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## Logical Families of Nonlocal Boxes

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The well known nonlocal PR Box [1] correlates outputs  $(a, b)$  to inputs  $(x, y)$  in a two-party correlation by means of the logical constraint equation  $a \oplus b = x \wedge y$ . It violates the CHSH Bell Inequality (BI) [2] maximally : giving the number 4 beyond the quantum limit  $2\sqrt{2}$ , this last being obtained for a maximally nonlocal entangled quantum state (Bell state). The PR box is also no-signaling.

Here we consider families of boxes with all possible combinations of logical functions  $f$  correlated by the equation :  $f_o(a, b) = f_i(x, y)$ . The indexes  $o$  (output) and  $i$  (input) correspond to different Boolean functions and the variables  $(x, y, a, b) \in \{0, 1\}$  are Boolean. There are 16 different Boolean functions for 2 binary variables ( $n = 2$ ), these are ordered with increasing binary number in the truth table :  $f_0$  has the truth table  $(0, 0, 0, 0)$ ,  $f_1$  is NOR  $(1, 0, 0, 0)$  and so on... For example OR ( $\vee$ ) is  $f_{14}(0, 1, 1, 1)$  and in the PR Box, XOR (exclusive or :  $\oplus$ ) is  $f_{o=6}(a, b)$   $(0, 1, 1, 0)$  and AND ( $\wedge$ ) is  $f_{i=8}(x, y)$   $(0, 0, 0, 1)$ .

We define the joint mean value for the possible outcomes of the box as a function of the marginal probabilities :  $C_{x,y}^{(i,o)} = \sum_{a,b} [P(a, b | f_o(a, b) = f_i(x, y)) \cdot A \cdot B]$  [3]. The measurement outcomes  $A = 2a - 1$  (Alice) and  $B = 2b - 1$  (Bob) give the values  $\pm 1$ . The Bell parameter considering the four input possibilities is :  $S^{(i,o)} = C_{00}^{(i,o)} + C_{01}^{(i,o)} + C_{10}^{(i,o)} - C_{11}^{(i,o)}$ . If  $S > 2$  we have nonlocality. There are actually four options by changing the place of the "-" in the expression of  $S^{(i,o)}$ . We calculated every case and kept the value of  $S$  with the maximal absolute value. We also tested no-signaling by the condition :  $P(a | x, y) = P(a | x)$  and  $P(b | x, y) = P(b | y)$ .

For  $n = 2$ , there are  $16 \times 16 = 256$  equations. A family of 16 equations giving maximal violation of the BI ( $|S| = 4$ ) and no-signaling, corresponds to the following combinations of logical functions : output  $o : \{6, 9\}$  and input  $i : \{1, 2, 4, 8\}$  (8 cases) and also  $o : \{6, 9\}$  and  $i : \{7, 11, 13, 14\}$  (8 cases). The PR Box is part of this family ( $o = 6, i = 8$ ). Another family of 32 equations giving a BI violation  $|S| = \frac{10}{3} \approx 3.33 > 2\sqrt{2} > 2$ , exceeding the quantum limit, but now signaling, corresponds to :  $o : \{7, 11, 13, 14\}$  and  $i : \{1, 2, 4, 8\}$ , and the converse  $o : \{1, 2, 4, 8\}$  and  $i : \{7, 11, 13, 14\}$ . An example of this kind of nonlocal logical box is :  $a \vee b = x \wedge y$  ( $o = 14, i = 8$ ).

For three-party correlations,  $n = 3$ , we have 256 logical functions. The expression of the joint outcome mean value  $C_{x,y,z}^{(i,o)}$  is similar as before. We use the Svetlichny BI [4], which is written as follows :  $M^{(i,o)} = C_{000}^{(i,o)} + C_{001}^{(i,o)} + C_{010}^{(i,o)} + C_{100}^{(i,o)} - C_{011}^{(i,o)} - C_{101}^{(i,o)} - C_{110}^{(i,o)} - C_{111}^{(i,o)}$ , if  $M > 4$  we have non-locality.

$n = 3$  corresponds to  $256 \times 256 = 65536$  equations. Non-locality is obtained for six different values :  $M_{NL} = 8; 6.4; 6; 5.33; 4.8; 4.57$  with the corresponding number of cases  $N_{case}(M_{NL}) = 71; 560; 112; 840; 896; 560$ . The cases of non-locality and non-signaling are less and occur only for  $M_{NL+NS} = 8; 6; 5.33$ .

In this work we show that in the case of two-party correlations, we have two families of nonlocal boxes : one with 16 boxes, comprising the PR box, and the other one, to our knowledge unknown up to now, giving 32 nonlocal boxes beyond the quantum limit but signaling. For three-party correlations the number increases and the picture is much more complex.

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