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Non-symmetrized noise in a quantum dot: interpretation in terms of photon emission and coherent superposition of scattering paths

R. Zamoum^{1,2}, M. Lavagna³, and A. Crépieux¹

¹ Aix Marseille Université, Université de Toulon,

CNRS, CPT UMR 7332, 13288 Marseille, France

² Faculté des sciences et des sciences appliquées,

Université de Bouira, rue Drissi Yahia, Bouira 10000, Algeria and

³ Université Grenoble Alpes INAC-SPSMS F-38000 Grenoble, France

 $CEA \ INAC-SPSMS \ F-38000 \ Grenoble, \ France$

We calculate the non-symmetrized current noise in a quantum dot connected to two reservoirs in the presence of interactions in the dot by using non-equilibrium Green function technique. We show that both the self-noise (inside a single reservoir) and the cross-noise (between both reservoirs) are expressed in terms of transmission amplitude and coefficient through the barriers. We identify the different photon emission processes involved in each contribution to the self-noise and highlight the fact that when there are several physical processes, the contribution results from a coherent superposition of scattering paths. Varying the gate and bias voltages, we discuss the profile of the differential Fano factor in the light of recent experiments.

Since the beginning of the 2000s, it is known that to characterize finite-frequency current fluctuations in quantum conductors, one has to measure and calculate the non-symmetrized noise which corresponds to the emission noise at positive frequency and to the absorption noise at negative frequency [1, 2]. This is related to the fact that the current operators do not commute for quantum systems [3]. In taking the symmetrized noise, one mixes the emission and absorption noises and the relevant information is lost. Despite a growing interest for non-symmetrized finite-frequency (NSFF) noise in quantum system both experimentally [4–12] and theoretically [13–27], there is no clear interpretation of the physical processes that contribute to the noise, even for non- interacting systems. Here we show that it makes sense to interpret each contribution to the NSFF selfnoise in terms of photon emission process, following the ideas developed by several authors [28–34]. To address this issue from a theoretical point of view, one can use either the scattering theory [35, 36] or the non-equilibrium Green function (NEGF) technique [37, 38]. Whereas the former method applies to non-interacting systems, the latter can be extended to interacting ones. However, the NEGF technique has been used until now to calculate the symmetrized noise only [39].

In this Letter, we present two methods to derive the non-symmetrized noise spectrum of a quantum dot (QD) connected to two reservoirs. The first method is based on the NEGF technique and allows us to express the NSFF self-noises and cross-noises in terms of the transmission amplitude and transmission coefficient through the barriers which themselves incorporate both transfer and interaction effects. The second method is based on a detailed analysis of all the different physical processes that contribute to the self-noises, paying attention to considering the coherent superposition of scattering paths when more than one process are involved. The cross-noises cannot be obtained within the second method, but for the selfnoises the two methods lead to the same expressions. We benchmark our results with the known results existing in the literature in some given limits, notably in the noninteracting limit when the scattering theory can be used.

We consider a single level QD connected to two reservoirs described by the Hamiltonian $H = H_L + H_R + H_T + H_D$, where $H_\alpha = \sum_{k \in \alpha} \varepsilon_k c_k^{\dagger} c_k$ is the Hamiltonian of the left ($\alpha = L$) and right ($\alpha = R$) reservoirs, $H_T = \sum_{\alpha = L,R} \sum_{k \in \alpha} (V_k c_k^{\dagger} d + h.c.)$ is the transfer Hamiltonian, and $H_D = \varepsilon_0 d^{\dagger} d$ is the Hamiltonian of the QD. $c_k^{\dagger}(d^{\dagger})$ and $c_k(d)$ are the creation and annihilation operators of one electron in the reservoirs (QD). The quantities ε_0 , ε_k and V_k are respectively the QD energy level, the energy of an electron with momentum k in the reservoirs and the transfer matrix element between the corresponding states. The spin degree of freedom can be included without any complication.

The objective is to calculate the NSFF noise defined as $S_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} S_{\alpha\beta}(t,0) e^{-i\omega t} dt$, where $S_{\alpha\beta}(t,0) = \langle \Delta \hat{I}_{\alpha}(t) \Delta \hat{I}_{\beta}(0) \rangle$ is the current correlator and $\Delta \hat{I}_{\alpha}(t) = \hat{I}_{\alpha}(t) - \langle \hat{I}_{\alpha} \rangle$ with \hat{I}_{α} , the current operator from the α reservoir to the central region through the α barrier given by: $\hat{I}_{\alpha} = (ei/\hbar) \sum_{k \in \alpha} (V_k c_k^{\dagger} d - V_k^* d^{\dagger} c_k)$, and $\langle \hat{I}_{\alpha} \rangle$ its average value.

Following Haug and Jauho [38], we first substitute the expression of the current operator in the correlator and get [40]

$$S_{\alpha\beta}(t,t') = \frac{e^2}{\hbar^2} \sum_{k \in \alpha k' \in \beta} \left[V_k V_{k'} G_1^{cd,>}(t,t') - V_k V_{k'}^* G_2^{cd,>}(t,t') - V_k^* V_{k'} G_3^{cd,>}(t,t') + V_k^* V_{k'}^* G_4^{cd,>}(t,t') - \langle \hat{I}_{\alpha} \rangle \langle \hat{I}_{\beta} \rangle , \qquad (1)$$

where $G_i^{cd,>}(t,t')$, with i = 1 to 4, are the greater components of the two-particle Green functions mixing c_k and d operators [41]. We introduce $G_i^{cd}(\tau, \tau')$, the contour-ordered (along the Keldysh contour C) counterparts of $G_i^{cd,>}(t,t')$ as well as $S_{\alpha\beta}(\tau,\tau')$, the contourordered counterpart of $S_{\alpha\beta}(t,t')$. We then derive and solve the equations of motion for $G_i^{cd}(\tau,\tau')$ in order to express them in terms of: (i) the contour-ordered oneparticle Green function for the disconnected reservoirs defined as $g_k(\tau,\tau') = -i\langle T_C c_k(\tau) c_{k'}^{\dagger}(\tau') \rangle_0$, where T_C is the contour-ordering operator, (ii) the contour-ordered one-particle Green function for the QD connected to the reservoirs, $G(\tau,\tau') = -i\langle T_C d(\tau) d^{\dagger}(\tau') \rangle$, and (iii) the contour-ordered two-particle Green functions for the QD, $G_i^{(2)}(\tau,\tau',\tau_1,\tau_2)$ [42]. After a series of manipulations, we get

$$\begin{aligned} \mathcal{S}_{\alpha\beta}(\tau,\tau') &= \delta_{\alpha\beta} \tilde{\mathcal{S}}_{\alpha}(\tau,\tau') + \frac{e^2}{\hbar^2} \sum_{k \in \alpha, k' \in \beta} |V_k V_{k'}|^2 \\ &\times \iint d\tau_1 d\tau_2 \Big[-g_k(\tau_1,\tau) g_{k'}(\tau_2,\tau') G_1^{(2)}(\tau,\tau',\tau_1,\tau_2) \\ &+ g_k(\tau_2,\tau) g_{k'}(\tau',\tau_1) G_2^{(2)}(\tau,\tau',\tau_1,\tau_2) \\ &- g_k(\tau,\tau_1) g_{k'}(\tau_2,\tau') G_3^{(2)}(\tau,\tau',\tau_1,\tau_2) \\ &- g_k(\tau,\tau_1) g_{k'}(\tau',\tau_2) G_4^{(2)}(\tau,\tau',\tau_1,\tau_2) \Big] - \langle \hat{I}_{\alpha} \rangle \langle \hat{I}_{\beta} \rangle , \end{aligned}$$

where $\tilde{S}_{\alpha}(\tau, \tau') = (e^2/\hbar^2) \sum_{k \in \alpha} |V_k|^2 [g_k(\tau', \tau)G(\tau, \tau') + g_k(\tau, \tau')G(\tau', \tau)]$. To go further, one needs to write and solve the equation of motion for the two-particle Green functions for the QD, which is an ambitious task. By making a Hartree-Fock level approximation to the QD two-particle Green functions which consists in factorizing it into a product of two QD one-particle Green functions, we get [43]

$$S_{\alpha\beta}(\tau,\tau') = \delta_{\alpha\beta}\tilde{S}_{\alpha}(\tau,\tau') + \frac{e^2}{\hbar^2} \sum_{k\in\alpha,k'\in\beta} |V_k V_{k'}|^2 \\ \times \iint d\tau_1 d\tau_2 \Big[-g_k(\tau_1,\tau)g_{k'}(\tau_2,\tau')G(\tau,\tau_2)G(\tau',\tau_1) \\ +g_k(\tau_2,\tau)g_{k'}(\tau',\tau_1)G(\tau,\tau')G(\tau_1,\tau_2) \\ +g_k(\tau,\tau_1)g_{k'}(\tau_2,\tau')G(\tau',\tau)G(\tau_1,\tau_2) \\ -g_k(\tau,\tau_1)g_{k'}(\tau',\tau_2)G(\tau_2,\tau)G(\tau_1,\tau') \Big] .$$
(3)

Next, we perform an analytical continuation of Eq. (3), we substitute the one-particle Green functions for the reservoirs and the QD by their expressions given in Ref. 38 and we make a Fourier transform, assuming that all the Green functions are time translation invariant (steady state). Moreover, the coupling strength between the QD and the reservoirs, $\Gamma_{\alpha}(\varepsilon) = 2\pi |V_{\alpha}(\varepsilon)|^2 \rho_{\alpha}(\varepsilon)$, with $\rho_{\alpha}(\varepsilon)$ the density of states in the reservoir α and $V_{\alpha}(\varepsilon)$ the Fourier transform of $V_{k\in\alpha}$, is assumed to be energy independent (exact in the wide band limit). In addition, we take symmetric coupling: $\Gamma_L = \Gamma_R = \Gamma$ [44]. Within these assumptions, we obtain the NSFF noise

$$S_{\alpha\beta}(\omega) = \frac{e^2}{h} \sum_{\gamma\delta} \int_{-\infty}^{\infty} d\varepsilon M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega) f_{\gamma}^e(\varepsilon) f_{\delta}^h(\varepsilon - \hbar\omega) , \quad (4)$$

where $f^e_{\gamma}(\varepsilon)$ is the Fermi-Dirac distribution function, $f^h_{\delta}(\varepsilon) = 1 - f^e_{\delta}(\varepsilon)$, and where the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)$ are listed in Table 1. They are expressed in terms of the transmission amplitude $t(\varepsilon) = i\Gamma G^r(\varepsilon)$, with $G^{r}(\varepsilon)$ the QD retarded Green function, and of the transmission coefficient $\mathcal{T}(\varepsilon) = t(\varepsilon)t^*(\varepsilon)$ through the barriers. The NSFF noise is made of four contributions, each of which being given by the integral over the energy ε of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)$ weighted by the product of the electron distribution function $f^e_{\gamma}(\varepsilon)$ and hole distribution function $f^e_{\delta}(\varepsilon - h\omega)$. Equation (4) is the central result of this Letter: when $\alpha = \beta$, it gives the expressions of the NSFF self-noises $S_{LL}(\omega)$ and $S_{RR}(\omega)$; when $\alpha \neq \beta$, it gives the expressions of the NSFF cross-noises $\mathcal{S}_{LR}(\omega)$ and $\mathcal{S}_{RL}(\omega)$. We emphasize the fact that Eq. (4) holds even in the presence of Coulomb interactions in the dot (included in $G^{r}(\varepsilon)$), the only assumption being the Hartree-Fock approximation.

Since $S_{\alpha\beta}(\omega) = S^*_{\beta\alpha}(\omega)$, the sum of $S_{\alpha\beta}(\omega)$ over the indexes α, β of the two reservoirs, is a real quantity equal to

$$\sum_{\alpha\beta} S_{\alpha\beta}(\omega) = \frac{e^2}{\hbar} \int_{-\infty}^{\infty} d\varepsilon |t(\varepsilon - \hbar\omega) - t(\varepsilon)|^2 \\ \times \sum_{\gamma\delta} f_{\gamma}^e(\varepsilon) f_{\delta}^h(\varepsilon - \hbar\omega) .$$
(5)

The above double sum is related to the charge fluctuations in the QD [19, 35]. Indeed, for a Breit-Wigner type transmission amplitude (non-interacting case), i.e. $t(\varepsilon) = i\Gamma/(\varepsilon - \varepsilon_0 + i\Gamma)$, Eq. (5) leads to $\sum_{\alpha\beta} S_{\alpha\beta}(\omega) =$ $\omega^2 S_{\rm Q}(\omega)$, with $S_{\rm Q}(\omega) = \int dt e^{-i\omega t} \langle \Delta \hat{Q}(t) \Delta \hat{Q}(0) \rangle$, the noise associated with the fluctuations of the charge $\hat{Q} =$ $ed^{\dagger}d$ in the QD. At zero-frequency or for an energyindependent transmission amplitude, $\sum_{\alpha\beta} S_{\alpha\beta}(\omega)$ vanishes. The charge fluctuations are non-zero only when $t(\varepsilon)$ acquires an energy dependence as it is the case for example when the system is coupled to an electromagnetic environment [45].

It is important to underline that our result for $S_{LL}(\omega)$ differs from the expression of the NSFF noise given in Ref. 46 in which the term $|t(\varepsilon) - t(\varepsilon - \hbar\omega)|^2$ is missing. However, when we symmetrize Eq. (4) with respect to the frequency, we get an expression that exactly coincides with the result obtained by Büttiker using scattering theory [47] in which the emblematic term $|t(\varepsilon) - t(\varepsilon - \hbar\omega)|^2$ is also present. Moreover at zero temperature, our result coincides with the expressions obtained by Hammer and Belzig in Ref. 22, and at equilibrium, the fluctuation-dissipation theorem holds since we get $S(\omega) = 2\hbar\omega N(\omega)G(\omega)$ with $N(\omega)$, the Bose-Einstein distribution function and $G(\omega)$, the ac-conductance (the reservoir indexes can be removed in that limit). This confirms the validity of our calculations. We have also checked that by using scattering theory (non-interacting

limit) we get an expression for the NSFF noise in terms of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)$, identical to that obtained in Eq. (4) using NEGF technique.

$M^{\gamma\delta}_{\alpha\beta}(\varepsilon,\omega)$	$\gamma = \delta = L$	$\gamma = \delta = R$	$\gamma = L, \delta = R$	$\gamma=R,\delta=L$
$\alpha = L$	$\mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$	$\mathcal{T}(arepsilon)\mathcal{T}(arepsilon-\hbar\omega)$	$\mathcal{T}(\varepsilon - \hbar\omega)[1 - \mathcal{T}(\varepsilon)]$	$\mathcal{T}(\varepsilon)[1-\mathcal{T}(\varepsilon-\hbar\omega)]$
$\beta = L$	$+ t(\varepsilon)-t(\varepsilon-\hbar\omega) ^2$			
$\alpha = R$	$\mathcal{T}(arepsilon)\mathcal{T}(arepsilon-\hbar\omega)$	$\mathcal{T}(arepsilon)\mathcal{T}(arepsilon-\hbar\omega)$	$\mathcal{T}(\varepsilon)[1-\mathcal{T}(\varepsilon-\hbar\omega)]$	$\mathcal{T}(\varepsilon - \hbar\omega)[1 - \mathcal{T}(\varepsilon)]$
$\beta = R$		$+ t(\varepsilon)-t(\varepsilon-\hbar\omega) ^2$		
$\alpha = L$	$t(arepsilon)t^*(arepsilon-\hbar\omega)$	$t^*(arepsilon)t(arepsilon-\hbar\omega)$	$t(\varepsilon)t(\varepsilon-\hbar\omega)[1-t^*(\varepsilon)]$	$t^*(\varepsilon)t^*(\varepsilon - \hbar\omega)[1 - t(\varepsilon)]$
$\beta = R$	$\times [(1 - t^*(\varepsilon))(1 - t(\varepsilon - \hbar\omega)) - 1]$	$\times [(1 - t(\varepsilon))(1 - t^*(\varepsilon - \hbar\omega)) - 1]$	$\times [1 - t^* (\varepsilon - \hbar \omega)]$	$\times [1 - t(\varepsilon - \hbar \omega)]$
$\alpha = R$	$t^*(arepsilon)t(arepsilon-\hbar\omega)$	$t(arepsilon)t^*(arepsilon-\hbar\omega)$	$t^*(\varepsilon)t^*(\varepsilon - \hbar\omega)[1 - t(\varepsilon)]$	$t(\varepsilon)t(\varepsilon-\hbar\omega)[1-t^*(\varepsilon)]$
$\beta = L$	$\times [(1 - t(\varepsilon))(1 - t^*(\varepsilon - \hbar\omega)) - 1]$	$\times [(1 - t^*(\varepsilon))(1 - t(\varepsilon - \hbar\omega)) - 1]$	$\times [1 - t(\varepsilon - \hbar \omega)]$	$\times [1 - t^*(\varepsilon - \hbar\omega)]$

TABLE I: Expressions of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega)$ appearing in Eq. (4).

Let us now move on to the second method that we develop to calculate the NSFF self-noises, i.e., $S_{LL}(\omega)$ and $\mathcal{S}_{RR}(\omega)$ which correspond at positive frequency to the emission noises in the left and right reservoir respectively. It consists in identifying the physical processes involved in the generation of current noises. For this, one must go back to the definition of the noise given above, stating that it is the Fourier transform of a two-particle correlator, say for instance $S_{LL}(t,0) = \langle \Delta \hat{I}_L(t) \Delta \hat{I}_L(0) \rangle$ for the self-noise in the left reservoir. From this and due to charge and energy conservation, one can see that $S_{LL}(\omega)$ corresponds to the transition probability from an initial state formed by adding a pair of one electron with energy ε and one hole with energy $\varepsilon - \hbar \omega$ in either the left or the right reservoir, to a final state where the electron-hole pair is located in the left reservoir and then recombines emitting a photon of energy $\hbar\omega$ on the left reservoir side. Fig. 1 illustrates all the possible processes along which the system transits from such an initial state to that final state. Note that in order to contribute to the noise, the physical process must let either the electron or the hole of the electron-hole pair experience an excursion into the QD.

We first discuss the contributions to $S_{LL}(\omega)$ when one starts from an initial state with an electron-hole pair located in the left reservoir, i.e., proportional to $f_L^e(\varepsilon)f_L^h(\varepsilon - \hbar\omega)$. In this case, there exists three distinct processes allowing the electron or the hole to experience an excursion into the QD: in the first process illustrated in Fig. 1(a₁), the hole of the initial electron-hole pair is reflected by the left barrier while its electron partner moves back and forth between the left reservoir and the QD before emitting a photon of energy $\hbar\omega$ by recombining with its hole partner in the left reservoir. The corresponding transition probability amplitude is $t_1 = t(\varepsilon)r^*(\varepsilon - \hbar\omega)$.



FIG. 1: Illustration of the processes that contribute to $S_{LL}(\omega)$ with emission of one photon with energy $\hbar \omega$ (yellow wave line) in the left reservoir. The full (empty) black circle represents an electron (hole) with energy ε ($\varepsilon - \hbar \omega$). The solid (dashed) red curved arrow line represents a transmission process with probability amplitude t (t^*). The solid (dashed) blue circular arrow line represents a reflection process with probability amplitude r (r^*). The coherent superposition of processes (a₁), (a₂) and (a₃) leads to the M_{LL}^{LL} contribution, process (b) to the M_{LL}^{RR} contribution, process (c) to the M_{LL}^{LR} contribution, and process (d) to the M_{LL}^{RL} contribution.

In the second process illustrated in Fig. 1(a₂), the hole of the electron-hole pair moves back and forth between the left reservoir and the QD while its electron partner is reflected by the left barrier before emitting a photon of energy $\hbar\omega$ by recombining with the hole present in the left reservoir. Its amplitude reads as $t_2 = r(\varepsilon)t^*(\varepsilon - \hbar\omega)$. Finally in the third process illustrated in Fig. 1(a₃), both the hole and the electron of the electron-hole pair move back and forth between the left reservoir and the QD before emitting a photon of energy $\hbar\omega$ by recombining together. Its amplitude is $t_3 = t(\varepsilon)t^*(\varepsilon - \hbar\omega)$. These three processes lead to the following contribution to the noise: $|t_1 + t_2 + t_3|^2$ corresponding to the coherent superposition of the three transmission processes in question. We have checked that $|t_1 + t_2 + t_3|^2$ is identical to $M_{LL}^{LL}(\varepsilon, \omega) = \mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon - \hbar\omega) + |t(\varepsilon) - t(\varepsilon - \hbar\omega)|^2$ using the relations $r(\varepsilon) = 1 - t(\varepsilon)$ and $t(\varepsilon) + t^*(\varepsilon) = 2\mathcal{T}(\varepsilon)$ guaranteed by the unitarity of the S-matrix in the scattering

theory.

The physical processes involved in the three other contributions to the noise $S_{LL}(\omega)$ can be identified in the same way. For the contribution proportional to $f_{R}^{e}(\varepsilon)f_{R}^{h}(\varepsilon-\hbar\omega)$, one has to start from an initial state where both the electron and the hole of the pair are in the right reservoir. There is a single process as illustrated in Fig. 1(b). Both the electron and the hole have to move across the entire structure from the right to the left reservoir through the QD before recombining to emit one photon of energy $\hbar\omega$ in the left reservoir. The latter process leads to the following contribution to the noise: $|t(\varepsilon)t^*(\varepsilon - \hbar\omega)|^2$ which is identical to $M_{LL}^{RR}(\varepsilon,\omega) = \mathcal{T}(\varepsilon)\mathcal{T}(\varepsilon-\hbar\omega)$. The same analysis can be carried out for the last two contributions proportional to $f_L^e(\varepsilon) f_R^h(\varepsilon - \hbar \omega)$ and $f_R^e(\varepsilon) f_L^h(\varepsilon - \hbar \omega)$ leading to the identification of processes (c) and (d) of Fig. 1 and to the expressions of $M_{LL}^{LR}(\varepsilon,\omega)$ and $M_{LL}^{RL}(\varepsilon,\omega)$.

Note that these processes describe the contributions to $S_{LL}(\omega)$ up to any order in the coupling strength Γ . In the weak coupling limit, the processes (a_1) , (a_2) , (c) and (d) are of order Γ^2 , whereas the processes (a_3) and (b) are of order Γ^4 . Similar physical processes can be found to explain the contributions to $S_{RR}(\omega)$. It is however not possible to find such a simple picture for the NSFF cross-noises $S_{LR}(\omega)$ and $S_{RL}(\omega)$ since they are not real quantities.



FIG. 2: Differential Fano factor F as a function of gate energy ε_0 and bias voltage V_S (both normalized by Γ) at $k_B T/\Gamma = 0.1$ for two values of the frequency : (a) $\hbar\omega_1/\Gamma = 0.1$ (low frequency) and (b) $\hbar\omega_2/\Gamma = 1$ (high frequency).

To illustrate our results, we calculate the differential Fano factor in the non-interacting limit considering a Breit-Wigner type transmission amplitude: $t(\varepsilon) = i\Gamma/(\varepsilon - \varepsilon_0 + i\Gamma)$. This quantity was introduced in recent experimental works [9] and defined as F =

 $|[d\mathcal{S}_{LL}/dV]_{V_S}/[ed\langle \hat{I}_L \rangle/dV]_{V_S-\hbar\omega/e}|$. It corresponds to the ratio of the derivative of the noise with bias voltage at $V = V_S$, to the differential conductance at V = $V_S - \hbar \omega / e$, where ω is the frequency at which the noise is measured. We plot F in Fig. 2 as a function of gate energy ε_0 and bias voltage V_S at two values of the frequency, $\hbar\omega_1/\Gamma = 0.1$ and $\hbar\omega_2/\Gamma = 1$. In agreement with the fact that the system cannot emit photon at energy larger than the supplied energy eV_S , we observe that F is strongly reduced for bias voltages V_S smaller than the frequency in absolute value (see the black horizontal band in both graphs of Fig. 2). At low frequency (Fig. 2(a)), F is always smaller than one, meaning that the noise is sub-Poissonian. At higher frequency (Fig. 2(b)), F takes values higher than one in some regions, meaning that the noise is super-Poissonian with an upper limit equal to F = 2 in agreement with Ref. [22]. Such an increase of F with increasing frequency has been measured in a carbon nanotube QD when placed in the Kondo regime [9]. Even if the analysis carried out in this Letter is not intended to describe the Kondo effect, it is worth noticing that this increase of F with increasing frequency constitutes a general trend. At both low and high frequencies, F is reduced in the bands surrounding the first bisectors, $\varepsilon_0 = \pm eV_S/2$ in the case of a symmetric profile of the chemical potentials on either sides of the QD, i.e. $\mu_{L,R} = \pm eV_S/2$. These bands correspond to a maximal conductance with a transmission close to one. The results reported in the two graphs show similarities with what has been observed experimentally in Ref. [9] with a characteristic pattern made of six areas of non-zero differential Fano factor separated by narrow bands in which the differential Fano factor is strongly reduced. Experimentally in the presence of the Kondo effect, the bands surrounding the bisectors are no longer straight but become curved resulting from the effect of the interactions.

In summary, we have derived the expression of the NSFF noises in a single level interacting QD connected to reservoirs using the NEGF technique. Both the self- and cross- noises are expressed in terms of the transmission amplitude t and coefficient \mathcal{T} in which the Coulomb interactions are included. We have identified the physical processes at the origin of the contributions to the selfnoises. We have shown that when the electron-hole pair is initially present in the reservoir emitting the photon, the contributions to the noise can be interpreted in terms of a coherent superposition of three distinct processes. On the contrary, when the electron and/or the hole of the pair is initially in the non-emitting reservoir, only one type of physical process exists and contributes to the noise. We have also shown that for a non-interacting QD, the differential Fano factor as a function of gate and bias voltages presents interesting features reminding of the experimental measurements. The methods presented and tested in this Letter can be used to treat many situations involving other transmission amplitudes and coefficients,

such as multiple channel systems, QD with multiple energy levels or quantum point contacts embedded in an electromagnetic environment.

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- R. Aguado and L.P. Kouwenhoven, Phys. Rev. Lett. 84, 1986 (2000).
- [2] R. Deblock, E. Onac, L. Gurevich, and L. Kouwenhoven, Science 301, 203 (2003).
- [3] G.B. Lesovik and R. Loosen, Pisma Zh. Eksp. Teor. Fiz.
 65, 280 (1997) [JETP Lett. 65, 295 (1997)].
- [4] E. Onac, F. Balestro, B. Trauzettel, C.F.J. Lodewijk, and L.P. Kouwenhoven, Phys. Rev. Lett. 96, 026803 (2006).
- [5] P.-M. Billangeon, F. Pierre, H. Bouchiat, and R. Deblock Phys. Rev. Lett. 96, 136804 (2006).
- [6] E. Onac, F. Balestro, L.H. Willems van Beveren, U. Hartmann, Y.V. Nazarov, and L.P. Kouwenhoven, Phys. Rev. Lett. 96, 176601 (2006).
- [7] E. Zakka-Bajjani, J. Ségala, F. Portier, P. Roche, D.C. Glattli, A. Cavanna, and Y. Jin, Phys. Rev. Lett. 99, 236803 (2007).
- [8] J. Basset, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. 105, 166801 (2010).
- [9] J. Basset, A.Yu. Kasumov, C.P. Moca, G. Zaránd, P. Simon, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. 108, 046802 (2012).
- [10] F.D. Parmentier, E. Bocquillon, J.-M. Berroir, D.C. Glattli, B. Placais, G. Fève, M. Albert, C. Flindt, and M. Büttiker, Phys. Rev. B 85, 165438 (2012).
- [11] C. Altimiras, O. Parlavecchio, P. Joyez, D. Vion, P. Roche, D. Esteve, and F. Portier, Phys. Rev. Lett. 112, 236803 (2014).
- [12] O. Parlavecchio, C. Altimiras, J.-R. Souquet, P. Simon, I. Safi, P. Joyez, D. Vion, P. Roche, D. Esteve, and F. Portier, Phys. Rev. Lett. **114**, 126801 (2015).
- [13] H.-A. Engel and D. Loss, Phys. Rev. Lett. 93, 136602 (2004).
- [14] A.V. Lebedev, A. Crépieux, and T. Martin, Phys. Rev. B 71, 075416 (2005).
- [15] T. Martin, in *les Houches Session LXXXI*, H. Bouchiat *et. al.* eds. (Elsevier 2005).
- [16] M. Creux, A. Crépieux, and T. Martin, Phys. Rev. B 74, 115323 (2006).
- [17] C. Bena and I. Safi, Phys. Rev. B 76, 125317 (2007).
- [18] I. Safi, C. Bena, and A. Crépieux, Phys. Rev. B 78, 205422 (2008).
- [19] E.A. Rothstein, O. Entin-Wohlman, and A. Aharony, Phys. Rev. B **79**, 075307 (2009).
- [20] C.P. Moca, P. Simon, C.H. Chung, and G. Zarand, Phys. Rev. B 83, 201303R (2011).
- [21] N. Gabdank, E.A. Rothstein, O. Entin-Wohlman, and A. Aharony, Phys. Rev. B 84, 235435 (2011).
- [22] J. Hammer and W. Belzig, Phys. Rev. B 84, 085419 (2011).

- [23] I. Safi and P. Joyez, Phys. Rev. B 84, 205129 (2011).
- [24] R. Zamoum, A. Crépieux, and I. Safi, Phys. Rev. 85, 125421 (2012).
- [25] S.Y. Müller, M. Pletyukhov, D. Schuricht, and S. Andergassen, Phys. Rev. B 87, 245115 (2013).
- [26] E.A. Rothstein, B. Horovitz, O. Entin-Wohlman, and A. Aharony, Phys. Rev. B 90, 245425 (2014).
- [27] B. Roussel, P. Degiovanni, and I. Safi, arXiv:1505.02116.
- [28] C.W.J. Beenakker and H. Schomerus, Phys. Rev. Lett. 86, 700 (2001).
- [29] J. Gabelli, L.-H. Reydellet, G. Fève, J.-M. Berroir, B. Plaçais, P. Roche, and D.C. Glattli, Phys. Rev. Lett. 93, 056801 (2004).
- [30] C.W.J. Beenakker and H. Schomerus, Phys. Rev. Lett. 93, 096801 (2004).
- [31] S. Gustavsson, M. Studer, R. Leturcq, T. Ihn, K. Ensslin, D.C. Driscoll, and A.C. Gossard, Phys. Rev. Lett. 99, 206804 (2007).
- [32] E. Zakka-Bajjani, J. Dufouleur, N. Coulombel, P. Roche, D.C. Glattli, and F. Portier, Phys. Rev. Lett. 104, 206802 (2010).
- [33] I.C. Fulga, F. Hassler, and C.W.J. Beenakker, Phys. Rev. B 81, 115331 (2010).
- [34] A.V. Lebedev, G.B. Lesovik, and G. Blatter, Phys. Rev. B 81, 155421 (2010).
- [35] Y.M. Blanter and M. Büttiker, Phys. Rep. 336, 2 (2000).
- [36] Sh. Kogan, in *Electronic Noise and Fluctuations in Solids*, Cambridge University Press (2009).
- [37] A.P. Jauho, N.S. Wingreen, and Y. Meir, Phys. Rev. B 50, 5528 (1994).
- [38] H.J.W. Haug and A.P. Jauho, in *Quantum kinetics in transport and optics of semiconductors*, Springer Series (2007).
- [39] F.M. Souza, A.P. Jauho, and J.C. Egues, Phys. Rev. B 78, 155303 (2008).
- [40] Note that Eq. (1) corresponds to the non-symmetrized current correlator whereas the expression given in Ref. [38] corresponds to the symmetrized one.
- [41] The greater components of the two-particle Green functions mixing c_k and d operators are defined as: $G_1^{cd,>}(t,t') = -\langle c_k^{\dagger}(t)d(t)c_{k'}^{\dagger}(t')d(t')\rangle$, $G_2^{cd,>}(t,t') = -\langle c_k^{\dagger}(t)d(t)d^{\dagger}(t')c_{k'}(t')\rangle$, $G_3^{cd,>}(t,t') = -\langle d^{\dagger}(t)c_k(t)c_{k'}^{\dagger}(t')d(t')\rangle$ and $G_4^{cd,>}(t,t') = -\langle d^{\dagger}(t)c_k(t)d^{\dagger}(t')c_{k'}(t')\rangle$.
- [42] The two-particle Green functions for the QD are defined as: $G_1^{(2)}(\tau, \tau', \tau_1, \tau_2) = -\langle T_C d(\tau) d(\tau') d^{\dagger}(\tau_1) d^{\dagger}(\tau_2) \rangle$, $G_2^{(2)}(\tau, \tau', \tau_1, \tau_2) = -\langle T_C d(\tau) d^{\dagger}(\tau') d(\tau_1) d^{\dagger}(\tau_2) \rangle$, $G_3^{(2)}(\tau, \tau', \tau_1, \tau_2) = -\langle T_C d^{\dagger}(\tau) d(\tau') d(\tau_1) d^{\dagger}(\tau_2) \rangle$ and $G_4^{(2)}(\tau, \tau', \tau_1, \tau_2) = -\langle T_C d^{\dagger}(\tau) d^{\dagger}(\tau') d(\tau_1) d(\tau_2) \rangle$.
- [43] The contributions contained in Eq. (3) correspond to the connected terms (the last four terms being the ones where the two integrals over τ_1 and τ_2 are intertwined). The last term $-\langle \hat{I}_{\alpha} \rangle \langle \hat{I}_{\beta} \rangle$ appearing in the r.h.s. of Eq. (2) is no longer present since it is exactly canceled by the disconnected terms (in which the two integrals over τ_1 and τ_2 can be done separately).
- [44] When one relaxes the assumption on symmetrical barriers, the expression of the noise in term of the transmission amplitude is much more complicate since one has to consider all the possible writings, distinguishing between left and right transmission amplitudes t_L and t_R .
- [45] F.D. Parmentier, A. Anthore, S. Jezouin, H. Le Sueur,

(2009).

U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Nat. Phys. 7, 935 (2011).

- [46] Y.V. Nazarov and Y.M. Blanter, in *Quantum Transport:* Introduction to Nanoscience, Cambridge University Press
- [47] M. Büttiker, Phys. Rev. B 45, 3807 (1992).