algorithm robustness to noise.
- Search for the relation between choice of H and robustness.
- Improvement concerning the filtering of the input-output signals.

THANK YOU