



HAL
open science

Macroscopic Modeling of Compressible Flow in Porous Media

Wafa Bendhaou, Fabien Bellet, Didier Lasseux, Benoît Goyeau

► **To cite this version:**

Wafa Bendhaou, Fabien Bellet, Didier Lasseux, Benoît Goyeau. Macroscopic Modeling of Compressible Flow in Porous Media. 7th International Conference on Porous Media & Annual Meeting, May 2015, Padova, Italy. hal-01263889

HAL Id: hal-01263889

<https://hal.science/hal-01263889>

Submitted on 28 Jan 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

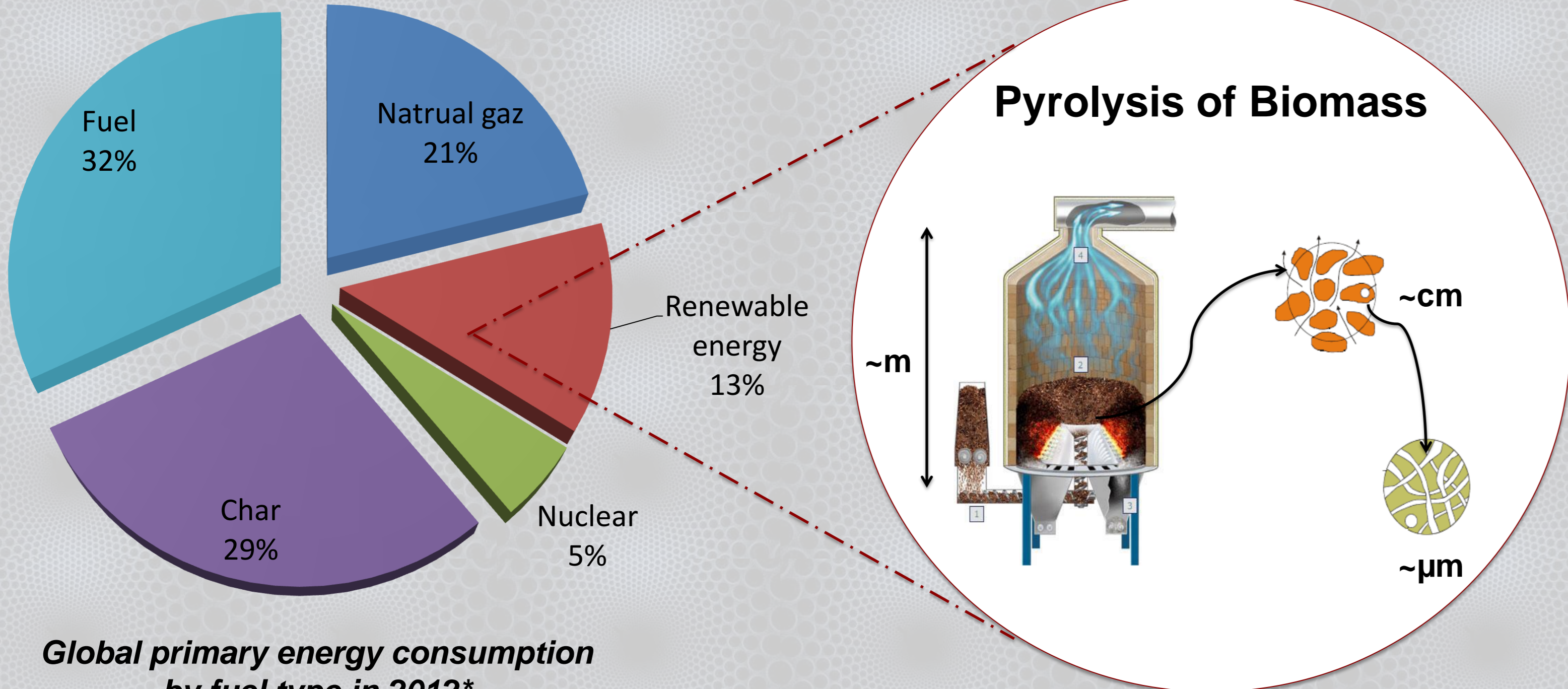
Macroscopic Modeling of Compressible Flow in Porous Media

Wafa Bendhaou*¹, Fabien Bellet¹, Didier Lasseux², Benoît Goyeau¹

1- Ecole Centrale Paris, Grande Voie des Vignes, 92290, Châtenay-Malabry, FR
2- Université Bordeaux, Esplanade des Arts et Métiers, 33405, Talence, FR

*Contact : wafa.bendhaou@ecp.fr

CONTEXT :



Model the process

Design large scale devices

Prove the competitiveness of this renewable energy

*Source: International Energy Agency (2014) World Energy Outlook – 2014

PROCESS CHARACTERISTICS :

Multiscale Phenomena

- The ratio between the macroscopic scale and the pore scale is about 10^6

Compressible Flow

- Temperature gradient
- Pressure gradient
- Fluid mixture

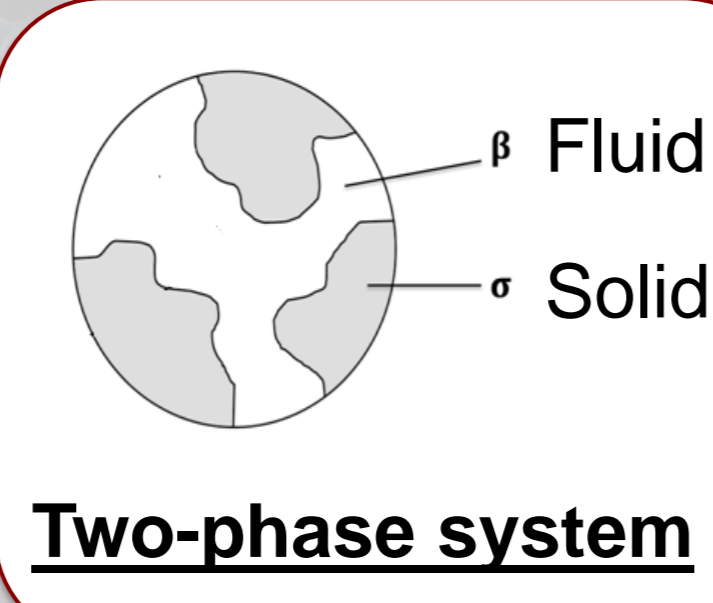
The computational cost of a Direct Numerical Simulation of this process is very High. Therefore, a homogenization technique, the Volume Averaging Method, is used to establish a macroscopic description of the process. This method leads to macroscopic equations and effective properties (permeability K_e ..).

OBJECTIVE :

Assess the compressibility effects on the macroscopic equations and the effective properties ?

ASSUMPTIONS :

- Compressible and isothermal flow
- Constant thermophysical properties
- Constant porosity
- Inertial effects are negligible



PORE SCALE EQUATIONS :

$$\rho_\beta = \rho(p_\beta)$$

$$\frac{\partial \rho_\beta}{\partial t} + \nabla \cdot (\rho_\beta \mathbf{v}_\beta) = 0$$

$$\frac{\partial}{\partial t} (\rho_\beta \mathbf{v}_\beta) = -\nabla (p_\beta - (\eta^b + \frac{\mu_\beta}{3}) \nabla \cdot \mathbf{v}_\beta) + \mu_\beta \nabla^2 \mathbf{v}_\beta$$

$$\mathbf{v}_\beta = 0 \quad \text{at } A_{\beta\sigma}$$

$$\mathbf{v}_\beta = \mathbf{f}(\mathbf{r}, t) \quad \text{at } A_{\beta e}$$

CLOSED MACROSCOPIC EQUATIONS :

$$\frac{\partial (\rho_\beta)^\beta}{\partial t} + \nabla \cdot [\mathbf{R}_\beta \cdot \langle \mathbf{v}_\beta \rangle^\beta] = 0$$

$$\frac{\partial}{\partial t} [\mathbf{R}_\beta \cdot \langle \mathbf{v}_\beta \rangle^\beta] = -\nabla \cdot [p_\beta]^\beta - (\eta^b + \frac{\mu_\beta}{3}) \nabla \cdot \langle \mathbf{v}_\beta \rangle^\beta + \mu_\beta \nabla^2 \langle \mathbf{v}_\beta \rangle^\beta - \mu_\beta \varepsilon_\beta \mathbf{K}_{\beta T}^{-1} \cdot \langle \mathbf{v}_\beta \rangle^\beta$$

Effective density

$$\mathbf{R}_\beta = \langle \rho_\beta \rangle^\beta \left[\mathbf{I} + \underbrace{\mu_\beta \chi_T (\mathbf{Bb})^\beta \cdot \langle \mathbf{v}_\beta \rangle^\beta + (\mu_\beta \chi_T)^2 \langle \mathbf{Cb} \rangle^\beta \cdot \langle \mathbf{v}_\beta \rangle^\beta}_{\text{Compressibility effects}} \right]$$

Effective permeability

$$\mathbf{K}_{\beta T}^{-1} = \mathbf{K}_{\beta i}^{-1} + \mathbf{K}_{\beta c}^{-1}$$

Compressibility effects

$$\mathbf{K}_{\beta c}^{-1} = \mu_\beta \chi_T \mathbf{F}_c \cdot \langle \mathbf{v}_\beta \rangle^\beta$$

$$\frac{1}{V_\beta} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot [-\mathbf{Ib} + \nabla \mathbf{B}] dA = -\varepsilon_\beta \mathbf{K}_{\beta i}^{-1}$$

$$\frac{1}{V_\beta} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot \left[-\mathbf{I} \left(\mathbf{c} - \frac{1}{3} \nabla \cdot \mathbf{C} \right) + \nabla \mathbf{C} \right] dA = -\varepsilon_\beta \mathbf{F}_c$$

-The Total permeability depends on the intrinsic permeability $\mathbf{K}_{\beta i}$ and a compressibility correction tensor $\mathbf{K}_{\beta c}$.

- $\mathbf{K}_{\beta i}$ depends only on the geometry of the porous medium. It can be calculated from the closure variables \mathbf{b} and \mathbf{B} that satisfy the incompressible sub-closure problem (0 order in Ma).

- $\mathbf{K}_{\beta c}$ depends on a geometrical parameter \mathbf{F}_c that is calculated from the closure variables \mathbf{c} and \mathbf{C} . $\mathbf{K}_{\beta c}$ depends also on the isothermal compressibility coefficient χ_T and the average velocity.

1- Pore scale equations

$$f(\langle \psi \rangle, \tilde{\psi}) = 0$$

2- Averaged equations

$$\langle f(\langle \psi \rangle, \tilde{\psi}) \rangle = 0$$

Volume averaging

4- Closed macroscopic equations

$$\langle f(\langle \psi \rangle, g(\langle \psi \rangle)) \rangle = 0$$

3- Closure

$$\tilde{\psi} = g(\langle \psi \rangle)$$

- Length scale constraint

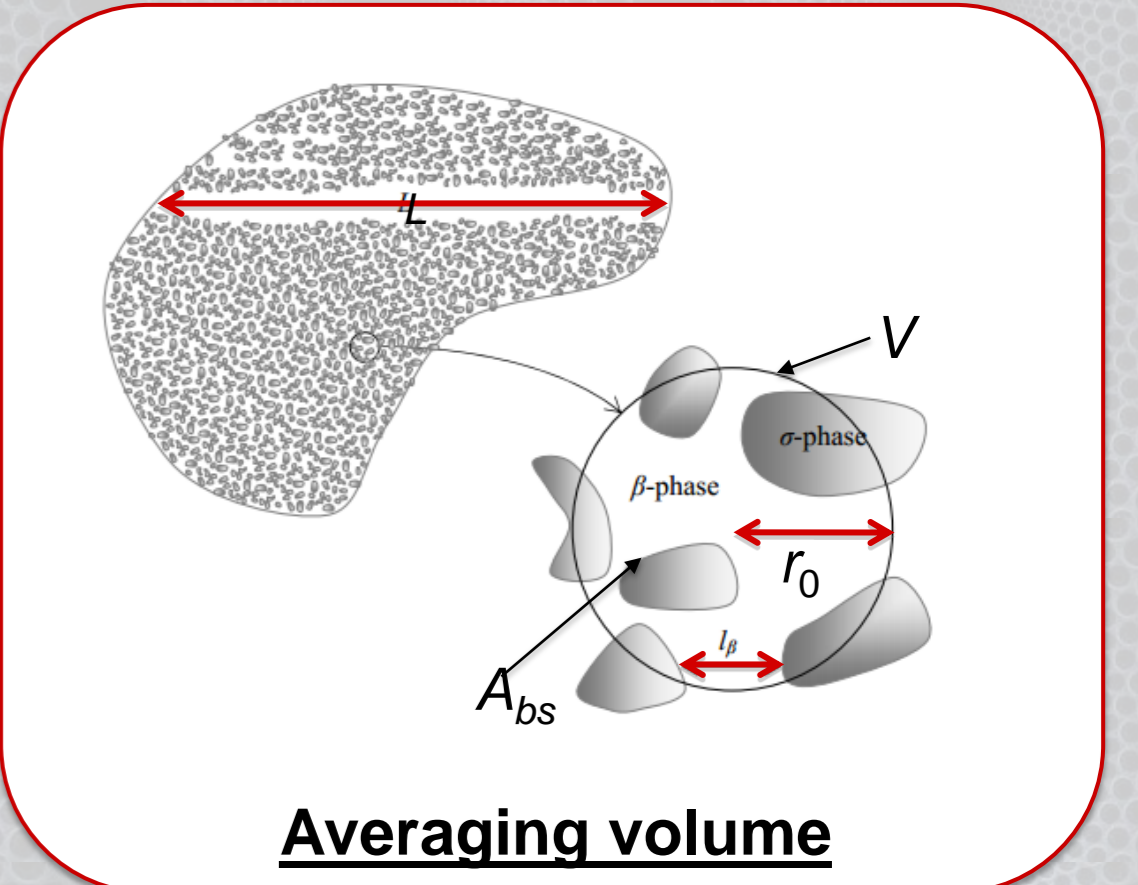
$$l_\beta \ll r_0 \ll L$$

- Superficial average

$$\langle \psi_\beta \rangle = \frac{1}{V} \int_{V_\beta} \psi_\beta dV$$

- Spatial decomposition (Gray)

$$\psi_\beta = \langle \psi_\beta \rangle^\beta + \tilde{\psi}_\beta$$



DIMENSIONLESS CLOSURE PROBLEM :

$$\tilde{\rho}_\beta^+ = \gamma Ma^2 \tilde{p}_\beta^+$$

$$(1 + \tilde{\rho}_\beta^+) \nabla^+ \cdot \tilde{\mathbf{v}}_\beta^+ + \mathbf{v}_\beta^+ \cdot \nabla^+ \tilde{\rho}_\beta^+ = 0$$

$$0 = -\nabla^+ \tilde{p}_\beta^+ + \frac{1}{Re} \left[\left(\frac{\eta^b}{\mu_\beta} + \frac{1}{3} \right) \nabla^+ \nabla^+ \cdot \tilde{\mathbf{v}}_\beta^+ + \nabla^+ \nabla^+ \tilde{\mathbf{v}}_\beta^+ \right]$$

$$- \frac{1}{V_\beta} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot \left(-\mathbf{I} \tilde{p}_\beta^+ + \frac{1}{Re} \left[\left(\frac{\eta^b}{\mu_\beta} + \frac{1}{3} \right) \nabla^+ \cdot \tilde{\mathbf{v}}_\beta^+ + \nabla^+ \tilde{\mathbf{v}}_\beta^+ \right] \right) dA$$

$$\tilde{\mathbf{v}}_\beta^+ = \frac{-\langle \mathbf{v}_\beta \rangle^\beta}{\| \langle \mathbf{v}_\beta \rangle^\beta \|} \quad \text{at } A_{\beta\sigma}$$

The Mach number Ma is a macroscopic parameter that quantifies the degree of the flow compressibility. The dimensionless spatial deviations are developed in power of Ma

2 SUB-CLOSURE PROBLEMS :

0 order in Ma

$$\nabla \cdot \tilde{\mathbf{v}}_{0\beta} = 0$$

$$0 = -\nabla \tilde{p}_{0\beta} + \mu_\beta \nabla^2 \tilde{\mathbf{v}}_{0\beta} - \frac{1}{V_\beta} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot [-\mathbf{I} \tilde{p}_{0\beta} + \mu_\beta \nabla \tilde{\mathbf{v}}_{0\beta}] dA$$

$$\tilde{\mathbf{v}}_{0\beta} = -\langle \mathbf{v}_\beta \rangle^\beta \quad \text{at } A_{\beta\sigma}$$

2nd order in Ma

$$\nabla \cdot \tilde{\mathbf{v}}_{2\beta} + \chi_T \langle \mathbf{v}_\beta \rangle^\beta \cdot \nabla \tilde{p}_{0\beta} = 0$$

$$0 = -\nabla \left[\tilde{p}_{2\beta} - (\eta^b + \frac{\mu_\beta}{3}) \nabla \cdot \tilde{\mathbf{v}}_{2\beta} \right] + \mu_\beta \nabla^2 \tilde{\mathbf{v}}_{2\beta}$$

$$- \frac{1}{V_\beta} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot \left[-\mathbf{I} \tilde{p}_{2\beta} - (\eta^b + \frac{\mu_\beta}{3}) \nabla \cdot \tilde{\mathbf{v}}_{2\beta} \right] + \mu_\beta \nabla \tilde{\mathbf{v}}_{2\beta} \right] dA$$

$$\tilde{\mathbf{v}}_{2\beta} = 0 \quad \text{at } A_{\beta\sigma}$$

CLOSURE VARIABLES :

$$\tilde{\mathbf{v}}_\beta = \tilde{\mathbf{v}}_{0\beta} + \tilde{\mathbf{v}}_{2\beta}$$

$$\tilde{p}_\beta = \tilde{p}_{0\beta} + \tilde{p}_{2\beta}$$

$$\tilde{\mathbf{v}}_{\beta 2} = \mu_\beta \chi_T \mathbf{C} : \langle \mathbf{v}_\beta \rangle^\beta \langle \mathbf{v}_\beta \rangle^\beta$$

$$\tilde{p}_{\beta 2} = \mu_\beta^2 \chi_T \mathbf{C} : \langle \mathbf{v}_\beta \rangle^\beta \langle \mathbf{v}_\beta \rangle^\beta$$

$$\tilde{\rho}_{\beta 2} = \chi_T \langle \rho_\beta \rangle^\beta \mu_\beta \mathbf{b} \cdot \langle \mathbf{v}_\beta \rangle^\beta$$

NUMERICAL RESULTS:

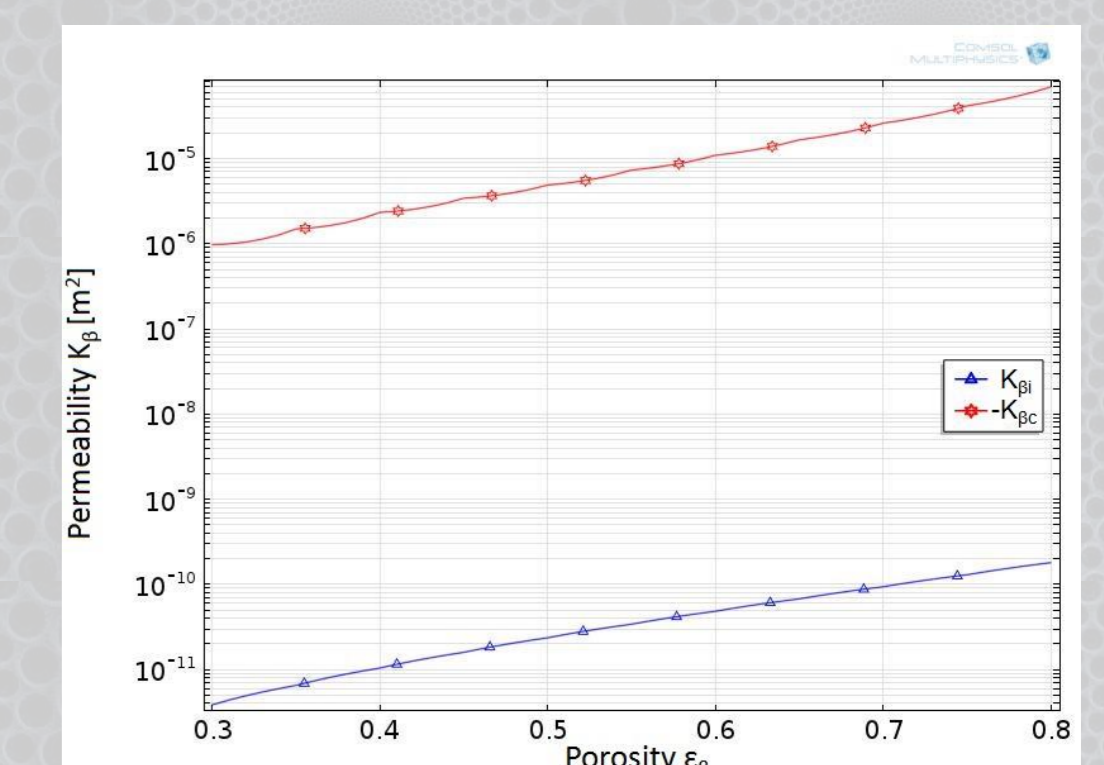
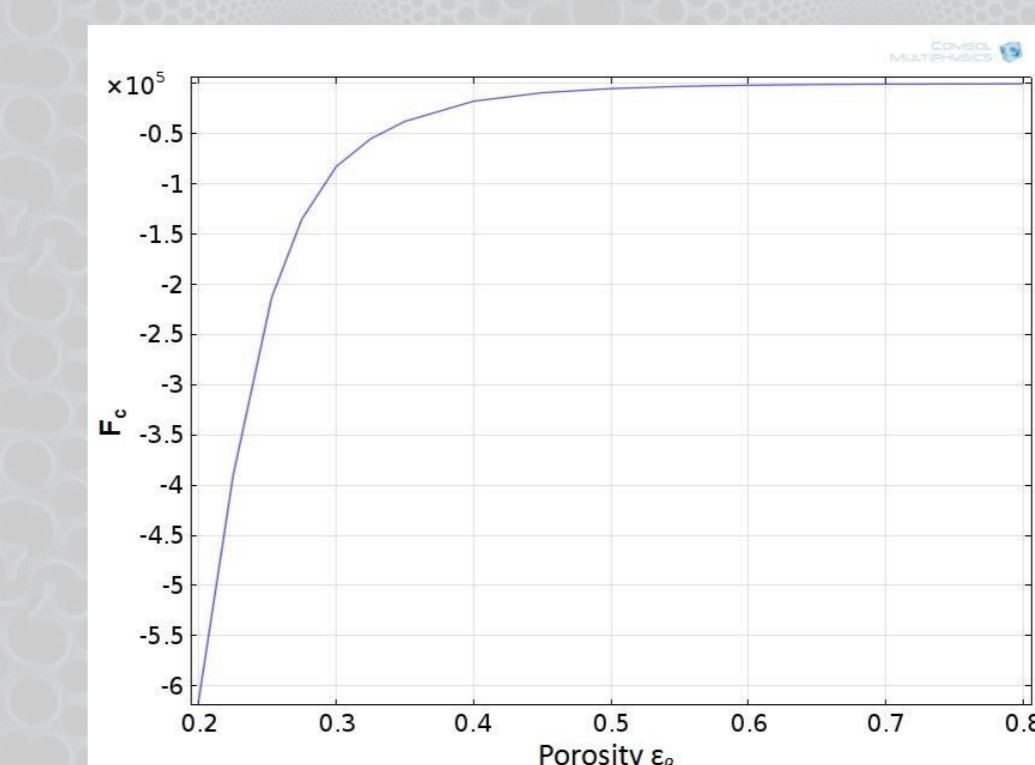
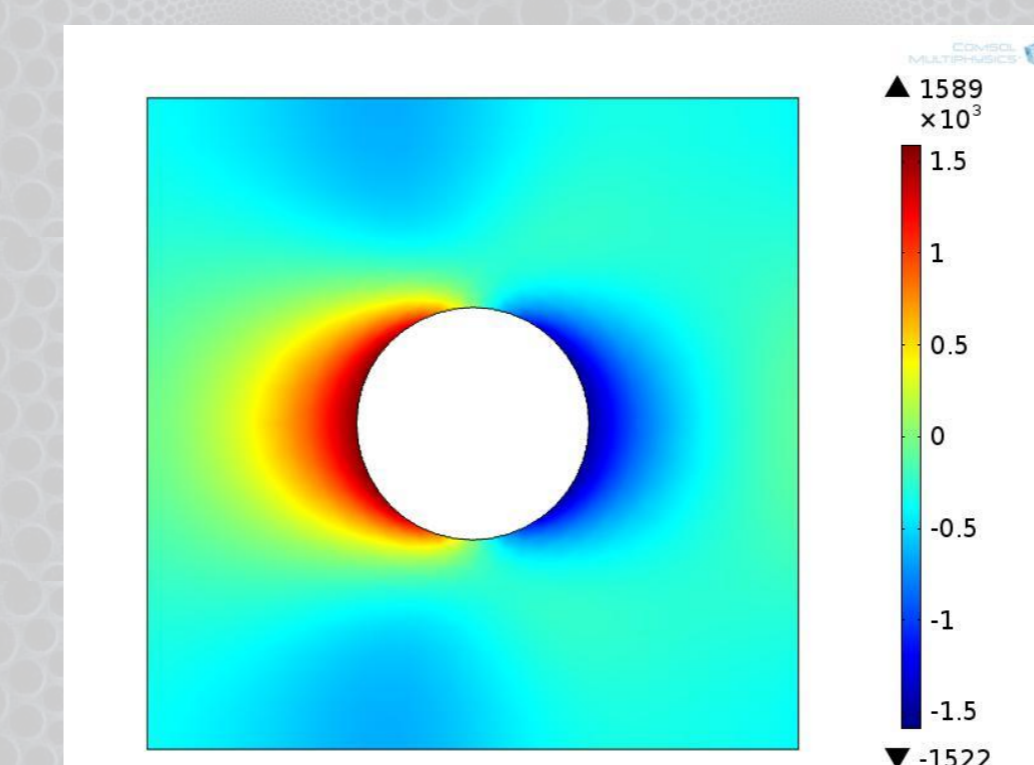
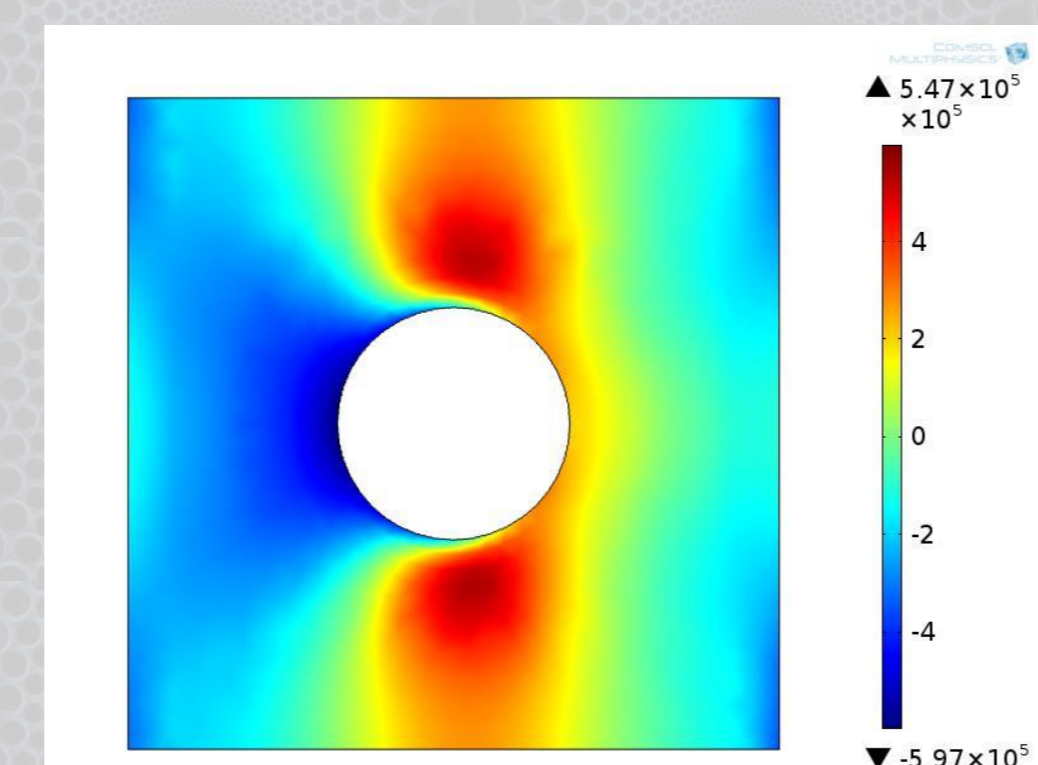
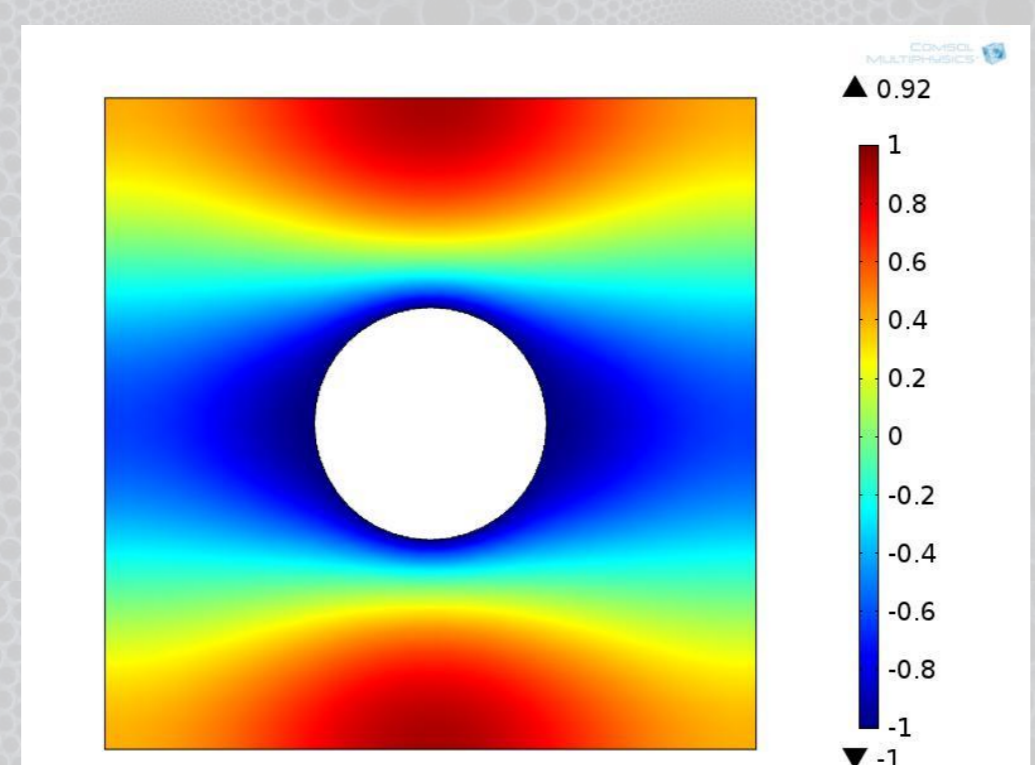
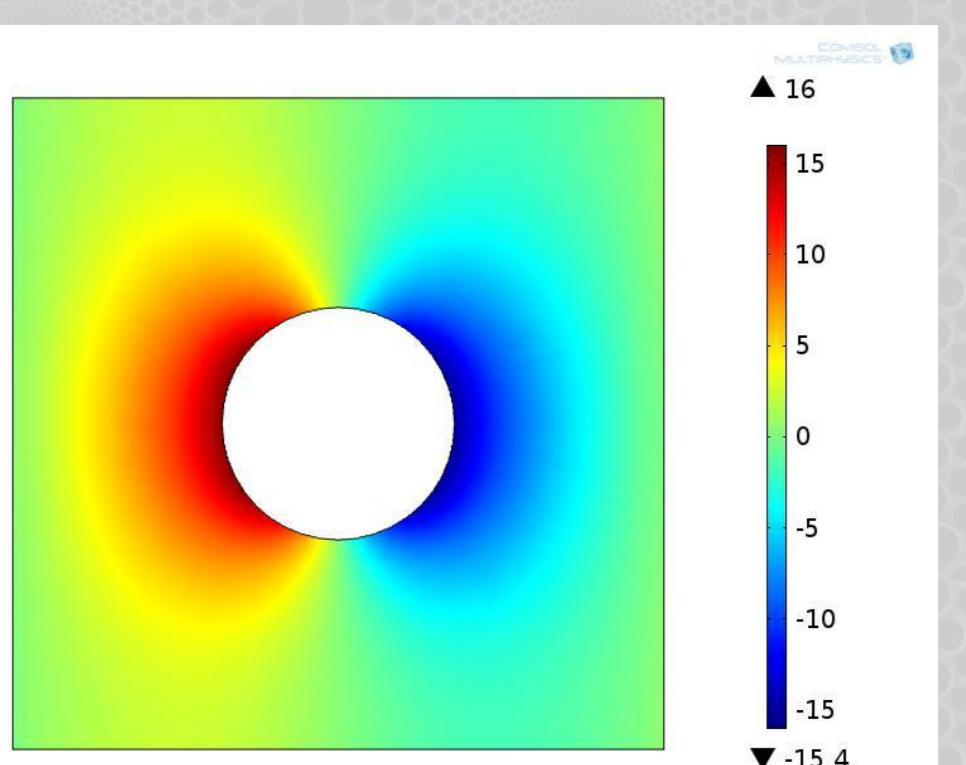


Fig 1 – Field of the closure variable \mathbf{b}_x

Fig 2 – Field of the closure variable \mathbf{B}_{xx}

Fig 3 – Field of the closure variable \mathbf{c}_{xx}

Fig 4 – Field of the closure variable \mathbf{C}_{xx}

Fig 5 – Variation of \mathbf{F}_c versus the porosity

Fig 6 – Variation of \mathbf{K}_β versus the porosity