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# Lower bounds for Scheduling Problems with production and consumption of resources

Abderrahim Sahli · Jacques Carlier · Aziz Moukrim

## 1 Introduction

The Event Scheduling Problem with Consumption and Production of Resources (ESPCPR) [4] is an extension of the Resource Constrained Project Scheduling Problem (RCPSP) where activities are replaced by events which can produce or consume the resources. ESPCPR consists in scheduling a group of events limited by precedence and resource constraints in order to minimize the makespan. Some other authors have worked on models similar to the ESPCPR. We can quote the works of Neumann and Schwindt [6] and of Laborie [5].

The aim of this paper is to introduce for ESPCPR new lower bounds whose calculation serves two goals. First, lower bounds can help in deleting useless nodes in a branch-and-bound tree in order to decrease the computation time. Second, they may help in evaluating the efficiency of heuristic methods when optimal solutions are not available.

The paper is organized as follows. We define in Section 2 the ESPCPR. In Section 3, we briefly present the Cumulative Scheduling Problem (CuSP) and we show how we can get a relaxation from ESPCPR to CuSP. In Section 4, we present our lower bounds and we conclude the paper in Section 5.

## 2 Problem description

An instance  $I = (X, U, a, v)$  of ESPCPR is defined by a set  $X = A \cup \{0, n+1\}$  of events where  $A = \{1, 2, \dots, n\}$  is the set of real events and 0 (resp.  $n+1$ ) is the fictitious beginning (resp. termination) event of the project,  $U$  is the set of precedence relations on the set  $X$  of events,  $a$  is the vector of resource production and consumption of events, and  $v$  is the matrix of *time lags*. The number of resource units produced or consumed by event  $i$  is defined by  $a_i$ , where  $a_0$  corresponds to the initial resource units of the project. If  $a_i < 0$ , then event  $i$  consumes  $|a_i|$  resource units, whereas if  $a_i > 0$ , it produces  $a_i$  resource units. A *schedule*  $S$  is a function assigning an occurrence time  $t_i$  to each event  $i \in X$ . The time lag from event  $i$  to event  $j$  is defined by  $v_{ij}$ . If  $v_{ij} \geq 0$ , then event  $j$  cannot occur before time  $t_i + |v_{ij}|$ ,

where  $t_i$  is the occurrence time of event  $i$ . If  $v_{ij} < 0$ , this implies that event  $i$  has to occur no later than time  $t_j + |v_{ij}|$ . At any time the resource availability has to remain positive or null. The makespan of a schedule  $S$  can be computed as  $C_{max} = t_{n+1}$ . A schedule is feasible if it satisfies all precedence and resource constraints. An optimal schedule is a feasible schedule which minimizes the makespan. In the case of multiple resources,  $K$  is the set of resources and  $a_i^k$  defines the quantity of resource  $k$  produced or consumed by event  $i$ , where  $k \in K$ .

### 3 Cumulative Scheduling Problem (CuSP)

In the Cumulative Scheduling Problem, a set  $I$  of  $n$  activities has to be scheduled without preemption in order to minimize the makespan. Each activity  $i$  has a processing time or duration  $d_i$ , a release date  $r_i$ , a tail or latency duration  $q_i$  and a resource requirement  $e_i$ , the availability of the resource is equal to  $R$ . Carlier and Pinson in [3] have adapted the Jackson's Pseudo-Preemptive Schedule (JPPS) to this problem in order to compute a lower bound with  $O(n \log n + nm \log m)$ -time complexity.

This problem is of prime interest for solving more complex scheduling problems like the ESPCPR. In fact, we can get a relaxation for ESPCPR based on CuSP as follows. We separate all events into two subsets. Let  $C$  contain all consumption events and  $P$  contain all production events. For each consumption event  $c$ , we compute its earliest starting time  $EST_c$  and store it as  $r_c$ . For each production event  $p$ , we compute its latest starting time  $LST_p$  and denote  $q_p = l_{0,n+1} - LST_p$ , which means latency duration or tail where  $l_{0,n+1}$  is the length of the longest path from the beginning event 0 to the termination event  $n+1$ . For a production event  $p$  and a consumption event  $c$ , if there exists a positive path from  $c$  to  $p$ , we denote  $l_{cp}$  the length of the longest path between  $c$  and  $p$ . So a bipartite graph  $G'$  is established  $G' = (C \cup P, U')$  where  $U' = \{(c, p) | c \in C, p \in P, l_{cp} \in \mathbb{N}^*\}$ , it defines a transportation problem ( $l_{cp}$  is the benefit).

A solution of the transportation problem is computed and transformed into a CuSP where each assignment of the resource is regarded as an activity. The resource flow between two events is converted into the resource required by the activity and  $l_{cp}$  into the duration of the activity.  $r_c$  is the release date and  $q_p$  is the tail.

Production and consumption events, which are not included in the solution, can also be transformed into activities by setting a hypothetic makespan  $C_{max}$ . Let  $P'$  (resp.  $C'$ ) be the set of the remaining production (resp. consumption) events and  $a'_p$  (resp.  $a'_c$ ) the new quantity to produce (resp. consume) by event  $p$  of  $P'$  (resp.  $c$  of  $C'$ ). For each production event  $p$  (resp. consumption event  $c$ ), it corresponds an activity  $i$  with release date  $r_i = 0$  (resp.  $r_i = LST_c$ ), a processing time  $d_i = EST_p$  (resp.  $d_i = C_{max} - LST_c$ ), a tail  $q_i = C_{max} - EST_p$  (resp.  $q_i = 0$ ) and a resource capacity requirement  $e_i = a'_p$  (resp.  $e_i = -a'_c$ ). The resource availability of this new instance which we denote  $CuSP(C_{max})$  is equal to  $\sum_{p \in P'} a'_p$ .

### 4 Lower bounds for ESPCPR

The first lower bound we present is a destructive bound based on JPPS that we compute as follows. First we fix a hypothetic makespan  $C_{max}$  and we extract an instance of the Cumulative Scheduling Problem  $CuSP(C_{max})$  as explained in the previous section. Then we apply JPPS to the corresponding instance to get a lower bound  $JPPS(C_{max})$ . If  $C_{max} < JPPS(C_{max})$  then  $C_{max} + 1$  is a lower bound for ESPCPR.

The second lower bound is based on the Shifting Algorithm. The Shifting Algorithm [1] [2] was introduced to solve the Financing Problem in polynomial time ( $O(n \log n)$ ). This problem is a special case of the ESPCPR, where the dates of production events are given. So to compute this bound, first we make a relaxation from ESPCPR to the Financing Problem by setting the production events at their earliest starting times and the consumption events at their latest starting times according to  $l_{0,n+1}$ . Then, we apply the Shifting Algorithm to the corresponding instance. At last, we take the makespan as a lower bound for ESPCPR.

Another destructive bound can be computed as follows. We fix the value of  $C_{max}$  and we set the date of production events at their earliest starting times and the date of consumption events at their latest starting times. If a resource conflict is detected then  $C_{max} + 1$  is a lower bound [2] [6].

The last lower bound is a destructive bound based on flow that we compute as follows. First, we introduce a bipartite graph  $G_I = (P \cup C, U_I)$ . The first part of the graph is the set of all production events  $P$  and the second part is the set of all consumption events  $C$ . We fix a hypothetic makespan  $C_{max}$  and we set the date of production events at their earliest starting times and the date of consumption events at their latest times. We consider an arc between a production event  $p$  and a consumption event  $c$ , if event  $p$  can start before event  $c$  which is obviously impossible if there exists a strictly positive path from  $c$  to  $p$ . If the flow problem defined by the graph  $G_I$  does not admit any solution then  $C_{max} + 1$  is a lower bound for ESPCPR.

## 5 Conclusion

In this paper, we have studied the Event Scheduling Problem with Consumption and Production of Resources (ESPCPR). We have shown how we can get a relaxation from ESPCPR to the Cumulative Scheduling Problem and the Financing Problem. Moreover, we have proposed three new lower bounds for this problem. We have tested them on the benchmark of Neumann and Schwindt [6]. Our lower bounds are very close to the optimal solution makespans. In fact, they reach them for more than 90.29% instances with 1.12% average deviation in percent. As a perspective, we aim to build a branch-and-bound method to solve the ESPCPR using our lower bounds.

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