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# An explicit optimal input design for first order systems identification

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Abstract: This paper focuses on the problem of closed loop online identification of the time constant in the single input single output (SISO) first order linear model. A new explicit approach for the simultaneous online optimal experiment design (OED) and model parameter identification is presented. Based on the observation theory and a model based predictive control (MPC) algorithm, this approach aims to solve an optimal control problem where input and output constraints may be specified. This constrained control objective aims to maximize the sensitivity of the model output with respect to the unknown model parameter (the time constant). The control law is derived explicitly offline and simple to be implemented: the input may be computed fast online while the unknown model time constant is estimated at the same time.

*Keywords:* Optimal experiment design, input design, identification, linear systems, observers, predictive control.

# 1. INTRODUCTION

When using dynamic models for simulation, control or optimization, all model parameters need to be numerically known. Among the dynamic models, the first order linear model is the simplest model: it is based on a static gain and a time constant. Methods to estimate the value of the parameters of a considered model concern the identification task (see Ljung (1999)), which relies on experimental data. OED may be useful if no data exist for identification, since it aims to design new experimental data (see Franceschini and Macchietto (2008)) which are often used offline for the identification procedure, hence decoupling these two procedures.

This paper focuses on the coupled online OED and parameter estimation for such a first order linear model, where the time constant (that may vary during the time) has to be estimated. For such estimation problem, one may first think to apply a step input. But, this input in not sufficiently persistent (i.e., when the steady is reached in that case after some time, the output does not contain enough information on the dynamic, and the time constant can not be estimated). Pseudo-random binary sequence (PRBS) is a better open loop approach, since it features a rich input signal. However, open loop control does not allow to maintain the process output is a prescribed region, for example if the process has still to be used in the meantime for production. Therefore, an input design for identification based on closed loop control is required. In Shouche et al. (2002), the authors developed a constrained MPC based on an auto regressive model with external inputs (ARX) with a persistent input constraint, where the parameters are estimated online by regression. This led to a non-convex simultaneous online identification and control problem.

The new method developed here for the SISO first order linear model is a particular development of a general approach (Qian et al. (2013, 2014)) developed for multivariable nonlinear models. In these papers, an approach combining the closed loop online parameter identification with OED has been proposed for the input design. Based on a MPC, it aims to compute online the optimal input that maximize a norm of the sensitivities of the process outputs with respect to the unknown model parameters, which are estimated at the same time by an observer. The predictive controller handles input and output constraints while its cost function is based on the sensitivity criterion. Here, based on this approach, for the SISO first order linear model, the problem is to design a controller and an estimator that both allow to estimate the unknown time constant (that may varies) and to maintain the process in a prescribed region. Based on the linearity of the model and assuming that the argument of the MPC is a single value (hence, the control horizon is tuned to one), an explicit controller can be derived offline, which avoid to solve any online optimization task.

This paper is structured as follows: Section 2 deals with some recalls on the previous general approach of Qian et al. (2013, 2014) developed for multivariable nonlinear models. Section 3 contains the proposed algorithm for the

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constrained optimal closed loop online identification of the time constant in a SISO first order linear model. In section 4, the simulation results demonstrate the efficiency of this approach.

### 2. PRELIMINARIES

In Qian et al. (2013, 2014), the algorithm features the following steps:

# Offline synthesis

- define a dynamic (nonlinear or linear) non autonomous system modelled by ordinary differential equations. This model contains known and unknown model parameters.
- define the sensitivity dynamic model of the model state with respect to the unknown model parameters.
- define an augmented dynamic model, where the state contains the model state and the unknown model parameters. It is used to define an observer that aims to estimate the unknown model parameters and (if possible) the model state.
- define a nonlinear MPC constrained optimization problem based on the maximization of the norm of the sensitivity dynamic model state to design a persistent input.

#### Online computations

- at each current time  $t_k$ , get the measure of the process output  $y_p(t_k)$
- then, based on the process input applied  $u(t_k)$  and  $y_p(t_k)$ , integrate the observer to get the current estimate of the unknown parameters
- then, based on the measure  $y_p(t_k)$ , the parameter estimations, the nonlinear MPC constrained optimization problem is solved since, in a general case, and due to the nonlinearity of the problem, the search for the optimal control  $u^{max}(t_k)$  to be applied at each sample time requires the online integration of the 2 prediction models used with the nonlinear MPC resolution.
- then,  $u^{max}(t_k)$  is applied from  $t_k + \epsilon$  to  $t_{k+1}$ , where  $\epsilon$  is the time necessary for the whole loop (i.e, take the measure, integrate the observer, solve the constrained optimization problem). Hence, the main cost in time is related to the resolution of the constrained optimization problem involving model integrations, which might be important compared to the sampling time.

# 3. NEW ALGORITHM

#### 3.1 Main ideas

For the previous approach, let us now consider the simplest SISO case: the well known first order model where  $x \in \mathbb{R}$  is the measured state, u is the measured input, with one unknown parameter (the time constant T > 0), the known static gain  $G \in \mathbb{R}^*$ ,  $y_p(t_k)$  is the measured process output at the current time  $t_k$  considered as the initial time:

$$\begin{cases} \dot{x}(t) = \frac{-1}{T}x(t) + \frac{G}{T}u(t), \ t > t_k \\ x(t = t_k) = x(t_k) = y_p(t_k) \end{cases}$$
(1)

In that case, the whole online numerical resolution may be simplified to get an explicit controller derived offline, hence reducing the online computational burden.

*Offline synthesis* The control law may be designed offline as follows:

- define the sensitivity dynamic model, in the time domain.
- define an augmented dynamic model, where the state contains the model state and the unknown time constant. It is used to define an observer that aims to estimate the unknown time constant.
- get the predicted step response of the sensitivity model into the prediction horizon  $N_p$ . It is based on the Laplace transform applied to the process model and to the sensitivity model: hence, the formula is parametrized for any initial conditions of the model state  $x(t_k)$  and the sensitivity model state  $x_s(t_k)$
- due to the convexity of the constrained quadratic problem, get the worst control law  $u_{min}(t_k)$  (that minimize the MPC cost function) and therefore the persistent control law  $u^{max}(t_k)$  (that maximize the MPC cost function as requested by the method) to be applied at each time  $t_k$ .

#### Online computations

- at each current time  $t_k$ , get the process measure  $y_p(t_k)$
- then, based on the process input applied  $u(t_k)$  and  $y_p(t_k)$ , integrate the observer to get the current estimate of the time constant  $\hat{T}(t_k)$
- then, based on the measure  $y_p(t_k)$  and the estimation  $\hat{T}(t_k)$ , the optimal control to apply  $u(t_k) = u^{max}(t_k)$  (constrained in magnitude) is computed, simply obtained from the evaluation of the explicit control law defined offline based on  $u_{min}(t_k)$  and the input bounds.
- then,  $u^{max}(t_k)$  is applied from  $t_k + \epsilon$  to  $t_{k+1}$ , where  $\epsilon$  is the time necessary for the whole loop (i.e, take the measure, integrate the observer, compute  $u^{max}(t_k)$ ). Hence, the main cost in time is the observer integration: it is low in that case compared to the sampling time (if it is larger than the order of the ms). Therefore, the proposed algorithm for the SISO first order model is a very fast control algorithm.

Following the existing general method given in Qian et al. (2013, 2014), the steps of the main ideas for this new input design are detailed in the following parts.

#### 3.2 Sensitivity model

Using the considered SISO model (1), the sensitivity model state  $x_s = \frac{\partial x}{\partial T}$  is given by:

$$\begin{cases} \dot{x}_s(t) = \frac{1}{T^2} x(t) - \frac{1}{T} x_s(t) - \frac{G}{T^2} u(t), \ t > t_k \\ x_s(t = t_k) = x_s(t_k) \\ x_s(t = 0) = 0 \end{cases}$$
(2)

# 3.3 Observer design

From the first order model (1), the augmented state is  $x_a(t) = [x(t) \frac{1}{T}]^T$  in the augmented model:

$$\begin{cases} \dot{x}_a(t) = A_a(y_p(t), u(t))x_a(t), \ t > t_k \\ \hat{y}(t) = C_a x_a(t), \ t > t_k \end{cases}$$
(3)

where:

$$A_a(y_p(t), u(t)) = \begin{pmatrix} 0 & -y_p(t) + Gu(t) \\ 0 & 0 \end{pmatrix} C_a = \begin{bmatrix} 1 & 0 \end{bmatrix} (4)$$

Remark 1. The system (3-4) is observable if  $(u(t), y_p(t))$  is a persistent input (Besançon (2007)).

Different observers based on the augmented model (3-4) may be used to estimate the unknown model parameter T (Besançon (2007)). A possible observer, which state is  $\hat{x}_a(t) = [\hat{x}(t) \frac{1}{\hat{T}(t)}]^T$ , may be written here as an extended Kalman filter (EKF):

$$\begin{aligned}
\dot{x}_{a}(t) &= A_{a}(y_{p}(t), u(t))\hat{x}_{a}(t) - \rho_{o}S_{o}^{-1}C_{a}^{T}(C_{a}\hat{x}_{a}(t) - y_{p}(t)), \ t > \\
\dot{S}_{o}(t) &= -\theta_{o}S_{o}(t) - A_{a}^{T}(t)S_{o}(t) - S_{o}(t)A_{a}(t) + \rho_{o}C_{a}^{T}C_{a}, \ t > t_{k} \\
\hat{x}_{a}(t = t_{k}) &= \hat{x}_{a}(t_{k}) = [y_{p}(t_{k}) \ \frac{1}{\hat{T}(t_{k})}] \\
S_{o}(t = t_{k}) &= S_{o}(t_{k})
\end{aligned}$$
(5)

where  $\rho_o > 1$ ,  $\theta_o > 0$  and  $S_o(t = 0)$  (a symmetric positive definite matrix) are the observer tuning parameters.

### 3.4 Sensitivity model step response

In the MPC framework, the optimization procedure aims to find the optimal  $u(t_k)$  to be applied during the next sample time through the maximization of the norm of the sensitivity dynamic model state. This requires here to evaluate  $x_s(t)$  into the prediction horizon  $N_p$ . Starting from the current time  $t = t_k$ , let us apply a step  $u(t_k)$ for the process input in the MPC procedure. We use the Laplace transform formula, for a function f(t) starting from any initial condition  $f(t_k)$  at any initial time  $t_k$ , is:

$$\begin{cases} L(\frac{df}{dt}(t)) = sL(f(t)) - f(t_k), \ t > t_k \\ \iff L(\frac{df}{dt}(t)) = sF(s) - f(t_k), \ t > t_k \end{cases}$$
(6)

Therefore, applying the Laplace transform to the system (1-2), at the current time  $t_k$  leads to:

$$X_s(s) = \frac{x_s(t_k)}{s + \frac{1}{T}} + \frac{(y_p(t_k) - GsU(s))}{T^2(s + \frac{1}{T})^2}, \ t > t_k.$$
 (7)

where  $X_s(s)$  is the Laplace transform of  $x_s(t)$ . Based on the Laplace transform of the input step function fed into (7), the response of the sensitivity model over the prediction horizon  $N_p$  is (in the Laplace domain):

$$X_s(s) = \frac{x_s(t_k)}{s + \frac{1}{T}} + \frac{(y_p(t_k) - Gu(t_k))}{T^2(s + \frac{1}{T})^2}, \ t > t_k$$
(8)

Hence, the predicted sensitivity response into the future requires the current process measure  $y_p(t_k)$ , the unknown model time constant T and of course the input value  $u(t_k)$ . Based on the usual inverse Laplace transform formula, the step response  $x_s(t)$  into the prediction horizon  $N_p$  may be written in the time domain, where the unknown parameter T is replaced by the last current estimation from the observer at  $t_k$   $(\hat{T}(t_k))$ :

$$x_{s}(t) = \dots \left( x_{s}(t_{k}) + \frac{(y_{p}(t_{k}) - Gu(t_{k}))(t - t_{k})}{\hat{T}(t_{k})^{2}} \right) e^{\frac{-(t - t_{k})}{\hat{T}(t_{k})}}, t > t_{k}$$
(9)

From (9), let us notice two well known particular cases for the first order system which have to be avoided to be able to estimate the time constant:

- (1) In open loop, starting from any initial condition  $(y_p(0), x_s(0) = 0)$ , if u(t) is kept constant, after a certain time, the sensitivity tends to 0.
- (2) Moreover, in the previous case, if the initial condition is a steady state (hence  $y_p(t) - Gu(t) = 0$ ,  $\forall t \ge 0$  and since  $x_s(0) = 0$ ), then the sensitivity is  $0 \forall t$ .

#### 3.5 Constrained optimal input design

In the present case, the MPC formulation of Qian et al. (2013, 2014) is written as:

$$\begin{cases} \max_{u_{inf} \le u(t_k) \le u^{sup}} J(u(t_k)) = \int_{t_k}^{t_{k+N_p}} (x_s(t))^2 dt & (10) \\ \text{where } x_s(t) \text{ is obtained from } (9) \end{cases}$$

where the control bounds  $u_{inf}$  and  $u^{sup}$  are adjusted offline according to the prescribed constraints on u and  $y_p$ :

$$\begin{cases}
 u_{inf,u} \leq u \leq u^{sup,u}, \ \forall t > 0 \\
 y_{inf} \leq y_p \leq y^{sup}, \ \forall t > 0
\end{cases}$$
(11)

In the following, if there is no such input constraints,  $u_{inf,u}$  is set to  $-\infty$  and  $u^{sup,u}$  is set to  $+\infty$ . If there is no such output constraints,  $y_{inf}$  is set to  $-\infty$  and  $y^{sup}$  is set to  $+\infty$ . In there is neither such input constraints nor output constraints, the problem is not well defined.

Since any step response of a first order system is monotonous and takes its maximum (or minimum) value at the steady state, and taking account of the sign of the static gain (it may be positive or negative) then the control bounds  $u_{inf}$  and  $u^{sup}$  in (10) may be simply tuned offline from (11) and the known static gain G:

$$\begin{cases}
 u^{sup,y} = max(\frac{y_{inf}}{G}, \frac{y^{sup}}{G}) \\
 u^{sup} = min(u^{sup,y}, u^{sup,u}) \\
 u_{inf,y} = min(\frac{y_{inf}}{G}, \frac{y^{sup}}{G}) \\
 u_{inf} = max(u_{inf,y}, u_{inf,u})
\end{cases}$$
(12)

Due to the input to output linearity of the sensitivity model, the sensitivity involved in the integration in (10) is quadratic in its optimization argument  $u(t_k)$ :

$$\begin{aligned}
& \max_{u_{inf} \leq u(t_k) \leq u^{sup}} J(u(t_k)) = \\
& \int_{t_k}^{t_{k+N_p}} f_2(t)u(t_k)^2 + f_1(t)u(t_k) + f_0(t)dt \\
& \text{where:} \\
& f_2(t) = e^{\frac{-2(t-t_k)}{\hat{T}(t_k)}} \left( \frac{G^2}{\hat{T}(t_k)^4} (t-t_k)^2 \right) \in \mathbb{R}^+ \\
& f_1(t) = e^{\frac{-(t-t_k)}{\hat{T}(t_k)}} \dots \\
& \left( -2(x_s(t_k) + \frac{y_p(t_k)}{\hat{T}(t_k)^2} (t-t_k)) \frac{G}{\hat{T}(t_k)^2} (t-t_k) e^{\frac{-(t-t_k)}{\hat{T}(t_k)}} \right) \\
& f_0(t) = e^{\frac{-(t-t_k)}{\hat{T}(t_k)}} \left( x_s(t_k) + \frac{y_p(t_k)}{\hat{T}(t_k)^2} (t-t_k) e^{\frac{-(t-t_k)}{\hat{T}(t_k)}} \right) \end{aligned}$$
(13)

Due to the positive sign of  $f_2(t)$ , the problem (13) is convex and may be rewritten showing characteristics of this parabola:

$$\begin{cases} \max_{u_{inf} \le u(t_k) \le u^{sup}} J(u(t_k)) = \dots \\ \dots C_{\infty}(t_k)(u(t_k) - u_{min}(t_k))^2 + J_{min}(t_k) \end{cases}$$
(14)

where  $c_{\infty}(t_k)$  is a positive real (obtained from the integration of  $f_2(t)$  in (13)), and where  $u_{min}(t_k)$  is the worst control  $u(t_k)$  to apply, since it minimizes at  $t_k$  the criteria J(.) (where  $J(.) = J_{min}(t_k) \ge 0$ , since J is the  $L^2$  norm of  $x_s$ ). Hence,  $u_{min}(t_k)$  has to be determined through the first order optimality condition:

$$\frac{\partial J}{\partial u(t_k)} = 0 \text{ at } u(t_k) = u_{min}(t_k) \tag{15}$$

which leads to:

$$u_{min}(t_k) = \frac{-\int_{t_k}^{t_{k+N_p}} f_1(t)dt}{2\int_{t_k}^{t_{k+N_p}} f_2(t)dt}$$
(16)

This leads to determine  $u_{min}(t_k)$  by the integration of the two dynamic models, starting at each  $t_k$  from the two initial conditions  $y_p(t_k)$  and  $x_s(t_k)$  and with the estimation  $\hat{T}(t_k)$ :

$$\begin{cases} u_{min}(t_k) = \frac{1}{G} y_p(t_k) \dots \\ \dots + \frac{\hat{T}(t_k)^2}{G} \frac{\int_{t_k}^{t_{k+N_p}} (t - t_k) e^{\frac{-2(t - t_k)}{\hat{T}(t_k)}} dt}{\int_{t_k}^{t_{k+N_p}} (t - t_k)^2 e^{\frac{-2(t - t_k)}{\hat{T}(t_k)}} dt} x_s(t_k) \end{cases}$$
(17)

By pursuing the integration and replacing  $t_{k+N_p}-t_k$  by the product  $N_pT_s$  (where  $T_s$  is the constant sampling time), this leads to:

$$\begin{pmatrix}
 u_{min}(t_k) = \frac{1}{G} \left( \hat{T}(t_k)^2 \gamma(.) x_s(t_k) + y_p(t_k) \right) \\
 \text{with} \\
 \gamma(N_p, T_s, \hat{T}(t_k), \nu(N_p, T_s, \hat{T}(t_k))) = \\
 \frac{2N_p T_s \nu(.) + \hat{T}(t_k)(\nu(.) - 1)}{2(N_p T_s)^2 \nu(.) + 2\hat{T}(t_k)N_p T_s \nu(.) + \hat{T}(t_k)^2(\nu(.) - 1)} \\
 \text{and} \\
 \nu(N_p, T_s, \hat{T}(t_k)) = e^{\frac{-2N_p T_s}{\hat{T}(t_k)}}$$
(18)

Due to the symmetry property of such a parabola with respect to the vertical line, which is passing through  $(u_{min}(t_k), J_{min}(t_k))$ , one may conclude with the optimal control law for  $u^{max}(t_k)$ :

$$u^{max}(t_k) = \begin{cases} u_{inf} \text{ if } u_{min}(t_k) > u_{inf} + \frac{u^{sup} - u_{inf}}{2} \\ u^{sup} \text{ otherwise} \\ \text{where:} \\ u^{sup} \text{ and } u_{inf} \text{ are defined in (12)} \\ \text{and } u_{min}(t_k) \text{ is defined in (18)} \end{cases}$$
(19)

Therefore, the optimal control law  $u^{max}(t_k)$  may be explicitly defined offline and is a bang bang control which value is evaluated online with the process measure and the time constant estimation, and is either the minimum of the maximum input value allowed. In as to be noted that the proposed solution is derived using the first order condition, and the constrained are enforced later resulting in bangbang control. In a general framework, this approach is not necessarily optimal. But here, since the first order condition condition is used to find the worst control (and not the best one), the optimal control is really found (and not a suboptimal solution).

#### 4. SIMULATION RESULTS

Simulations allow to compare 2 controller performances:

- case OL: an open loop PRBS input,
- case CL: the optimal closed loop controller with input constraints and output constraints based on the algorithm (19).

The time constant  $T_p$  of the simulated process is assumed to vary in 9 periods, between 3 values (50, 80 and 100s) as step functions, with 3 different length for each step (1000, 500 and 250s). Runs are done with Matlab<sup>1</sup> under the following conditions (table 1):

Table 1. Simulation conditions

	Parameter	Value (unit)	
Time	$T_{final}$	5250~(s)	
	$T_s$	1 (s)	
Model and Process	G	10 (-)	
	$y_p(0)$	0 (-)	
	$T_p(0)$	80~(s)	
Observer	$\rho_o$	1.02 (-)	
	$\theta_o$	0.05 (-)	
	$S_o$	$\begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} (-)$	
	$\hat{T}(0)$	20~(s)	
Controller	$[u_{inf,u}, u^{sup,u}]$	[0, 1]	
	$[u_{inf}, u^{sup}]$	[0, 4]	

After some trial-errors, the prediction horizon  $N_p$  is tuned at each  $t_k$  according to:

$$N_p(t_k)T_s = \hat{T}(t_k)$$

Simulations results are summarized in table 2: they allow to see that for the 2 cases, in spite of a 300 % initial error given by the initial condition of the time constant fed into the observer, the estimation of the time varying process time constant is done with a good accuracy for each step. But the estimations is better in the closed

<sup>&</sup>lt;sup>1</sup> www.mathworks.com



Fig. 1. Time constant: time varying target and estimation (with closed loop control with constraints on u and y)



Fig. 2. Time constant: time varying target and estimation (with open loop control with constraints on u)

loop (mean error over  $\begin{bmatrix} \frac{T_{final}}{100}, T_{final} \end{bmatrix}$  is 3.6%, see Fig. 1) rather than 4.62% in open loop where it is more noisy (Fig. 2). Let us notice that in open loop, at the beginning, a very large estimation error occurs, which is not the case in closed loop (the short period where it occurs is not in  $\left[\frac{T_{final}}{100}, T_{final}\right]$ ). The drawback of the OL is that the output is uncontrolled (ranging between 0 and 6.14 (Fig. 3)). If such output values are not acceptable, it is preferable to use the proposed closed loop control algorithm CL with both input constraints and output constraints (Fig. 4) where the output ranges between 0.05 and 3.65 (hence always within the bounds of the output constraints). Such output constraints impact  $u_{inf}$  and  $u^{sup}$ and hence the control values  $u_{min}(t)$  and  $u^{max}(t)$  applied (see Fig. 5). The sensitivity maximized in CL (see Fig. 6) is 3 times higher in average than in OL (see Fig. 7).

Table 2. Results analysis

Criteria	Input	
	OL	CL
Maximum value taken by $y_p$	6.14	3.655
Minimum value taken by $y_p$	0.0	0.050
Mean sensitivity over		
$\left[\frac{T_{final}}{100}, T_{final}\right] \left(\frac{1}{s}\right)$	0.004	0.011
Mean time constant estimation error		
over $\left[\frac{T_{final}}{100}, T_{final}\right]$ (%)	4.62	3.60



Fig. 3. Output (with open loop control with constraints on u)



Fig. 4. Output (with closed loop control with constraints on u and y)



Fig. 5. Input (with closed loop control with constraints on u and y), top:  $u^{max}$ , bottom:  $u_{min}$ 

#### 5. CONCLUSIONS

In this paper, an explicit bang bang controller has been proposed for the estimation of the time constant in a first order linear model. This input design relies on an observer (solved online with the input and output measures) and a constrained MPC problem, which exact optimal solution is derived explicitly offline. Hence, online computational cost is very low since it concerns mostly the observer integration. Perspectives are concerned with the optimal tuning of the control horizon and the robustness of this approach with respect to output noise and model uncertainties.

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Fig. 6. Parameter sensitivity (with closed loop control with constraints on u and y)



Fig. 7. Parameter sensitivity (with open loop control with constraints on u)

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