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**HAL Id: hal-01259059**

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# Isotropic non-ideal cloaks providing improved invisibility by adaptive segmentation and optimal refractive index profile from ordering isotropic materials

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**Abstract:** Mimicking the ideal cloak, which is anisotropic and inhomogeneous, can be achieved by alternating homogeneous isotropic materials, whose permittivity and permeability of each isotropic coating can be determined from effective medium theory. An improved two-fold method is proposed by optimally discretizing the cloak and re-ordering the combination of the effective parameters of each layer to form a smooth step-index profile. The roles of impedance matching and index matching are investigated for cloaking effects. Smoothing the index profile leads to better invisibility than that obtained by smoothing the impedance profile, since the forward scattering can be further diminished. Nonlinear-transformation-based spherical ideal cloaks are studied, and improved design method is explored together with different segmentation schemes. Significant improvement in invisibility is always observed for the optimal segmentation in virtual space with the proposed two-fold design method no matter how nonlinear the coordinate transformation is.

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**OCIS codes:** (290.5839) Scattering, invisibility; (160.1190) Materials: Anisotropic optical materials; (230.3205) Invisibility cloaks

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## 1. Introduction

Coordinate transformation to manipulate electromagnetic fields [1] has received great attention [2, 3]. Perfect cloaks can be designed based on this transformation-optics (TO) technique which exploits the invariance of Maxwell's equations under a coordinate transformation provided that the material properties are changed accordingly. Thus a specific transformation in spaces can be regarded as an alteration in material's parameters. This idea however cannot be directly employed in the case of elastic cloaks [4]. There are also some non-TO-based cloak design methods based on scattering cancelation [5, 6] and homogenization technique [7, 8]. Nevertheless, such invisibility performance based on the long-wavelength limit severely relies on the core-shell ratio and the parameters of the cloaked material, and the wave still penetrates into the cloaked region.

TO-based cloaks in regular [9, 10, 11] and irregular [12, 13] shapes have been investigated. Significant reduction in the scattering has been experimentally verified for 2D cloaks, albeit for narrow bandwidths in the microwave regime and for objects at most a few wavelengths in diameter [14]. Flat cloaks with broader bandwidths have been proposed recently [15]. Those TO-based cloaks are designed in a forward manner, namely, one needs to know the transformation function beforehand so as to obtain the cloak's parameters. More recently, an implicit design

method in an inverse perspective was developed which can bypass the knowledge of transformation functions first (can be found finally) and all the parameters were uniquely determined [16]. Inspired by the classic spherical cloak [1] based on a linear coordinate transformation, the expressions of electromagnetic fields were explicitly presented [17] in terms of spherical Bessel functions based on extended Mie theory [18]. It was analytically shown that the singularity at cloak's inner surface corresponds to surface voltages [19] in spherical cloaks, which explains why the wave cannot either leave or enter the cloaked region.

In addition to those TO-based cloaks designed by linear coordinate transformations, nonlinear high-order transformation is being considered which is believed to possess more degrees of freedom in designing cloaks [11, 20, 21]. Also, from the newly proposed "inverse" mechanism of cloak design [16], the nonlinear-transformation-based cloaks, especially the power quadratic cloaks, will exhibit much better invisibility under the identical discretization scheme, and more importantly such improvement is even stable when the discretization is not very high (e.g., see Figs. 9 and 10 in [16]). Due to the fact that ideal cloaks (from linear or nonlinear transformation) require their parameters to be anisotropic and inhomogeneous, the analysis of cloak-wave interaction becomes more difficult and so does the real fabrication. For instance, the nonlinear-transformation-based (NTB) cloaks in [11] cannot be solved by the method proposed for classic spherical cloak [17]. Therefore a discretized non-ideal model has to be implemented in which each layer is homogeneously anisotropic. Considering the manufacture cost and difficulty, the discretization has to be reasonably finite, and thus the choices of coordination transformation [11] have significant impacts upon the success of designing practical non-ideal cloaks. However, after discretizing a NTB cloak, the model still has multiple anisotropic layers. By replacing each anisotropic layer with piecewise-homogeneous isotropic layers [22], then the non-ideal cloak becomes isotropic and in turn it facilitates the theoretical analysis of wave interactions as well as the experimental realization (e.g., transmission-line method verification [23, 24] to approximate the effective parameters).

In addition to the invisibility performance exhibited by the multilayered isotropic cloak as designed in [22], Ref.[22] also reveals that the reduction in backscattering is significant while scattering reduction in forward direction is not obvious compared with the bare PEC sphere. In this paper, we develop a two-fold method to achieve better invisibility than Ref.[22], i.e., self-adaptive optimal discretization of the NTB spherical cloaks, followed by re-ordering effective piecewise isotropic mediums. This improved method not only works for the improved invisibility of nonlinear spherical cloaks in general but also significantly minimize the forward scattering after applying the effective medium theory. The little sacrifice in the backscattering, due to the mismatch in impedance, is found to be very economic since the total cross section can be further reduced owing to the matching of indices compared with the results in [22]. The numerical results prove and validate the advantages of the current method.

## 2. NTB spherical cloaks and design method for improved invisibility

The configuration of a NTB spherical cloak and the schematic discretizations into isotropic multilayers are depicted in Fig. 1. The incident wave propagates along  $z$  axis with the electric field polarized along  $x$  axis. The core region ( $0 < r < R_1$ ) is a perfect electric conductor (PEC) and the shell region ( $R_1 < r < R_2$ ) is filled by the NTB cloak, characterized by the relative parameters  $\bar{\boldsymbol{\epsilon}}(r) = \epsilon_r(r)\hat{r}\hat{r} + \epsilon_t(r)(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi})$  and  $\bar{\boldsymbol{\mu}} = \mu_r(r)\hat{r}\hat{r} + \mu_t(r)(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi})$  ( $\epsilon_r(r) = \mu_r(r)$  and  $\epsilon_t(r) = \mu_t(r)$ ). The prescribed function  $f(r)$  to transform the original virtual space  $r'$  to the compressed physical space  $r$  is designed to be of a nonlinear class

$$r' = f(r) = \frac{r^x - R_1^x}{R_2^x - R_1^x} R_2, \quad (1)$$

where  $x$  denotes the nonlinearity of the spatial compression. Based on the well-established TO method [1, 3], the parameters of such NTB spherical cloaks can be simply determined

$$\varepsilon_r = \mu_r = \frac{R_2(r^x - R_1^x)^2}{x(R_2^x - R_1^x)r^{x+1}} \quad (2a)$$

$$\varepsilon_t = \mu_t = \frac{xR_2r^{x-1}}{R_2^x - R_1^x}. \quad (2b)$$

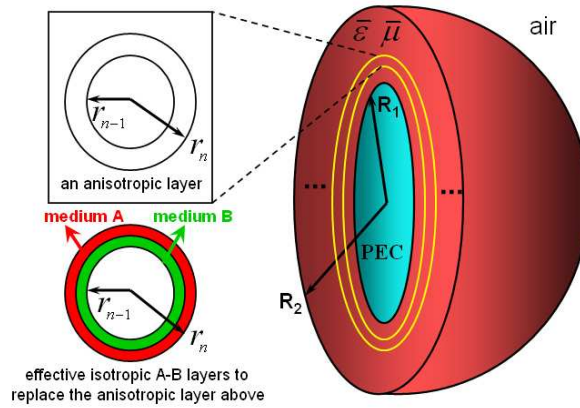


Fig. 1. The geometry of a NTB spherical cloak, which is to be discretized into totally  $M$  anisotropic layers (the upper-left illustration shows the geometry of  $n$ -th anisotropic layer whose width is  $r_n - r_{n-1}$ ). The thickness of each anisotropic layer may not be the same to maintain good invisibility. Each anisotropic layer will be divided into 2 isotropic layers of equal thickness, characterized by medium A and medium B as the lower-left illustration shows. Therefore the whole non-ideal cloak becomes an AB $\cdots$ AB isotropic multilayered structure with the total layer number of  $2M$ . In this paper,  $M = 40$  and  $R_2 = 2R_1 = 2\lambda$  are chosen.

For those NTB cloaks whose parameters are shown as in Eq.(2), it is clear that there is no analytical solution to model the wave interaction with the cloak such as [17]. Instead, one can directly consider a multilayered anisotropic structure and use scattering matrix method to determine the cloak's scattering properties [11] where the scattering theorem in each anisotropic layer has been studied [18]; or one can alternatively apply the effective medium theory and then consider an isotropic multilayered structure [22]. Even though the scattering is significantly reduced compared to the scattering without the cloak, there are still two critical drawbacks: (1) when the transformation is nonlinear, equally dividing the physical space as the discretization scheme will give rise to unwanted perturbation because the parameters are varying very fast in certain regions; (2) albeit effective medium theory makes the model isotropic and thus easier to realize, it is still quite visible in the forward direction due to the large shadow.

Note that for the classic spherical cloak [1], its prescribed function  $f(r)$  is a linear straight line with respect to  $r$  (i.e.,  $x = 1$  in Eq. (1)), and thus it does not make difference whether one equally discretizes the cloak in either physical (i.e.,  $\Omega(r)$ ,  $R_1 < r < R_2$ ) or virtual (i.e.,  $\Omega'(r')$ ,  $0 < r' < R_2$ ) spaces. On the contrary, given a nonlinear prescribed function  $r' = f(r)$  between the virtual space and the physical space, the self-adaptive stepwise segmentation in physical space  $r$ —finer segmentation is needed where  $f(r)$  turns to be more steep—is desired to mimic the transformation function. Hence it is obvious to find that the most economic way is to equally discretize the virtual space  $r'$  into  $M$  layers and project them onto physical space

$r$  via the transformation curve. Then we can obtain a set of  $r_n$  which automatically control the steps according to the slope of  $f(r)$ , e.g., in the region where  $f(r)$  is changing dramatically, more discretized anisotropic layers are assigned, and less for the region where  $f(r)$  seems flat. In this paper, this self-adaptive scheme is adopted throughout unless stated otherwise (e.g., for comparison purposes in Figs. 7 and 8), and its advantage is also justified by comparing with the cloaking performance obtained from directly dividing the physical space  $r$  into  $M$  layers of identical thickness. By projecting the equal segmentation in  $r'_n = R_2 \cdot n/M$  onto the physical  $r$ , one has

$$r_n = f^{-1}(r'_n) = \left[ \frac{R_2^x - R_1^x}{R_1^x} \frac{n}{M} + 1 \right]^{1/x} \cdot R_1, \quad n=1, 2, \dots, M, \quad (3)$$

for the upper-left illustration in Fig. 1.

In the mean time, an improved set of effective mediums is proposed, which not only makes the anisotropic cloak isotropic but also lowers down the forward scattering and total scattering cross section dramatically. Since it is already known that  $\epsilon_r(r) = \mu_r(r)$  and  $\epsilon_t(r) = \mu_t(r)$ , the **old** set of material parameters of isotropic medium-A and medium-B dielectrics can be obtained [22]:

$$\epsilon_A = \mu_A = \epsilon_t + \sqrt{\epsilon_t^2 - \epsilon_t \epsilon_r} \quad (4a)$$

$$\epsilon_B = \mu_B = \epsilon_t - \sqrt{\epsilon_t^2 - \epsilon_t \epsilon_r}. \quad (4b)$$

However, based on the Sten's formula [25], we find the following **new** set

$$\epsilon_A = \mu_B = \epsilon_t + \sqrt{\epsilon_t^2 - \epsilon_t \epsilon_r} \quad (5a)$$

$$\epsilon_B = \mu_A = \epsilon_t - \sqrt{\epsilon_t^2 - \epsilon_t \epsilon_r}, \quad (5b)$$

which also satisfy effective medium theory

$$\zeta_t = (\zeta_A + \zeta_B)/2 \quad (6a)$$

$$\zeta_r = 2\zeta_A\zeta_B/(\zeta_A + \zeta_B), \quad \zeta = \epsilon \text{ or } \mu. \quad (6b)$$

To maintain the accuracy of the effective medium theory,  $M$  cannot be too small (we choose  $M = 40$  herein). Nevertheless, based on the optimization algorithm, a much smaller  $M$  can be used as shown in [26] for cylindrical cases but a subtle deviation in optimized parameters will impair the invisibility performance greatly.

It is also important to note that, during the discretization from one anisotropic layer to two isotropic A-B layers, we have to choose a position within  $r_{n-1}$  and  $r_n$  to determine the anisotropic parameters in Eq. (2), and thus we assume the thin shell in the upper-left illustration in Fig. 1 has uniform anisotropic parameters. Now, one can apply either Eq. (4) or Eq. (5) to derive the isotropic parameters needed in the lower-left illustration in Fig. 1. However, it is found that this pre-chosen position within a thin shell cannot be randomly put even if each discretized shell itself is thin enough to be treated as a homogeneous anisotropic shell. In Fig. 2, we consider three simple choices of this pre-chosen position for the  $n$ -th anisotropic shell, i.e., the left ( $r = r_{n-1}$ ); the middle ( $r = (r_{n-1} + r_n)/2$ ); and the right ( $r = r_n$ ). It can be seen that, those three choices in Fig. 2 will lead to significantly distinct performances in scattering reduction as shown in Fig. 3.

More importantly, as observed in Fig. 3, for the **new** set of effective isotropic materials, selecting the parameters corresponding to the middle point **M** of each discretized anisotropic

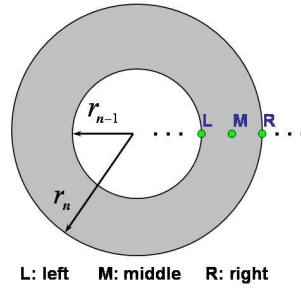


Fig. 2. The shadowed region denotes the  $n$ -th discretized anisotropic layer as in the upper-left illustration in Fig. 1. Before converting an discretized anisotropic layer into 2 isotropic sub-layers, we consider three positions generally (i.e., left  $\rightarrow r = r_{n-1}$ ; middle  $\rightarrow r = (r_{n-1} + r_n)/2$ ; right  $\rightarrow r = r_n$ ). In each case, this anisotropic layer's parameters, determined from Eq. (2), are uniform and constant within.

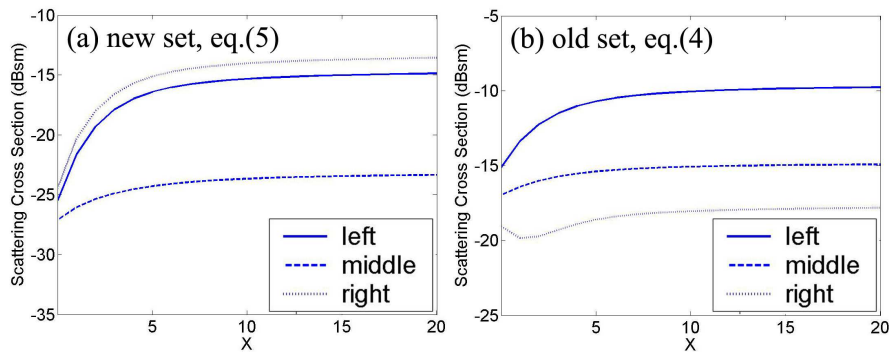


Fig. 3. The scattering cross section of the NTB spherical cloak versus the transformation nonlinearity, approximated by an isotropic non-ideal cloak. Three choices of positions (**L**, **M**, **R** in Fig. 2) under two sets of isotropic materials (a) *new* set and (b) *old* set are investigated and compared.

layer as a uniform anisotropic layer will dramatically outperform the other two choices (i.e., the left **L** and right **R**) especially the coordinate transformation becomes more nonlinear, and this invisibility improvement is quite stable along with the increase in the nonlinearity factor  $x$ . On the contrary, for the *old* set, selecting the right point **R** in Fig. 2 gives rise to lowest total scattering cross section. Nevertheless, for nonlinear transformation based cloaks, the choice of the middle point **M** under the *new* set leads to the most improved invisibility for isotropic non-ideal cloaks. Therefore, the choice of middle point  $M$  is adopted in the following.

### 3. Numerical results and verifications of improved cloaking effects

We will consider  $x = 0.1$ ,  $x = 4$ , and  $x = 20$  in Eq. (1) corresponding to small, medium, and large nonlinear transformations, respectively. In Fig. 4, the bistatic radar cross section (RCS) is plotted to demonstrate the invisibility as well as the improvement of the *new* isotropic cloak compared with the *old* one. It can be observed that the non-ideal cloak made of either *old* set of effective isotropic layers in Eq. (4) or *new* set in Eq. (5) are able to reduce the scattering induced by the original bare PEC sphere. However the forward scattering of the *old* set becomes even larger than that of a bare PEC sphere, which is not desired (*see* blue lines in Fig. 4). It can be overcome by considering the *new* set which further lowers down the forward scattering by



more than 25 dBsm, and this huge improvement is stable even when the spatial compression is quite nonlinear (i.e., very large values of  $x$ ), thanks to the proposed self-adaptive discretization scheme. There is a small sacrifice in backscattering of the improved *new* set compared with the backscattering of the *old* set, but it is evidently worthwhile because the backscattering of the *new* set is still negligible.

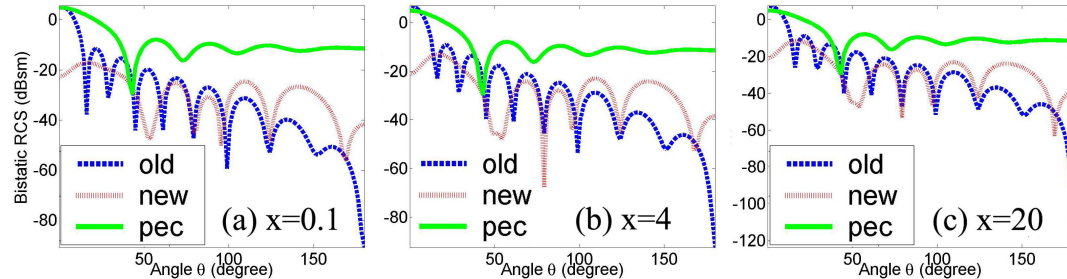


Fig. 4. Bistatic RCS for (a)  $x=0.1$ ; (b)  $x=4$ ; and (c)  $x=20$ . The RCS is normalized by  $\lambda_0^2$ , where  $\lambda_0$  denotes the wavelength in free space. The curves characterized by *old* and *new* correspond to the isotropic non-ideal cloaks designed by Eq. (4) and Eq. (5), respectively.

From Fig. 5, we consider the forward as well as total scattering cross sections (SCS) under various nonlinear transformations for  $f(r)$ . It is clear to see that the *new* set leads to much lower scattering particularly in forward direction, which in turn brings down the total SCS. It can be concluded that the reduction in forward scattering significantly contributes to the total scattering suppression, and the proposed self-adaptive scheme in Eq.(3) makes the improved invisibility pronounced continuously for NTB spherical cloaks at arbitrary nonlinearity  $x$  in spatial transformation.

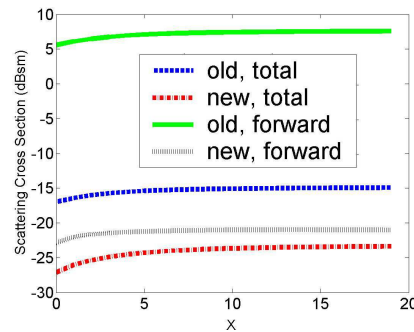


Fig. 5. Total and forward scattering cross sections for isotropic non-ideal cloaks designed by the *old* set (blue and green lines) and *new* set (red and black lines) versus the nonlinearity  $x$  in the coordinate transformation.

To verify the cloaking effects, we thus present the near-field distribution of the electric field in the presence of the NTB spherical cloak enclosing a PEC core. Here we only focus on two extreme cases, i.e, when  $x$  is very small and  $x$  is very large. Fig. 6 reveals that even though the far-field diagrams for  $x = 0.1$  and  $x = 20$  are quite similar as Fig. 4 shows, their corresponding near fields, especially in the cloak shell region, are distinct. In Fig. 6, it is shown that when  $x$  increases, the field intensity inside the cloak shell will be increased significantly while the nearly undisturbed wave fronts still hold outside the cloak ( $r > R_2$ ) as shown in the inset of Fig. 6(b). In addition, those high-intensity areas will be more squeezed into the region towards

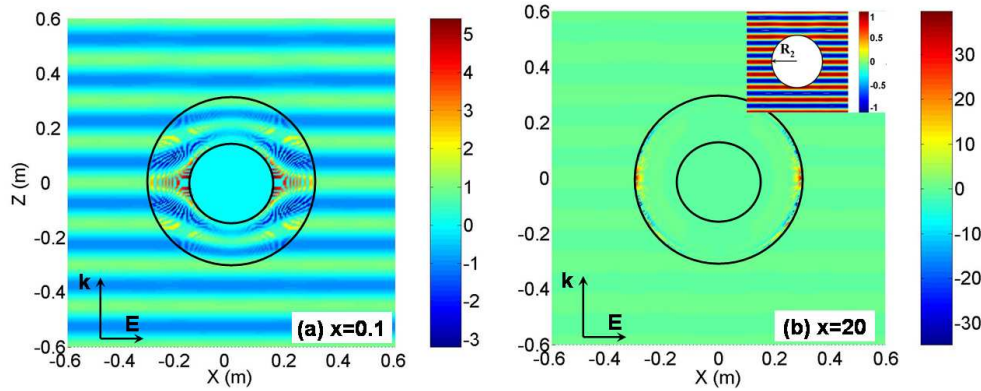


Fig. 6. The real part of  $E_x$  on the  $x$ - $z$  plane of the improved cloaks designed by *new* set for (a) $x=0.1$ ; (b)  $x=20$ . The inset in (b) only shows the total electric field outside the cloak (i.e.,  $r > R_2$ ) excluding the region in white ( $0 < r < R_2$ ). The working frequency is chosen to be 2GHz.

the outer radius  $R_2$  when  $x$  is getting larger. The highly nonlinear transformation based cloak can be realized as an isotropic non-ideal cloak, restore the invisibility after the self-adaptive discretization, and produce improved invisibility by using the *new* set of effective mediums.

To provide more insight into this two-fold improved method, the self-adaptive discretization based on equally dividing virtual space (denoted by “virtual”) and the traditional discretization based on equally dividing physical space (denoted by “physical”) are compared with each other together with the comparison between *new* and *old* sets of isotropic parameters in respective cases as shown in Fig. 7.

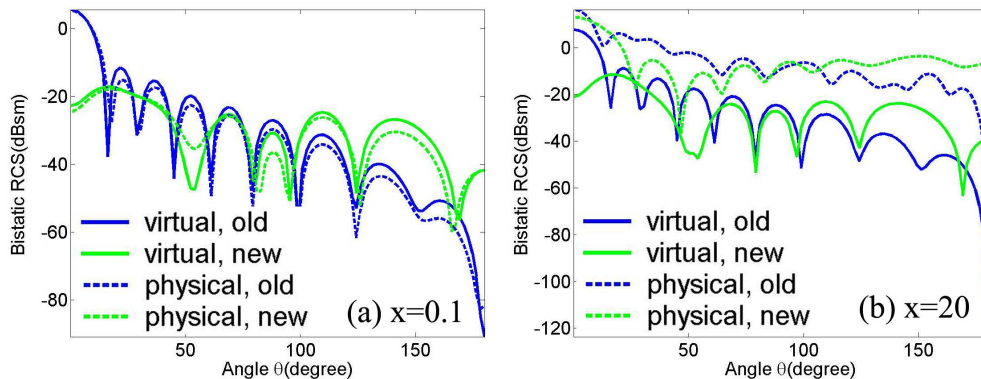


Fig. 7. The bistatic RCSs of the isotropic cloaks designed by *new* and *old* sets for (a) $x=0.1$ ; (b)  $x=20$ . In each set, “virtual” and “physical” cases of discretizations are compared.

It is interesting to point out that, in Fig. 7(a), the improved invisibility is primarily induced by the *new* set of parameters designed in Eq. (5) while the difference in choosing “virtual” or “physical” discretization scheme is negligible. It is due to the fact that when  $x = 0.1$  the transformation function  $f(r)$  is nearly linear against  $r$ . Under such circumstance, if one equally discretizes the ideal NTB cloak in virtual space and then projects onto physical space, the stepping intervals will be almost the same as those obtained from equally dividing the physical space directly. However, if  $x$  becomes large enough, namely,  $f(r)$  is quite nonlinear, one has to

adopt the current two-fold method to achieve improved invisibility: i) “virtual” discretization scheme is used; ii) the new *set* of effective parameters is used. One can observe that the “virtual” scheme is even more important than adopting the *new* set of parameters by inspecting blue solid line and green dashed line in Fig. 7(b): even though the former (blue solid line in Fig. 7(b)) uses *old* set in Eq. (4), both the total and forward scattering are much smaller than the latter (green dashed line in Fig. 7(b)) based on the *new* set in Eq. (5). Of course, it is evident that our current two-fold method (“virtual, new” corresponding to the green solid line) clearly outperforms all other cases in terms of scattering reduction.

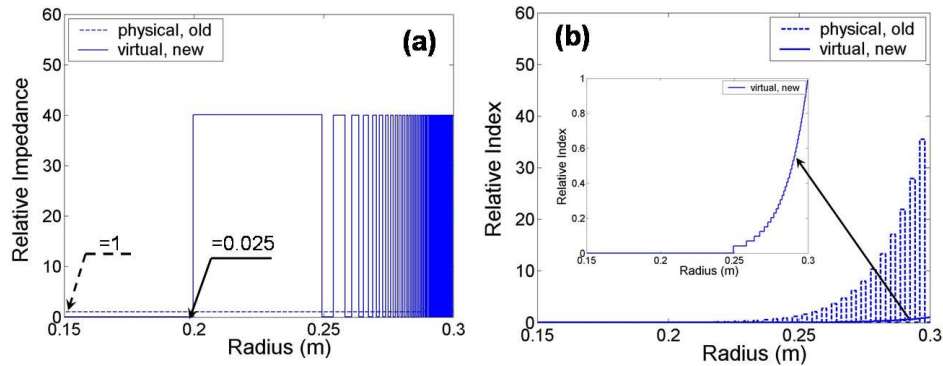


Fig. 8. The impedance (a) and index (b) profiles of  $x = 20$  isotropic cloaks for two cases: 1) the traditional *old* set when the ideal  $x = 20$  cloak is equally divided in “physical” space; 2) the current two-fold method

Of particular interest is the study of the roles of impedance and index matching in such an isotropic non-ideal cloak for  $x = 20$  as shown in Fig. 7(b). We compare the impedance and index profiles for the worst (“physical, old”) and the best candidates (“virtual, new”) in Fig. 8. It can be seen in Fig. 8(a) that the impedance is always matched with that of free space for “physical, old” case designed by equally dividing the physical space and using Eq. (4) while the relative impedance of isotropic cloak for “virtual, new” based on our two-fold method has only two values, i.e., 0.025 and 40 which changes in an alternating manner in the cloak region. In Fig. 8(b), our two-fold method leads to a much smoother index profile which matches that of the free space at the outer radius  $R_2$ . It can be seen that, for these isotropic non-ideal cloaks realized from ideal NTB anisotropic cloaks, a smooth index profile would be more crucial in achieving improved invisibility than a smooth impedance profile. The index matching will diminish the forward scattering further down though a negligible increase in backscattering may occur, while the impedance match is mainly efficient in removing the backscattering. Therefore, the effective medium of each interlayer has to be properly determined to take into account the slope of  $f(r)$  and as the index profile.

#### 4. Conclusion

In this paper, we propose a two-fold method to design isotropic cloaks with significantly improved invisibility by considering the slope of coordinate transformation function and the index profile of the cloak. Such improved method can automatically adjust the interval of stepwise discretization according to a given nonlinear coordinate transformation, which leads to the suppression of total scattering but with a spoiled forward scattering. Then the following *new* set of effective medium with oscillating impedance profile but smooth index profile further lowers down the scattering in forward scattering with a rewarding sacrifice in backscattering which is

still weak enough to be ignored.

### **Acknowledgement**

The authors would like to express their acknowledgment of support by Grant R-263-000-574 from the National University of Singapore. This research was supported in part by CNRS (France) in the framework of the Program INTERNATIONAL PROJECT FOR SCIENTIFIC COOPERATION (PICS) France-Singapore.