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► **To cite this version:**

Lionel Bichet, Frédéric Dubois, Yann Monerie, Céline Pélissou, Frédéric Perales. Eigenerosion: extension and applications to heterogeneous media. CFRAC 2015, Jun 2015, Cachan, France. hal-01262580

**HAL Id: hal-01262580**

**<https://hal.science/hal-01262580>**

Submitted on 26 Jan 2016

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# *Eigenerosion: extension and applications to heterogeneous media*

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In 1921, Griffith introduced the concept of energy release rate in brittle fracture: below a certain amount of energy, no crack extension can occur. A variational formulation of this idea has been proposed by Francfort and Marigo [1] in the late 90's whose a regularization (1) has been recently developed by Schmidt *et al.* [4].

$$F_\epsilon(u, \epsilon^*, t) = \underbrace{\int_{\Omega} W(\epsilon(u) - \epsilon^*) dV}_{\text{strain}} - \underbrace{\int_{\Gamma_f} \bar{T} \cdot u dS}_{\text{load}} + G_c \underbrace{\frac{|\mathcal{C}_\epsilon|}{2\epsilon}}_{\text{crack}} \quad (1)$$

where  $\bar{T}$  is the applied traction on the part  $\Gamma_f$  of the boundary of the domain  $\Omega$ ,  $W$  is the elastic strain energy density of the body,  $\epsilon(u)$  is the strain operator of linear elasticity,  $\epsilon^*$  is an eigendeformation field [2],  $G_c$  is the critical energy-release rate and  $\epsilon$  is a small length which defines  $\mathcal{C}_\epsilon$  an  $\epsilon$ -neighbourhood to the crack,  $|\mathcal{D}|$  is the measure of any domain  $\mathcal{D}$ . When  $\epsilon$  tends to zero the standard Griffith's theory is recover.

The minimization of the functional (1) within the finite element framework was firstly realized by Pandolfi and Ortiz [3] in the context of homogeneous elastic media. These authors proposed to model the crack propagation associated to the eigenerosion using a 'killing element' method (deletion of elements verifying the crack propagation criterion). Two physical quantities areinvolved in this process.

- A **variation of energy**.

$$-\Delta E_K = \underbrace{\frac{1}{2} u^T \Delta S_K u}_{\text{element contribution}} + \underbrace{\frac{1}{2} u^T \Delta S_K S^{-1} \Delta S_K u}_{\text{structure contribution}} \quad (2)$$

where  $\Delta S_K$  is the stiffness matrix of the  $K$ -th element,  $S$  is the stiffness matrix of the meshed body where the element  $K$  is removed and  $u$  the displacement field before the erosion or equivalently before the assumed crack propagation. For sake of simplicity, [3] have taken into account only the element contribution in the equation (2).

- A crack length increment. For a crack occupying a domain  $C$ , the size of its extension is evaluated by a neighbourhood process [3]:

$$\delta A(K) = \frac{|(C \cup K)_\epsilon \setminus C_\epsilon|}{2\epsilon}, \quad (3)$$

where  $(C \cup K)_\epsilon$  is the current neighbourhood of the crack,  $C_\epsilon$  is the previous configuration of the crack-neighbourhood and  $K$  is the current killed element. When  $\epsilon$  tends to 0,  $\delta A(K)$  approaches the physical increment of the crack.

In the present work, 1/ we investigate a criterion with a both local-global energy, 2/ we propose an extension of the eigerosion method to fracture dynamics, 3/ we point out and clarify some technical problems related to the crack length increment (new crack-neighbourhood process, crack-boundary interactions, crack-crack interactions) and 4/ we extend the eigerosion method to heterogeneous media. Each development is validated on benchmark.

The extension to heterogeneous media is based on a local predictor-corrector method. The prediction phase consists in determining the elements that are candidates to the erosion process, i.e. that maximize the ratio  $-\Delta E_K / [\delta A(K) G_c(K)]$  over the structure, where  $G_c(K)$  denotes the critical energy release rate of the  $K$ -th element depending on the underlying phase. This extension to heterogeneous media is applied to an engineering problem: the study of the crack tortuosity in concrete media. The effect of two distinct parameters is studied: the aggregates volume fraction and the size polydispersity of the aggregates.

Moreover, the extension of the method to the 3D case is actually studied.

## References

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