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Frame-based Gaussian beam shooting from experimental far field data

I.F. Arias Lopez*

C. Letrou†

Abstract — Frame decomposition is conventionally used to perform decomposition of fields radiated by large planar apertures into a half plane, in a rigorous and stable way. An algorithm based on a spectral partitioning concept has been developed to extend frame-based Gaussian beam launching to omnidirectional and non directive sources. Frame decomposition is applied in six planes in the spectral domain, starting from the knowledge of the radiated far field, which is partitioned through a “partition of unity” procedure. This communication outlines this method, then focuses on its application and test in a real life context where initial far field data are obtained from measurements.

1 INTRODUCTION

Frame theory provides a rigorous framework to represent source field distributions in the form of superpositions of translated and phase shifted Gaussian windows [1, 2]. These Gaussian windows radiate fields in the form of paraxial Gaussian beams, in as much as their spectrum is sufficiently localized. Frame decomposition has thus been introduced as the initial discretization step in some Gaussian beam shooting (GBS) algorithms [3, 4]. It was also introduced to re-expand diffracted fields on planar obstacles or apertures, through a re-expansion algorithm performed in the course of Gaussian beam tracking in multi-reflecting environments [5].

Until recently, only planar source distributions were amenable to frame decomposition, and frame based GBS was better suited to highly directive radiating apertures than to less directive ones, due to inaccuracies occurring in paraxial beam expressions for highly tilted beams.

In order to generalize frame-based Gaussian beam shooting to a wider range of source distributions and of antennas, with preserved accuracy and rigor, we have developed an algorithm based on a 3d spectral domain partition.

Given an antenna far field, its plane wave spectrum (PWS) is known in any plane, hence in six planes, taken parallel to cube faces, radiating into

six half-spaces. A partition of unity is then formulated which allows to synthesize the 3d far field from the summation of fields radiated by these six spectra, each multiplied by a partitioning window. Frame decomposition is then applied to these “partial spectra”, in each of the six planes, enabling GBS from all the planes. Fields radiated by the six partial PWS are summed to obtain the 3d radiated fields.

Section 2 gives an outline of the “spectral partitioning” formulation. Section 3 presents its application to experimental horn fields, the initial far field data being obtained from measurements.

2 FORMULATION

In this section, frame decomposition is first presented as applied to functions of one variable, to introduce frame parameters and frame theory concepts, then to functions of two variables, in order to discretize field components of radiating planar distributions. Finally, six decompositions in six different planes are combined in order to discretize fields radiated by any type of source.

Harmonic time dependence $e^{-i\omega t}$, with ω the angular frequency, is assumed and suppressed in equations. The Fourier transform of a function $g \in L^2(\mathbb{R})$, denoted \tilde{g} , is defined as:

$$\tilde{g}(k_x) = \int_{-\infty}^{+\infty} g(x) e^{-ik_x x} dx$$

2.1 Frame based Gaussian beam shooting

In the $L^2(\mathbb{R})$ Hilbert space, the set of Gaussian functions

$$w_{mn}(x) = w(x - m\bar{x}) e^{in\bar{k}_x(x - m\bar{x})}, \quad (m, n) \in \mathbb{Z}^2$$

$$\text{with } w(x) = \sqrt{\frac{\sqrt{2}}{L}} e^{-\pi \frac{x^2}{L^2}}$$

is a frame if and only if $\bar{x}\bar{k}_x = 2\pi\nu$ with $\nu < 1$ (oversampling factor) [1]. \bar{x} and \bar{k}_x are respectively the spatial and spectral domain translation steps.

Frames are complete sets hence any function $f \in L^2(\mathbb{R})$ can be expressed as a weighted summation of the frame windows:

$$f = \sum_{(m,n) \in \mathbb{Z}^2} a_{mn} w_{mn} \quad (1)$$

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with the (non unique) a_{mn} complex coefficients called “frame coefficients”. These coefficients can be calculated by projecting the function on a “dual frame” of functions [2, 4].

By Fourier transform of equation (1), the decomposition of \tilde{f} on the frame obtained by translating the Gaussian window \tilde{w} is easily obtained as:

$$\tilde{f} = \sum_{(m,n) \in \mathbb{Z}^2} a_{mn} e^{imn\bar{k}_x \bar{x}} \tilde{w}_{mn} \quad (2)$$

As is visible from equations (1) and (2), frame coefficients can be calculated either in the initial domain or in the transformed one.

Frames in $L^2(\mathbb{R}^2)$ are defined as product frames: $w_{mnpq} = w_{mn}w_{pq}$, $(m, n, p, q) \in \mathbb{Z}^4$. If a y -polarized field distribution radiating into the $z > 0$ half-space is decomposed on such a frame, with A_{mnpq} the frame coefficients, then the radiated field at any point M with $z > 0$ is obtained as:

$$\vec{E}(M) = \sum_{m,n,p,q} A_{mnpq} e^{i(mnl_x \kappa_x + pqly \kappa_y)} \vec{B}_{mnpq}(M)$$

where \vec{B}_{mnpq} is the field radiated by the plane wave spectrum (PWS) associated to the w_{mnpq} frame window used to discretize the $E_y(x, y)$ function. For spectrally narrow frame windows, the \vec{B}_{mnpq} radiated field can be expressed in the form of a Gaussian beam (ray type expression with complex curvature), based on a paraxial approximation in the spectral domain [3, 4].

2.2 Spectral partitioning

In this subsection, partial PWS are defined in six different planes, by multiplying the PWS of the antenna in each plane by a “partitioning function”. The partitioning functions are introduced to obtain the field at any observation point outside of the reactive region of the radiating source by summing up the fields calculated by GBS from several of these planes, based on frame decomposition of the partial PWS.

2.2.1 Notations

The far field radiated by the antenna is assumed to be known in all (θ, ϕ) directions in the global coordinate system $(O, \hat{x}, \hat{y}, \hat{z})$. Six coordinate systems $\mathcal{S}_j = (O, \hat{x}_j, \hat{y}_j, \hat{z}_j)$, $j = 1, \dots, 6$, are introduced, and plane wave spectra are defined in the six $z_j = 0$ planes, called $\mathcal{P}_j = (O, \hat{x}_j, \hat{y}_j)$. Figure 1 gives an exploded view of the \mathcal{P}_j planes and of their associated spectral variables (k_{x_j}, k_{y_j}) . The PWS of the

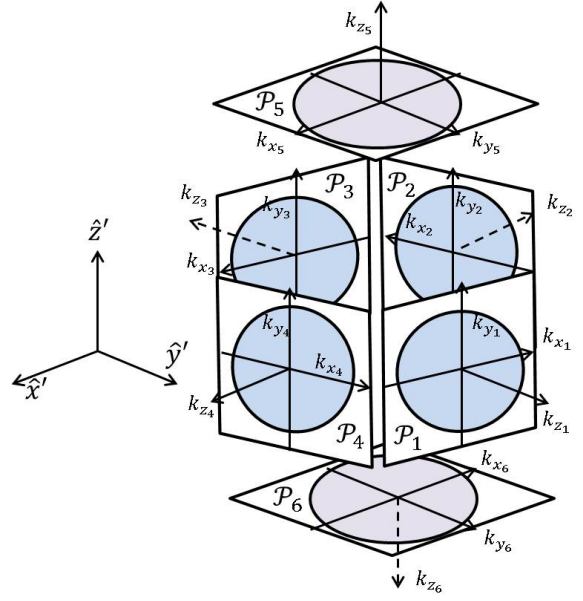


Figure 1: \mathcal{P}_j planes and associated spectral variables.

antenna in the \mathcal{P}_j plane is denoted $\vec{E}^{(j)}(k_{x_j}, k_{y_j})$ and radiates into the $z_j > 0$ half-space, noted \mathcal{H}_j .

Any point M in the 3d space, except for the origin O of all \mathcal{P}_j planes, belongs to exactly three different \mathcal{H}_j , $j \in J$ half-spaces.

2.2.2 Partition of unity relation

The classical asymptotic expression of the radiated far field in a given half-space as a function of a PWS radiating into this half-space, makes it easy to deduce each $\vec{E}^{(j)}$ PWS from the antenna far field in the \mathcal{H}_j half-space.

Partial plane wave spectra (after partitioning) are defined in the \mathcal{P}_j planes as:

$$\vec{E}^{\chi_j}(k_{x_j}, k_{y_j}) = \vec{E}^{(j)}(k_{x_j}, k_{y_j}) \chi_j(k_{x_j}, k_{y_j})$$

with $\chi_j(k_{x_j}, k_{y_j})$, $j = 1, \dots, 6$ partitioning functions.

In order to obtain the radiated far field of the antenna by summing up the fields radiated by all these partial PWS, the partitioning functions must verify [6]:

$$\sum_{j \in J} \chi_j(k_{x_j}, k_{y_j}) = 1$$

for any direction in the 3d space, defined by a wavevector \vec{k} . J is the set of indices such that $k_{z_j} > 0$, and (k_{x_j}, k_{y_j}) are the components of \vec{k} along the \hat{x}_j and \hat{y}_j vectors.

2.2.3 Partition of unity functions

The two variable partitioning functions are obtained:

- as the product of two one-variable partitioning functions of variables (k_{x_j} and k_{y_j}), in planes \mathcal{P}_j , $j = 1, \dots, 4$ (“lateral” planes);
- as a one-variable partitioning function of $k_{r_j} = \sqrt{k_{x_j}^2 + k_{y_j}^2}$, in planes \mathcal{P}_j , $j \in \{5, 6\}$ (planes oriented toward “bottom” and “top”).

With this choice, the χ_5 and χ_6 functions are rotationally symmetric around the z' axis. We also impose that $\chi_1 = \chi_3$ and $\chi_2 = \chi_4$.

The one-variable partitioning functions are derived from the Hann window, in order to minimize the widening effect in space of spectral truncation [7]. Figures 2 and 3 present the χ_1 and χ_5 functions, respectively.

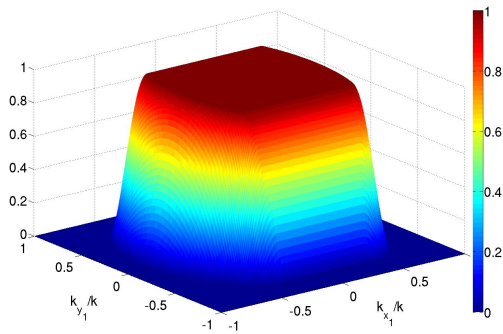


Figure 2: Two-variable χ_1 partitioning function.

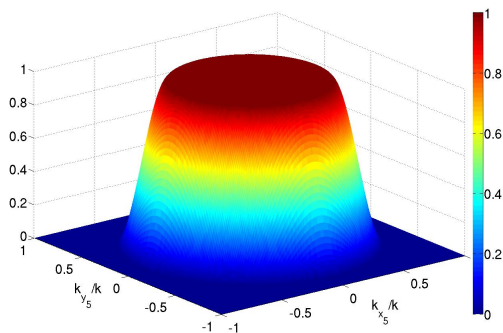


Figure 3: Two-variable χ_5 partitioning function.

3 NUMERICAL RESULTS

The algorithm presented in the previous section has been validated in the case of analytically

defined omnidirectional radiation patterns (half-wavelength dipole, dipole array). The far field of such antennas is synthesized by GBS from 6 planes where spectral partitioning has been applied to the initially known antenna PWS [6].

In real life contexts, antennas are generally known by their far field radiation patterns, and more accurate simulations (of propagation for instance) are obtained if measured antenna patterns are used to characterize the antenna radiation, instead of closed form theoretical expressions. We have applied the frame based GBS algorithm including spectral partitioning to the measured far field of a horn.

Frame coefficient computation involves a discrete Fourier transform in the spectral domain, and the required data points are on a regular grid in each of the six planes used for GBS, but by no means on a regular grid on a sphere centered on the antenna. For this reason, the measured far field data need to be interpolated in the course of frame coefficient calculation.

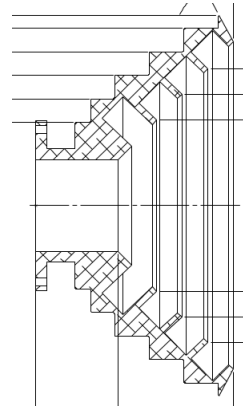


Figure 4: Corrugated horn (cut).

The measured horn is a corrugated horn shown in Figure 4. Its aperture diameter is 24 centimeters, and it was tested at 6 GHz on a sphere of radius 5 meters, in the far field of the antenna, with a 2 degree step along ϕ and θ . A rectangular horn is used as testing antenna. Co- and cross-polarizations (linear ones) patterns were measured.

Figures 5 and 6 present the measured and synthesized far field of this horn, which compare well. Figures 7 and 8 present a comparison of 1d patterns, for one cartesian component in the far field. Except for inaccuracies in measured data (alignment), the two patterns compare very well again.

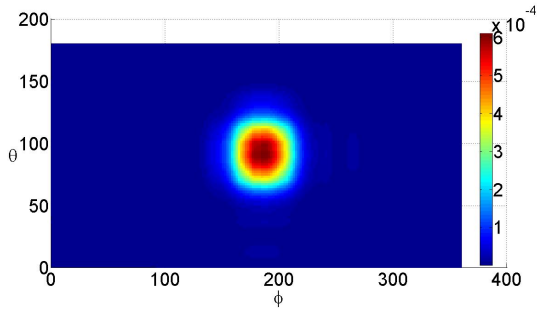


Figure 5: Horn measured far field magnitude.

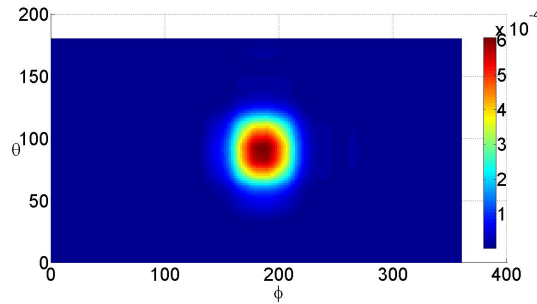


Figure 6: Horn far field magnitude synthesized by GBS.

4 CONCLUSION

The “spectral partitioning” algorithm has been shown to efficiently complement the usual frame-based Gaussian beam shooting method, in the case of non directive antenna fields. The method has been applied to experimentally known patterns (discretized measured data). The accuracy of the fields computed by GBS is assessed by comparison with the initial measured data in the far field. Results assessing the ability of the method to perform far to “near” (intermediate zone) field transformation will be presented at the conference.

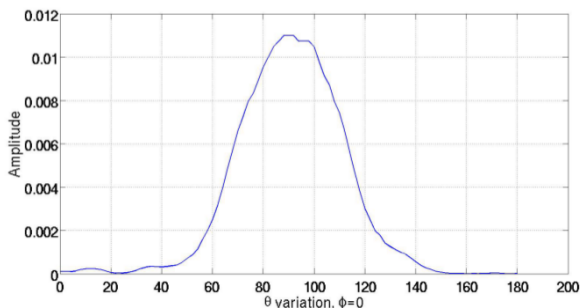


Figure 7: Horn measured far field component.

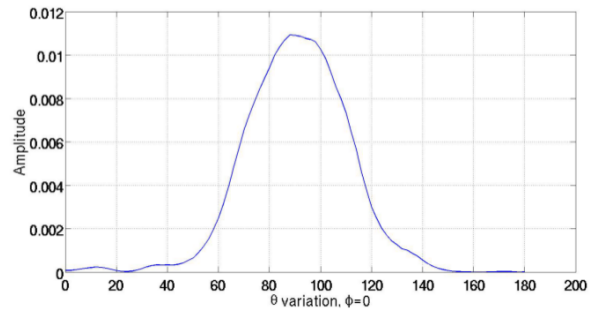


Figure 8: Horn far field component synthesized by GBS.

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