Feedback Enhances Simultaneous Energy and Information Transmission in Multiple Access Channels

Selma Belhadj Amor, Samir M. Perlaza, Ioannis Krikidis, and H. Vincent Poor

***Abstract*—In this paper, the fundamental limits of simultaneous information and energy transmission in the two-user Gaussian multiple access channel with feedback are fully characterized. A simple achievability scheme based on power-splitting and Ozarow's scheme is shown to be optimal. Finally, the maximum individual information rates and the information sum-capacity that are achievable given a minimum energy rate constraint of b energy-units per channel use at the input of the energy harvester are identified. An interesting conclusion is that for a fixed information transmission rate, feedback can at most double the energy transmission rate with respect to the case without feedback.**

I. INTRODUCTION

For decades, a common engineering practice has been to exclusively use radio frequency (RF) signals for information transmission. However, this practice has been shown to be suboptimal [1]. Indeed, an RF signal carries both energy and information. From this standpoint, a variety of modern wireless systems suggest that RF signals can be simultaneously used for information and energy transmission [2]. Nevertheless, information and energy transmission are often conflicting tasks and thus subject to a trade-off between the information transmission rate (bits per channel use) and the energy transmission rate (energy-units per channel use). This trade-off is evidenced in finite constellation schemes, as highlighted in Popovski *et al.* [3]. Consider the noiseless transmission of a 4-PAM signal over a point-to-point channel in the alphabet $\{-2, -1, 1, 2\}$. If there is no received energy rate constraint, one can clearly convey 2 bits per channel use by choosing all available symbols with equal probability. However, if one requires the received energy rate to be for instance the maximum possible, the maximum transferable information rate is 1 bit per channel use. This is basically because communication takes place using only the symbols carrying the maximum energy. From this simple example, it is easy to see how additional energy rate constraints may change the overall performance of the network. In the context of multi-user channels, very little is known about the fundamental limits of simultaneous energy and information transmission (SEIT). Indeed, most of the existing results in this area have approached SEIT from a signal-processing or networking point of view and focused mainly on feasibility aspects.

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A. SEIT in Multiple Access Channels (MACs)

In the particular case of the discrete memoryless multiple access channel (DM-MAC), the trade-off between information rate and energy rate has been studied in [4]. Therein, Fouladgar *et al.* characterized the information-energy capacity region of the two-user DM-MAC, when a minimum energy rate is required at the input of the receiver. Such a constraint changes the dynamic of the communication system in the sense that it requires additional transmitter coordination to achieve the targeted energy rate. Recently, Belhadj Amor *et al.* studied SEIT in the Gaussian MAC (G-MAC) without feedback and derived the information-energy capacity region as well as the maximum individual and sum rates that can be achieved subject to a minimum energy rate b. Other types of energy rate constraints for the G-MAC have also been investigated. For instance, Gastpar [5] considered the G-MAC under a maximum received energy rate constraint. Under this assumption, channel-output feedback has been shown not to increase the capacity region. However, in the G-MAC under a minimum energy rate constraint, the effect of feedback is not yet well understood from an energy transmission perspective. More generally, the use of feedback in the K-user G-MAC, even without energy rate constraints, has been shown to be of limited impact in terms of sum-rate improvement. This holds even in the case of perfect feedback. More specifically, the use of feedback in the G-MAC increases the sum-capacity by at most $\frac{\log_2(K)}{2}$ bits per channel use [6]. Hence, the use of feedback is difficult to justify from the point of view of information transmission.

B. Contributions

This paper studies the fundamental limits of SEIT in the two-user G-MAC with feedback (G-MAC-F). It shows that when the goal is to simultaneously transmit both information and energy, feedback can significantly improve the global performance of the system in terms of both information and energy transmission rates. One of the main contributions is the identification of all the achievable information and energy transmission rates in bits per channel use and energy-units per channel use, respectively. More specifically, the information-energy capacity region with feedback is fully characterized and it is shown to be achievable by a simple scheme based on power-splitting and Ozarow's capacity achieving scheme [7]. As a byproduct, the maximum individual information rates and the information sum-capacity that are achievable when there exists a minimum energy rate constraint are fully identified.

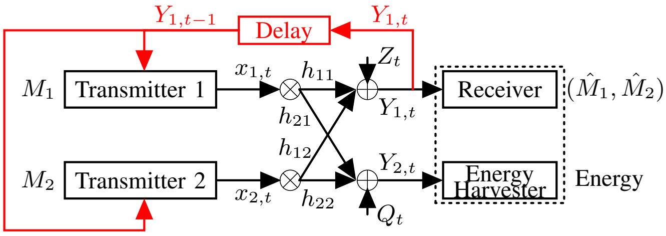

Fig. 1. Two-user memoryless Gaussian MAC with feedback and energy harvester.

Two of the most important observations in this work are: (a) the information-energy capacity region of the G-MAC without feedback is a proper subset of the information-energy capacity region of the G-MAC-F, that is, the former is strictly contained in the latter; and (b) feedback can at most double the energy rate for a fixed information rate.

II. Gaussian Multiple Access Channel with Feedback

Consider the two-user memoryless G-MAC-F in Fig. 1. Transmitter 1 and transmitter 2 aim at respectively sending the message indices M_1 and M_2 to the receiver. At each channel use $t \in \mathbb{N}$, $X_{i,t}$ denotes the real symbol sent by transmitter i, with $i \in \{1,2\}$. Let $n \in \mathbb{N}$ denote the blocklength. The symbols $X_{i,1}, \ldots, X_{i,n}$ satisfy an average *input power constraint*

$$\frac{1}{n} \sum_{t=1}^{n} \mathsf{E}\left[X_{i,t}^2\right] \leqslant P_i, \tag{1}$$

where the expectation is over the message indices and P_i denotes the average transmit power of transmitter i in energy-units per channel use. The receiver observes the real channel output

$$Y_{1,t} = h_{11}X_{1,t} + h_{12}X_{2,t} + Z_t, \tag{2}$$

and the energy harvester (EH) observes

$$Y_{2,t} = h_{21}X_{1,t} + h_{22}X_{2,t} + Q_t, \tag{3}$$

where h_{1i} and h_{2i} are the corresponding constant non-negative channel coefficients from transmitter i to the receiver and EH, respectively. The channel coefficients must satisfy the following $\mathcal{L}_2$-norm condition: $\forall j \in \{1,2\}, \|\mathbf{h}_j\|^2 \leqslant 1$, with $\mathbf{h}_j \triangleq (h_{j1}, h_{j2})^{\mathsf{T}}$ to ensure the principle of conservation of energy. The noise terms Z_t and Q_t are realizations of two identically distributed zero-mean unit-variance real Gaussian random variables. In the following, there is no particular assumption on the joint distribution of Q_t and Z_t.

A perfect feedback link from the receiver to transmitter i allows at each channel use t, the observation of the channel output $Y_{1,t-1}$ at both transmitters.

The G-MAC-F above is fully described by the signal to noise ratios (SNRs) – SNR_{ji}, with $\forall (i,j) \in \{1,2\}^2$, which are defined as follows:

$$\mathrm{SNR}_{ji} \triangleq |h_{ji}|^2 P_i, \tag{4}$$

given the normalization over the noise powers.

Within this context, two main tasks are to be accomplished simultaneously: information transmission and energy transmission.

A. Information Transmission

The goal of the communication is to convey the independent messages M_1 and M_2 from transmitters 1 and 2 to the common receiver. The messages M_1 and M_2 are independent of the noise terms $Z_1, \ldots, Z_n$, $Q_1, \ldots, Q_n$ and uniformly distributed over the sets $\mathcal{M}_1 \triangleq \{1, \ldots, \lfloor 2^{nR_1} \rfloor\}$ and $\mathcal{M}_2 \triangleq \{1, \ldots, \lfloor 2^{nR_2} \rfloor\}$, where R_1 and R_2 denote the information rates. The existence of feedback links allows the t-th symbol of transmitter i to be dependent on all previous channel outputs $Y_{1,1}, \ldots, Y_{1,t-1}$ as well as its message index M_i. More specifically,

$$X_{i,1} = f_{i,1}^{(n)}(M_i) \quad \text{and} \tag{5}$$

$$X_{i,t} = f_{i,t}^{(n)}(M_i, Y_{1,1}, \ldots, Y_{1,t-1}), \quad t \in \{2, \ldots, n\}, \tag{6}$$

for some encoding functions $f_{i,1}^{(n)} \colon \mathcal{M}_i \to \mathbb{R}$ and $f_{i,t}^{(n)} \colon \mathcal{M}_i \times \mathbb{R}^{t-1} \to \mathbb{R}$. The receiver produces an estimate $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) = \Phi^{(n)}(Y_1^n)$ of the message-pair (M_1, M_2) via a decoding function $\Phi^{(n)} \colon \mathbb{R}^n \to \mathcal{M}_1 \times \mathcal{M}_2$, and the average probability of error is

$$P_{\text{error}}^{(n)}(R_1, R_2) \triangleq \Pr\left\{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) \neq (M_1, M_2)\right\}. \tag{7}$$

B. Energy Transmission

The expected energy transmission rate (in energy-units per channel use) at the EH is

$$B^{(n)} \triangleq \frac{1}{n} \sum_{t=1}^{n} \mathsf{E}\left[Y_{2,t}^2\right], \tag{8}$$

where the expectation is over the message indices. The goal of the energy transmission is to guarantee that the average energy rate $B^{(n)}$ is not less than a given (constant) energy rate B that must satisfy $0 < B \leqslant 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$, for the problem to be feasible. Hence, the probability of energy outage is defined as follows:

$$P_{\text{outage}}^{(n)}(B) = \Pr\left\{B^{(n)} < B - \epsilon\right\}, \tag{9}$$

for some $\epsilon > 0$ arbitrarily small.

C. Simultaneous Energy and Information Transmission

The G-MAC-F in Fig. 1 is said to operate at the information-energy rate triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ considered if (a) reliable communication at information rates R_1 and R_2 is ensured; and (b) the average energy rate during the whole block-length is not lower than B. Under these conditions, the information-energy rate triplet (R_1, R_2, B) is said to be achievable.

Definition 1 (Achievable Rates)**.** *The triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ is achievable if there exists a sequence of encoding and decoding functions $\left\{\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \Phi^{(n)}\right\}_{n=1}^{\infty}$ such that $\limsup_{n\to\infty} P_{\text{error}}^{(n)}(R_1, R_2) = 0$ and $\limsup_{n\to\infty} P_{\text{outage}}^{(n)}(B) = 0$ for any $\epsilon > 0$.*

From Def. 1, it is clear that for any achievable triplet (R_1, R_2, B), whenever the targeted energy rate B is smaller than the minimum energy rate required to guarantee reliable communications at the information rates R_1 and R_2, the energy rate constraint is vacuous. This is mainly because the energy rate constraint is always satisfied, and thus the transmitter can exclusively use the available power budget for increasing the information rate. Alternatively, when the energy rate B must be higher than what is strictly necessary to guarantee reliable communication, the transmitters face a trade-off between information and energy rates. Often, increasing the energy transmission rate implies decreasing the information transmission rates and *vice-versa*. This trade-off is accurately modeled by the notion of *information-energy capacity region*.

Definition 2 (Information-Energy Capacity Region)**.** *The information-energy capacity region of the G-MAC-F* $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ *is the closure of all achievable information-energy rate triplets* (R_1, R_2, B).

III. Main Results

A. Information-Energy Capacity Region with Feedback

The information-energy capacity region of the G-MAC-F is fully characterized by the following theorem.

Theorem 1 (Information-Energy Capacity Region with Feedback)**.** *The perfect feedback information-energy capacity region* $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ *of the G-MAC-F is the set of non-negative information-energy rate triplets* (R_1, R_2, B) *that satisfy*

$$
\begin{aligned}
R_1 &\leqslant \frac{1}{2}\log_2\left(1 + \beta_1\ \mathrm{SNR}_{11}\left(1-\rho^2\right)\right) && \text{(10a)}\\
R_2 &\leqslant \frac{1}{2}\log_2\left(1 + \beta_2\ \mathrm{SNR}_{12}\left(1-\rho^2\right)\right) && \text{(10b)}\\
R_1 + R_2 &\leqslant \frac{1}{2}\log_2\Big(1 + \beta_1\ \mathrm{SNR}_{11} + \beta_2\ \mathrm{SNR}_{12}\\
&\quad +2\rho\sqrt{\beta_1 \mathrm{SNR}_{11}\beta_2\mathrm{SNR}_{12}}\Big) && \text{(10c)}\\
B &\leqslant 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho\sqrt{\beta_1\mathrm{SNR}_{21}\ \beta_2\mathrm{SNR}_{22}}\\
&\quad +2\sqrt{(1-\beta_1)\mathrm{SNR}_{21}\ (1-\beta_2)\mathrm{SNR}_{22}}, && \text{(10d)}
\end{aligned}
$$

with $(\rho, \beta_1, \beta_2) \in [0,1]^3$.

Proof: The proof of Theorem 1 is presented in [8]. ■

From an achievability standpoint, the parameters β_1 and β_2 in Theorem 1 might be interpreted as the fractions of power that transmitter 1 and transmitter 2 allocate for information transmission, respectively. The remaining fraction of power $(1-\beta_i)$ is allocated by transmitter i for exclusively transmitting energy to the EH. The information transmission follows Ozarow's perfect feedback capacity-achieving scheme in [7]. The energy transmission is accomplished by random symbols that are known at both transmitters and the receiver. More specifically, transmitter i generates two signals: an information-carrying (IC) signal with average power $\beta_i P_i$ energy-units per channel use; and a no-information-carrying (NIC) signal with power $(1-\beta_i)P_i$ energy-units per channel use. The role of the NIC signal is exclusively energy transmission from the transmitter to the EH. Conversely, the role of the IC signal is twofold: information transmission from the transmitter to the receiver and energy transmission from the transmitter to the EH.

The parameter ρ is the average Pearson correlation coefficient between the IC signals sent by both transmitters. This parameter plays a fundamental role in both information transmission and energy transmission. If $\beta_1 \neq 0$ and $\beta_2 \neq 0$, let $\rho^\star(\beta_1, \beta_2)$ be the unique solution in $(0,1)$ to the following equality:

$$
\begin{aligned}
&1 + \beta_1\ \mathrm{SNR}_{11} + \beta_2\ \mathrm{SNR}_{12} + 2\rho\sqrt{\beta_1\mathrm{SNR}_{11}\beta_2\mathrm{SNR}_{12}}\\
&\quad = \left(1 + \beta_1\ \mathrm{SNR}_{11}(1-\rho^2)\right)\left(1 + \beta_2\ \mathrm{SNR}_{12}(1-\rho^2)\right); \quad (11)
\end{aligned}
$$

otherwise, let $\rho^\star(\beta_1, \beta_2) = 0$.

Note that for any power-splitting $(\beta_1, \beta_2) \in (0,1]^2$, the left hand side of (11) is monotonically increasing in ρ whereas the right hand side is monotonically decreasing in ρ. This implies that $\rho^\star(\beta_1, \beta_2)$ is a maximizer of the sum-rate. More specifically, at $\rho = \rho^\star(\beta_1, \beta_2)$, the sum of (10a) and (10b) is equal to (10c) and it corresponds to the sum-capacity of the G-MAC-F.

The Pearson correlation factor between the NIC signals of both transmitters does not appear in Theorem 1 because maximum energy transmission occurs using NIC signals that are fully correlated, and thus the corresponding Pearson correlation coefficient is one. Without loss of optimality, NIC signals can be chosen to be independent of the messages M_1 and M_2 as well as the noise sequences, and known by both the receiver and the transmitters. Hence, NIC signals can be independent of the IC signals and more importantly, the interference they create at the receiver can be easily eliminated via successive interference cancellation.

Under these assumptions, this coding scheme guarantees the achievability of non-negative rate pairs (R_1, R_2) satisfying (10a)-(10c).

At the EH, both the IC and NIC signals contribute to the total energy harvested (8). The IC signal is able to convey at most $\beta_1\mathrm{SNR}_{21} + \beta_2\mathrm{SNR}_{22} + 2\rho\sqrt{\beta_1\mathrm{SNR}_{21}\beta_2\mathrm{SNR}_{22}}$ energy-units per channel use, while the NIC signal is able to convey at most $(1-\beta_1)\mathrm{SNR}_{21} + (1-\beta_2)\mathrm{SNR}_{22} + 2\sqrt{(1-\beta_1)\mathrm{SNR}_{21}(1-\beta_2)\mathrm{SNR}_{22}}$ energy-units per channel use. The sum of these two contributions as well as the contribution of the noise at the EH justifies the upper-bound on the energy transmission rate in (10d).

Remark 1. *The information-energy capacity region without feedback* $\mathcal{E}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ *derived in [9, Theorem 1] is identical to* $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ *in the case in which channel inputs are chosen to be mutually independent, i.e.,* $\rho = 0$. *Thus, for any non-zero* SNR_{11}, SNR_{12}, SNR_{21}, *and* SNR_{22}, *it holds that*

$$
\begin{aligned}
&\mathcal{E}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})\\
&\qquad \subset \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22}). \quad (12)
\end{aligned}
$$

Note that the inclusion here is strict. For instance, any rate triplet (R_1, R_2, B) *for which* $R_1 + R_2$ *equals the perfect feedback sum-capacity cannot be achieved without feedback.*

B. Information Transmission with Feedback Subject to Minimum Energy Rate Constraint b

Let $0 \leqslant b \leqslant 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$ denote the minimum energy rate that must be guaranteed at the input of the EH in the G-MAC-F with parameters SNR_{11}, SNR_{12}, SNR_{21}, and SNR_{22}. In the following, the maximum individual information rates as well as the information sum-capacity that are achievable given a minimum energy rate constraint of b energy-units per channel use at the input of the EH are identified.

1) Maximum Individual Information Rates with Feedback and with Minimum Energy Rate Constraint b: The maximum individual information rate $R_i^{\mathrm{FB}}(b)$, with $i \in \{1,2\}$, is the solution to an optimization problem of the form

$$R_i^{\mathrm{FB}}(b) = \max_{(r_i, r_j, c) \in \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22}) : c \geqslant b} r_i. \quad (13)$$

Proposition 1 (Maximum Individual Information Rates)**.** *For a given required minimum energy rate b, transmitter i's maximum individual information rate with feedback coincides with its maximum individual information rate without feedback and is given by*

$$R_i^{\mathrm{FB}}(b) = \frac{1}{2}\log_2\left(1 + \beta^{\star}(b)\mathrm{SNR}_{1i}\right), \quad i \in \{1,2\}, \quad (14)$$

with $\beta^{\star}(b) \in [0,1]$ *defined as follows:*

$$\beta^{\star}(b) = 1 - \left(\frac{\left(b - (1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22})\right)^+}{2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}}\right)^2. \quad (15)$$

Proof: The proof of Proposition 1 is provided in [8]. ■

The rate $R_i^{\mathrm{FB}}(b)$ is achieved by transmitter i, for instance, when transmitter j uses all its available power for exclusively transmitting energy to the EH ($\beta_j = 0$) by using common randomness; and transmitter i uses a power split in which the part of power dedicated for exclusively transmitting energy to the EH, $1 - \beta_i$, is the fraction needed to satisfy

$$1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{(1-\beta_1)\mathrm{SNR}_{21}(1-\beta_2)\mathrm{SNR}_{22}} \geqslant b, \quad (16)$$

with equality, that is, $\beta_i = \beta^{\star}(b)$ (when $\beta_j = 0$).

2) Information Sum-Capacity with Feedback and with Minimum Energy Constraint b: The perfect feedback information sum-capacity $R_{\mathrm{sum}}^{\mathrm{FB}}(b)$ is the solution to an optimization problem of the form

$$R_{\mathrm{sum}}^{\mathrm{FB}}(b) = \max_{(r_1, r_2, c) \in \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22}) : c \geqslant b} r_1 + r_2. \quad (17)$$

Proposition 2 (Information Sum-Capacity)**.** *The perfect feedback information sum-capacity of the G-MAC subject to a minimum energy rate constraint b that must be guaranteed at the input of the EH is*

1) $\forall b \in \left[0, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}\right]$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(b) = \frac{1}{2}\log_2(1 + \mathrm{SNR}_{11} + \mathrm{SNR}_{12} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{11}\mathrm{SNR}_{12}}); \quad (18)$$

2) $\forall b \in \left(1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}\right)$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(b) = \frac{1}{2}\log_2(1 + \beta^{\star}(b)\mathrm{SNR}_{11}) \quad (19)$$

$$+ \frac{1}{2}\log_2(1 + \beta^{\star}(b)\mathrm{SNR}_{12}); \quad (20)$$

3) $\forall b \in \left[1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}, \infty\right]$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(b) = 0, \quad (21)$$

with $\beta^{\star}(b)$ *defined in* (15) *and* $\rho^{\star}(1,1)$ *is the unique solution in* $(0,1)$ *to* (11) *when* $\beta_1 = \beta_2 = 1$.

Proof: The proof of Proposition 2 is presented in [8]. ■

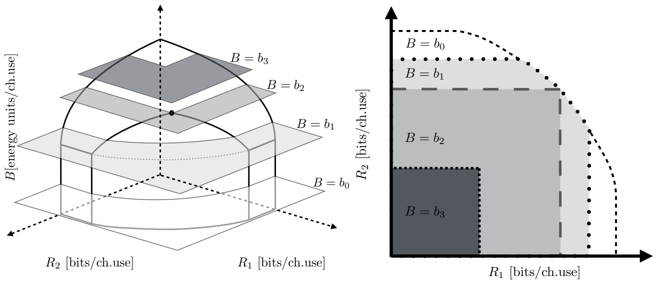

Fig. 2. Intersection of the planes $B = b_0$, $B = b_1$, $B = b_2$, and $B = b_3$ with the information-energy capacity region $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$, with $\mathrm{SNR}_{11} = \mathrm{SNR}_{12} = \mathrm{SNR}_{21} = \mathrm{SNR}_{22}$, $b_0 \in \left[0, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}\right]$, $b_1 \in \left[1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}\right]$, $b_2 = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$ and $b_3 \in \left[1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}},\right]$.

Note that even if feedback does not increase the maximal individual rates that can be achieved for a given received energy rate b, it increases the sum-rate that can be achieved (See Proposition 2 in comparison to [9, Theorem 2]).

Fig. 2 shows a general example of the intersection of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$, in the Cartesian coordinates (R_1, R_2, B), with a plane $B = b$ when $\mathrm{SNR}_{11} = \mathrm{SNR}_{12} = \mathrm{SNR}_{21} = \mathrm{SNR}_{22}$.

Case 1: In the case in which $b \in [0, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}]$, then $\beta^{\star}(b) = 1$, and thus the energy constraint does not add any additional bound on the individual rates and the sum-rate other than (10a), (10b), and (10c). In fact, in this case, any intersection of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$, in the Cartesian coordinates (R_1, R_2, B), with a plane $B = b$ corresponds to the set of triplets (R_1, R_2, b), in which the corresponding pairs (R_1, R_2) form a set that is identical to the information capacity region of the G-MAC-F, denoted by $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12})$, which is achievable by using Ozarow's scheme without any power-splitting, i.e., $\beta_1 = \beta_2 = 1$. In this case, transmitting information using all the available power budget is always enough to satisfy the energy constraint. (See the intersection of the plane $B = b_0$ and the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$, with $b_0 \in \left[0, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}\right]$, in Fig. 2.)

Case 2: In the case in which $b \in (1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}]$, it follows that $1 - \left(\rho^{\star}(1,1)\right)^2 \leqslant \beta^{\star}(b) < 1$, and thus the energy constraint limits the individual rates. That is, transmitter i's individual rate is bounded away from $\frac{1}{2}\log_2(1 + \mathrm{SNR}_{1i})$. Let $\mathcal{B}(b) \subset \mathbb{R}_+^2$ be a box of the form

$$\mathcal{B}(b) = \Big\{(R_1, R_2) \in \mathbb{R}_+^2 : R_i \leqslant \frac{1}{2}\log_2\left(1 + \beta^{\star}(b)\mathrm{SNR}_{1i}\right), i \in \{1,2\}\Big\}. \quad (22)$$

Any intersection of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ with a plane $B = b$ is a set of triplets (R_1, R_2, b) for which the corresponding pairs (R_1, R_2) satisfy $(R_1, R_2) \in \mathcal{B}(b) \cap \mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12})$, which form a proper subset of $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12})$. It is important to highlight that in this case, this intersection always includes the triplet (R_1, R_2, b), with $R_1 + R_2 = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_{11} + \mathrm{SNR}_{12} + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_{11}\mathrm{SNR}_{12}}\right)$, i.e., the information sum-capacity. That is, the power-split $\beta_1 = \beta_2 = 1$ is always feasible. (See the corresponding intersections of the planes $B = b_1$ and $B = b_2$ with the volume $\mathcal{E}_{\rm FB}\left(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22}\right)$, where $b_1 \in (1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}]$ and $b_2 = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$, in Fig. 2).

Case 3: In the case in which $b \in (1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}, 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}]$, it follows that $0 \leqslant \beta^\star(b) < 1 - \left(\rho^\star(1,1)\right)^2$, and thus the individual rates are limited by $R_i < \frac{1}{2}\log_2\left(1 + \left(1 - \left(\rho^\star(1,1)\right)^2\right)\mathrm{SNR}_i\right)$. This immediately implies that any intersection of the volume $\mathcal{E}_{\rm FB}\left(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22}\right)$ with a plane $B = b$ is a set of triplets (R_1, R_2, b) for which the corresponding pairs (R_1, R_2) satisfy $(R_1, R_2) \in \mathcal{B}(b)$. This is basically due to the fact that $\mathcal{B}(b) = \mathcal{B}(b) \cap \mathcal{C}_{\rm FB}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12})$, since $\mathcal{B}(b) \subset \mathcal{C}_{\rm FB}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12})$. In this case, there exists a loss of sum-rate induced by the fact that at least one of the fractions β_1 and β_2 is smaller than one. More specifically, for these values of b, $R_i < \frac{1}{2}\log_2\left(1 + (1 - (\rho(1,1)^\star)^2)\mathrm{SNR}_{1i}\right)$ for at least one $i \in \{1, 2\}$, and thus this set does not contain the information sum-capacity rate pair. Indeed, for any $b > 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$, the set $\mathcal{B}(b)$ monotonically shrinks with b. This is clearly shown by Fig. 2.

Finally, note that when $b = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2(\rho^\star(1,1) + \epsilon)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$, for some $\epsilon > 0$, it holds that $\beta^\star(b) = 1 - \left(\rho^\star(1,1) + \epsilon\right)^2$. Substituting this into (19) and taking the limit when ϵ tends to 0, by the definition of $\rho^\star(1,1)$, the resulting value is given by (18) and thus $R_{\rm sum}^{\rm FB}(b)$ is a continuous function in b. Clearly, the maximum energy rate is achieved when $\beta_1 = \beta_2 = 0$, which implies that no information is conveyed from the transmitters to the receiver.

C. Energy Transmission Enhancement with Feedback

In this subsection, the enhancement of the energy transmission rate due to the use of feedback is quantified when the information sum-rate is the information sum-capacity without feedback. Denote by $B_{\rm NF} = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}$ the maximum energy rate that can be guaranteed at the EH in the G-MAC (without feedback) when the information sum-rate corresponds to the information sum-capacity without feedback. Denote also by $B_{\rm F}$ the maximum energy rate that can be guaranteed at the EH in the G-MAC-F when the information sum-rate is the information sum-capacity without feedback. The exact value of $B_{\rm F}$ is given by the following lemma.

Lemma 1. *The maximum energy rate $B_{\rm F}$ that can be guaranteed at the EH in the G-MAC-F when the information sum-rate is the information sum-capacity without feedback is*

$$B_{\rm F} = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\sqrt{(1-\gamma)\mathrm{SNR}_{21}\mathrm{SNR}_{22}}, \quad (23)$$

with $\gamma \in (0, 1)$ defined as follows:

$$\gamma = -\frac{\mathrm{SNR}_{11} + \mathrm{SNR}_{12}}{2\mathrm{SNR}_{11}\mathrm{SNR}_{12}} + \sqrt{\left(\frac{\mathrm{SNR}_{11} + \mathrm{SNR}_{12}}{2\mathrm{SNR}_{11}\mathrm{SNR}_{12}}\right)^2 + 2\left(\frac{\mathrm{SNR}_{11} + \mathrm{SNR}_{12}}{2\mathrm{SNR}_{11}\mathrm{SNR}_{12}}\right)}. \quad (24)$$

Proof: The proof of Lemma 1 is presented in [8]. ■

The following theorem provides an upper bound on $\frac{B_{\rm F}}{B_{\rm NF}}$.

Theorem 2 (Maximum Energy Rate Improvement with Feedback)**.** *Feedback can at most double the energy rate. That is,*

$$1 \leqslant \frac{B_{\rm F}}{B_{\rm NF}} < 2. \quad (25)$$

Proof: The proof of Theorem 2 follows immediately from Lemma 1. ■

IV. Conclusion and Extensions

This paper has characterized the information-energy capacity region of the two-user G-MAC-F and measured the energy transmission enhancement induced by the use of feedback. What is important to mention here is that SEIT requires additional transmitter cooperation/coordination. From this viewpoint, any technique that allows transmitter cooperation (i.e., feedback, conferencing, etc.) is likely to provide performance gains in SEIT in general multi-user networks. For instance, the results on the energy transmission enhancement induced by feedback in the two-user G-MAC-F can be extended to the arbitrary K-user G-MAC-F with $K \geqslant 3$. However, such a cooperation is usually not natural, especially if the transmitters do not share common information or are not co-located. Not surprisingly, this requires the transmitters to be "altruistic" and be always willing to cooperate to improve the overall system throughput. Consequently, the fundamental limits on SEIT take different forms depending on whether or not the network is centralized. In a decentralized network [10], each decision maker aims to maximize its own individual reward and its individual choice does not necessarily achieve the capacity of the network. In other words, the individual choice is not necessarily optimal from a global viewpoint. Hence, the information-energy capacity results are not sufficient to describe the fundamental limits on SEIT in decentralized networks.

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