

**This document must be cited according
to its final version which is published in a journal as:**

**C. Afri, M. Nadri, P. Dufour,
“Observer design for a ternary distillation column with side stream”,
53rd IEEE Conference on Decision and Control (CDC),
Los Angeles, CA, USA, pp. 6383-6388, december 15-17, 2014.
[DOI : 10.1109/CDC.2014.7040390](https://doi.org/10.1109/CDC.2014.7040390)**

**You downloaded this document from the
CNRS open archives server, on the webpages of Pascal Dufour:
<http://hal.archives-ouvertes.fr/DUFOUR-PASCAL-C-3926-2008>**

Observer Design for a Ternary Distillation Column with Side Stream

Chouaib Afri Madiha Nadri Pascal Dufour

LAGEP

Université de Lyon, F-69622, Lyon, France – Université Lyon 1, Villeurbanne, France – CNRS,
UMR 5007, LAGEP, France.

53rd Annual Conference on Decision and Control
Paper 1058

Problem statement

Goal

- Improving purity control in distillation columns.
- Optimizing installation costs and energy.

Problem statement

Goal

- Improving purity control in distillation columns.
- Optimizing installation costs and energy.

Constraints

- High cost of the measures.
- Nonlinear coupling between controlled physical quantities.

Problem statement

Goal

- Improving purity control in distillation columns.
- Optimizing installation costs and energy.

Constraints

- High cost of the measures.
- Nonlinear coupling between controlled physical quantities.

Existing controllers

- Decentralised PID controllers

Problem statement

Goal

- Improving purity control in distillation columns.
- Optimizing installation costs and energy.

Constraints

- High cost of the measures.
- Nonlinear coupling between controlled physical quantities.

Existing controllers

- Decentralised PID controllers
 - Based on a simple model \Rightarrow Poor control performances.
 - High energy consumption.

Problem statement

Goal

- Improving purity control in distillation columns.
- Optimizing installation costs and energy.

Constraints

- High cost of the measures.
- Nonlinear coupling between controlled physical quantities.

Existing controllers

- Decentralised PID controllers
 - Based on a simple model \Rightarrow Poor control performances.
 - High energy consumption.

Solution

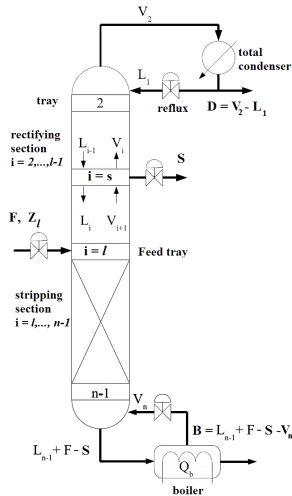
Need for and advanced controller using a model based state estimation.

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results
- 7 Conclusion and perspectives

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results
- 7 Conclusion and perspectives



n Number of trays (18 tray).

i Tray index $i=1, \dots, n$.

l Feed Tray.

s Extraction tray.

Z Feed flow molar fraction.

F Molar feed rate.

S Extracted molar flow rate.

L Reflux molar flow rate.

V Vapour molar flow rate.

FIGURE: Ternary distillation column with side stream.

Modelling assumptions

Classical assumptions¹

- Constant column internal pressure.
- Constant molar retention.
- L_i 's are equal, V_i 's are equal, for all $i = 1, \dots, n$.
- Trays walls are adiabatic.
- Liquid and outgoing vapour of each tray are in thermal equilibrium.
- Homogeneous mixture in each tray.
- Feed flow rate is continuous and in the liquid phase.
- We assume a theoretical boiler and total condenser.

Special assumption

- Assume a non-ideal mixture².

-
1. Leads to solving an ODE system
 2. Leads to solving an ADE system

Outline

- 1 Ternary distillation column
- 2 **Dynamical model**
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results
- 7 Conclusion and perspectives

Material balance equations

Total condenser, $i = 1$:

$$N_1 \frac{dx_{j,1}}{dt} = V_f(y_{j,2} - x_{j,1})$$

Upper rectifying section, $i = 2, \dots, s$:

$$N_i \frac{dx_{j,i}}{dt} = L(x_{j,i-1} - x_{j,i}) + V_f(y_{j,i+1} - y_{j,i})$$

Lower rectifying section, $i = s+1, \dots, l-1$:

$$N_i \frac{dx_{j,i}}{dt} = (L - S)(x_{j,i-1} - x_{j,i}) + V_f(y_{j,i+1} - y_{j,i})$$

Feed tray, $i = l$:

$$N_l \frac{dx_{j,l}}{dt} = (L - S)(x_{j,l-1} - x_{j,l}) + F(Z_{j,l} - x_{j,l}) + V_f(y_{j,l+1} - y_{j,l})$$

Stripping section, $i = l+1, \dots, n-1$:

$$N_i \frac{dx_{j,i}}{dt} = (F + L - S)(x_{j,i-1} - x_{j,i}) + V_f(y_{j,i+1} - y_{j,i})$$

Boiler, $i = n$:

$$N_n \frac{dx_{j,n}}{dt} = (F + L - S)(x_{j,n-1} - x_{j,n}) + V_f(x_{j,n} - y_{j,n})$$

Thermodynamic constraints, $i = 1, \dots, n$:

$$(\sum_{j=1}^3 y_{j,i}) - 1 = 0$$

j Component index, $j=1$ for Benzene, $j=2$ for Toluene, $j=3$ for o-Xylene.

N Molar retention.

x Liquid phase molar fraction.

y Vapour phase molar fraction.

Vapour phase molar fractions

Remark

The correlation between physical quantities from

- Antoine formula.
- Vapour Liquid Equilibrium (VLE).
- Murphree efficiency.

allowed to get the vapour molar fractions $y_{j,i}$ functions of liquid molar fractions $x_{j,i}$.

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results**
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results
- 7 Conclusion and perspectives

Trays number	$n = 18,$
Side stream and feed trays	$s = 6, l = 9$
Liquid molar retention in condenser	$N_1 = 20 \text{ mol}$
Liquid molar retention in boiler	$N_{18} = 20 \text{ mol}$
Liquid molar retention in each tray	$N_i = 8 \text{ mol}$
Liquid and vapour Murphree efficiency	$e^l = 1, e^v = 1$
Internal pressure	$P_T = 760 \text{ mmHg}$
Feed flow rate	$F = 1.67 \text{ mol/min}$
Benzene composition in feed rate	$Z_{Ben,l} = 0.6$
Toluene composition in feed rate	$Z_{Tol,l} = 0.3$
Feed temperature	$T_l = 379,32 \text{ K}$
Side stream liquid flow rate	$S = 0.167 \text{ mol/min}$
Steady state vapour flow rate	$V_f = 7.0086 \text{ mol/min}$
Steady state liquid flow rate	$L = 6.0017 \text{ mol/min}$
Steady state quality of Benzene in top product	$x_{1,1} = 0.95$
Steady state quality of Benzene in bottom product	$x_{1,18} = 8.87 \times 10^{-5}$
Steady state quality of Toluene in top product	$x_{2,1} = 0.05$
Steady state quality of Toluene in bottom product	$x_{2,18} = 0.67$

TABLE: Operating point and initial steady state.

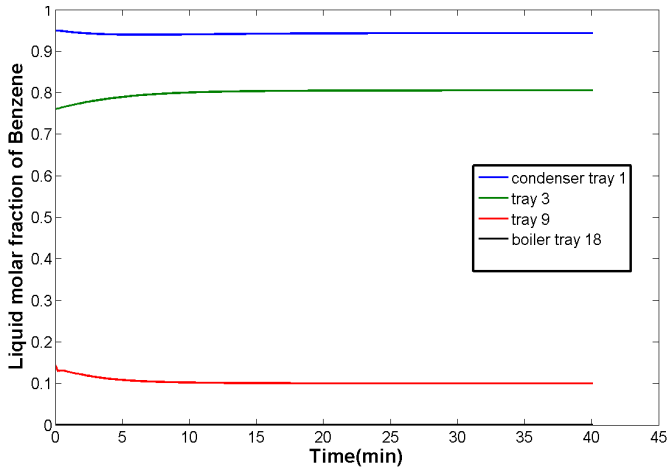


FIGURE: Response of the dynamic model starting from the experimental steady state, in 4 locations : liquid molar fractions of Benzene.

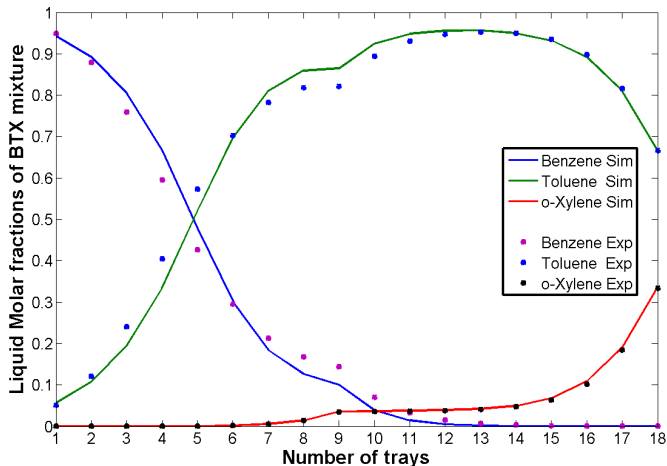


FIGURE: Final molar fraction profiles : simulation and experimental.

Advanced controller problem

In addition to the output measures, advanced controllers require more informations about plan.

Advanced controller problem

In addition to the output measures, advanced controllers require more informations about plan.

In our case

- Column internal liquid molar fractions.
- Column internal temperatures.

are important to design such controllers.

Advanced controller problem

In addition to the output measures, advanced controllers require more informations about plan.

In our case

- Column internal liquid molar fractions.
- Column internal temperatures.

are important to design such controllers.

But It is very expensive to measure all this physical quantities.

Advanced controller problem

In addition to the output measures, advanced controllers require more informations about plan.

In our case

- Column internal liquid molar fractions.
- Column internal temperatures.

are important to design such controllers.

But It is very expensive to measure all this physical quantities.

Solution

Estimate them from the measurement of some.

State space representation

We consider the following new notations

$$\left\{ \begin{array}{ll} x_i^1 = x_{1,i}; & 1 \leq i \leq l-1 \\ x_i^2 = x_{1,n-i+1}; & 1 \leq i \leq n-l-1 \\ \bar{x}_i^1 = x_{2,i}; & 1 \leq i \leq l-1 \\ \bar{x}_i^2 = x_{2,n-i+1}; & 1 \leq i \leq n-l-1 \\ z_i^1 = T_i; & 1 \leq i \leq l-1 \\ z_i^2 = T_{n-i+1}; & 1 \leq i \leq n-l-1 \\ u^T = (L, V_f, F, S, Z_{1,l}, Z_{2,l})^T \\ y_1 = (x_1^1, x_1^2)^T = (x_{1,1}, x_{1,n})^T \\ y_2 = (\bar{x}_1^1, \bar{x}_1^2)^T = (x_{2,1}, x_{2,n})^T, \end{array} \right.$$

where

y_1 & y_2 Outputs system contains the top and bottom measures of light component (Benzene) and intermediate component (Toluene) respectively.

u Inputs system, S , F , $Z_{1,l}$ and $Z_{2,l}$ are uncontrolled inputs, V_f and L are manipulated inputs.

x & \bar{x} State variables.

State space representation

The system belongs to the class of fully interconnected implicit systems written in the following compact format :

$$\begin{cases} \dot{x}(t) = f(x(t), \bar{x}(t), z(t), u(t)) & x \in \mathbb{R}^n, \quad u \in U \\ \dot{\bar{x}}(t) = \bar{f}(x(t), \bar{x}(t), z(t), u(t)) & \bar{x} \in \mathbb{R}^n \\ \varphi(x(t), \bar{x}(t), z(t)) = 0 & z \in \mathbb{R}^d \\ y(t) = [h(x(t)) \quad \bar{h}(\bar{x}(t))]^T & y \in \mathbb{R}^p, \end{cases}$$

where,

$$\begin{aligned} h(x) &= [x_1^1(t), x_1^2(t)]^T ; \bar{h}(\bar{x}) = [\bar{x}_1^1(t), \bar{x}_1^2(t)]^T \\ x &= (x_1^1, \dots, x_{l-1}^1, x_1^2, \dots, x_{n-l+1}^2)^T ; \bar{x} = (\bar{x}_1^1, \dots, \bar{x}_{l-1}^1, \bar{x}_1^2, \dots, \bar{x}_{n-l+1}^2)^T \\ f &= \begin{bmatrix} f^1(\bar{x}, x, z, u) \\ f^2(x, \bar{x}, z, u) \end{bmatrix} ; \bar{f} = \begin{bmatrix} \bar{f}^1(\bar{x}, x, z, u) \\ \bar{f}^2(x, \bar{x}, z, u) \end{bmatrix} ; \varphi = \begin{bmatrix} \varphi^1(x^1, \bar{x}^1, z^1) \\ \varphi^2(x^2, \bar{x}^2, z^2) \end{bmatrix} \end{aligned}$$

State space representation

f , \bar{f} and φ are \mathcal{C}^1 functions with respect to their arguments and satisfy the following triangular structure :

$$f^i = \begin{bmatrix} f_1^i(x_1^i, x_2^i, \bar{x}^i, z_2^i, u) \\ \vdots \\ f_k^i(x_{k-1}^i, x_k^i, x_{k+1}^i, \bar{x}^i, z_k^i, z_{k+1}^i, u) \\ \vdots \\ f_{n_i-1}^i(x_{n_i-2}^i, x_{n_i-1}^i, x_{n_i}^i, \bar{x}^i, z_{n_i-1}^i, z_{n_i}^i, u) \\ f_{n_i}^i(x, \bar{x}, z, u) \end{bmatrix} \quad \bar{f}^i = \begin{bmatrix} \bar{f}_1^i(\bar{x}_1^i, \bar{x}_2^i, x^i, z_2^i, u) \\ \vdots \\ \bar{f}_k^i(\bar{x}_{k-1}^i, \bar{x}_k^i, \bar{x}_{k+1}^i, x^i, z_k^i, z_{k+1}^i, u) \\ \vdots \\ \bar{f}_{n_i-1}^i(\bar{x}_{n_i-2}^i, \bar{x}_{n_i-1}^i, \bar{x}_{n_i}^i, \bar{x}^i, z_{n_i-1}^i, z_{n_i}^i, u) \\ \bar{f}_{n_i}^i(\bar{x}, x, z, u) \end{bmatrix}$$

$$\varphi^i = \begin{bmatrix} \varphi_1^i(x_1^i, \bar{x}_1^i, z_1^i) \\ \vdots \\ \varphi_k^i(x_{k-1}^i, x_k^i, x_{k+1}^i, \bar{x}_{k-1}^i, \bar{x}_k^i, \bar{x}_{k+1}^i, z_k^i) \\ \vdots \\ \varphi_{n_i}^i(x_{n_i}^i, \bar{x}_{n_i}^i, z_{n_i}^i) \end{bmatrix}$$

where $n_1 = l - 1$, and $n_2 = n - l + 1$.

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer**
- 6 Observer simulation results
- 7 Conclusion and perspectives

State Estimation

Estimation problem

By knowing the outputs $y(t)$ and inputs $u(t)$, how we can estimate the unknowns $(x(t), z(t))$?

State Estimation

Estimation problem

By knowing the outputs $y(t)$ and inputs $u(t)$, how we can estimate the unknowns $(x(t), z(t))$?

Solution

State space observer design.

Estimation problem

Two approaches are possible

1. An implicit observer

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), \hat{\hat{x}}(t), \hat{z}(t), u(t)) - K_{\theta}(\hat{y}_1(t) - y_1(t)) \\ \dot{\hat{\hat{x}}}(t) = \bar{f}(\hat{x}(t), \hat{\hat{x}}(t), \hat{z}(t), u(t)) - \bar{K}_{\theta}(\hat{y}_2(t) - y_2(t)) \\ \varphi(\hat{x}(t), \hat{\hat{x}}(t), \hat{z}(t)) = 0 \\ \hat{y}(t) = [h(\hat{x}(t)) \quad \bar{h}(\hat{\hat{x}}(t))]^T \end{cases}$$

Principle : Resolution of algebro-differential equations \Rightarrow use of iterative optimisation techniques for algebraic equations resolution

Estimation problem

Two approaches are possible

1. An implicit observer

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), \hat{x}(t), \hat{z}(t), u(t)) - K_{\theta}(\hat{y}_1(t) - y_1(t)) \\ \dot{\hat{x}}(t) = \bar{f}(\hat{x}(t), \hat{x}(t), \hat{z}(t), u(t)) - \bar{K}_{\theta}(\hat{y}_2(t) - y_2(t)) \\ \varphi(\hat{x}(t), \hat{x}(t), \hat{z}(t)) = 0 \\ \hat{y}(t) = [h(\hat{x}(t)) \quad \bar{h}(\hat{x}(t))]^T \end{cases}$$

Principle : Resolution of algebro-differential equations \Rightarrow use of iterative optimisation techniques for algebraic equations resolution

Disadvantages

- Optimization can increase the computing time.
- The algorithm may become difficult to implement on-line for complex systems and/or fast dynamic systems.

Observer algorithm

2. An explicit observer :

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}, \hat{x}, \hat{z}, u) + K_{\theta}(\hat{y}_1 - y_1) \\ \dot{\hat{x}} = \bar{f}(\hat{x}, \hat{x}, \hat{z}, u) + \bar{K}_{\theta}(\hat{y}_2 - y_2) \\ \dot{\hat{z}}(t) = - \left(\frac{\partial \varphi}{\partial z} \Big|_{(\hat{x}, \hat{z}, \hat{x})} \right)^{-1} \left(\left[\frac{\partial \varphi}{\partial x} \Big|_{(\hat{x}, \hat{z})}, \frac{\partial \varphi}{\partial \bar{x}} \Big|_{(\hat{x}, \hat{z})} \right] \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}} \end{bmatrix}^T + \Lambda \varphi(\hat{x}, \hat{x}, \hat{z}) \right), \end{cases}$$

where,

$$K_{\theta} = \text{diag}(-r_1 \Delta_{\theta \delta_1} S_1^{-1} C_1^T, -r_2 \Delta_{\theta \delta_2} S_2^{-1} C_2^T) ; \bar{K}_{\theta} = \text{diag}(-\bar{r}_1 \bar{\Delta}_{\theta \delta_1} \bar{S}_1^{-1} \bar{C}_1^T, -\bar{r}_2 \bar{\Delta}_{\theta \delta_2} \bar{S}_2^{-1} \bar{C}_2^T),$$

Principle : Ordinary differential equations (ODE) resolution.

Advantage

Compared to the implicit observer, the time of calculus = time of ODE integration.

Observer algorithm

$$S_n = \begin{bmatrix} s_{11} & s_{12} & 0 & \cdots & 0 \\ s_{12} & s_{22} & s_{23} & \cdots & 0 \\ & & \ddots & \ddots & \vdots \\ 0 & s_{23} & & \ddots & \\ \vdots & \cdots & \ddots & \ddots & \\ 0 & \cdots & 0 & s_{n-1n} & s_{nn} \end{bmatrix}, A_n(t) = \begin{bmatrix} 0 & a_1(t) & 0 & \cdots & 0 \\ 0 & 0 & a_2(t) & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & a_{n-1}(t) \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}$$

S satisfies the inequality

$$\forall t \leq 0, A_n^T(t)S_n + S_n A_n(t) - \rho C_n^T C_n \leq -\eta I_n$$

The a_i 's are unknowns and satisfy : $\forall t \geq 0, \alpha_1 \leq a_i(t) \leq \alpha_2$ where,
 $\alpha_1, \alpha_2 > 0$.

r_1 & r_2 positive constants tuning parameters of the observer greater than 1

Δ_θ is a diagonal matrix of $(\theta, \theta^2, \dots, \theta^{n_i})$

θ tuning positive constant observer parameter.

ρ & η are positive constants.

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results**
- 7 Conclusion and perspectives

Trays number	$n = 18,$
Side stream and feed trays	$s = 6, l = 9$
Liquid molar retention in condenser	$N_1 = 20 \text{ mol}$
Liquid molar retention in boiler	$N_{18} = 20 \text{ mol}$
Liquid molar retention in each tray	$N_i = 8 \text{ mol}$
Liquid and vapour Murphree efficiency	$e^l = 1, e^v = 1$
Internal pressure	$P_T = 760 \text{ mmHg}$
Feed flow rate	$F = 1.67 \text{ mol/min}$
Benzene composition in feed rate	$Z_{Ben,l} = 0.6$
Toluene composition in feed rate	$Z_{Tol,l} = 0.3$
Feed temperature	$T_l = 379,32 \text{ K}$
Side stream liquid flow rate	$S = 0.167 \text{ mol/min}$
Steady state vapour flow rate	$V_f = 7.0086 \text{ mol/min}$
Steady state liquid flow rate	$L = 6.0017 \text{ mol/min}$
Steady state quality of Benzene in top product	$x_{1,1} = 0.95$
Steady state quality of Benzene in bottom product	$x_{1,18} = 8.87 \times 10^{-5}$
Steady state quality of Toluene in top product	$x_{2,1} = 0.05$
Steady state quality of Toluene in bottom product	$x_{2,18} = 0.67$

TABLE: Operating point and initial steady state.

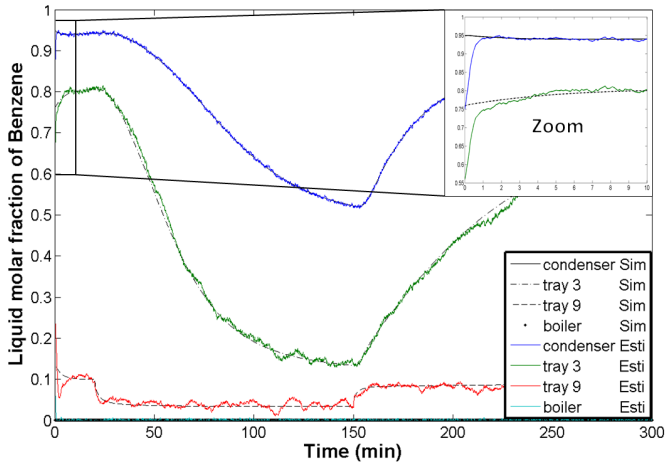


FIGURE: True and observed trays temperature in the presence of initial estimations errors and output added noises.

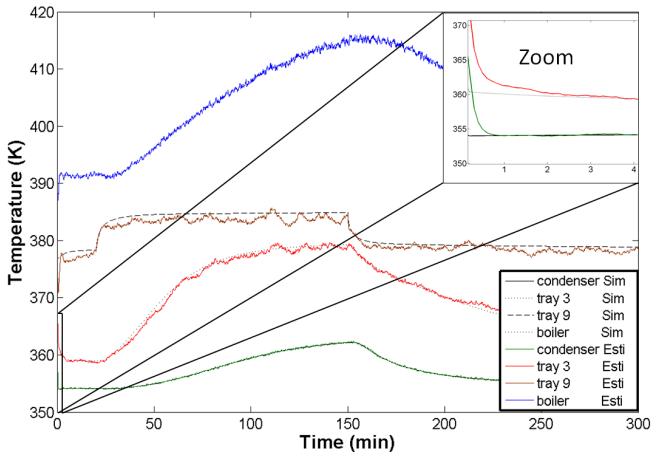


FIGURE: True and observed Benzene molar fractions in the presence of initial estimations errors and output added noises.

Outline

- 1 Ternary distillation column
- 2 Dynamical model
- 3 Model simulation results
- 4 Advanced controllers based on state model
- 5 Explicit observer
- 6 Observer simulation results
- 7 Conclusion and perspectives

Conclusion :

- Mathematical model validation describing the dynamic of ternary mixture fractions in a distillation column with 18 trays.
- High gain explicit observer performances validation.

Perspectives :

- Synthesis of an observer based on trays temperature measurements.
- Synthesis of a suitable control law.

Thanks for your attention

Thanks for your attention

About our research team "Nonlinear systems and Processes (SNLEP)" :
sites.google.com/site/snlepteam

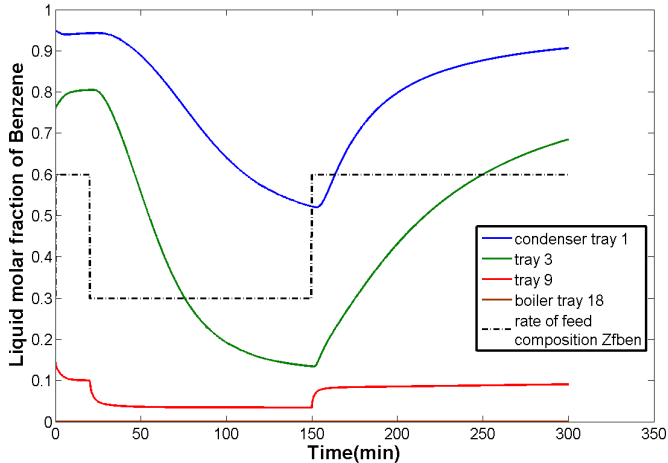


FIGURE: Open loop response to the feed disturbances .