



Methodology of Network Analysis Using Xtan The
Second Geometrization Approach
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I) Introduction

Introduction

Using previous works on the second geometrization, we can now submit a method to use xTAN formalisme in order to study theoretically networks (xTAN means extended tensorial analysis of networks). A first application consists in the identification of risk in electromagnetic compatibility. But on this example, many others can be developed in various jobs including multiphysic ones. We detail the methodology through each of its steps : network graphs, equations, jacobian matrix and Kron's metric, linked parametrized surface, metric and sources, susceptibility spectrum, excitation spectrum and finally analysis.

I) Introduction

Introduction

First tests to use geometry in network analysis was stopped. Due to formulation too much far from the available geometry theory, the Kron's initial equations do not give solution for this objective. **Second geometrization** encloses Kron's equations in standard differential geometry. As a consequence, it becomes possible to **study networks** under a general topological approach. We present here how to reach this kind of **system of equations**. After what we can submit a methodology that can be applied for all kind of problems. Finally we give some first simple examples in order to show how the technique can be used to analyse theoretically networks. In conclusion we speak of future works.

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Keywords

Kron method ; submanifold ; parametrized surfaces ; second geometrization.

II) Basic mechanism to go from Kron's relations to xTAN

Graphs and equations

We won't develop here the Kron's method. Many publications are today available, including its specialization for EMC (MKME method : modified Kron's method for EMC which enclosed generalized interactions as chords). We suppose known the formalism. It leads, once the problem defined using **graphs**, to a **group of equations** giving the system behaviors.

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Function Ψ

This system has the form :

$$\Psi = \{\psi_k(x^1, x^2, \dots, x^k) = e_k\} \quad (1)$$

Variables of this function, are **generalized variables** in multiphysics, like currents, locations, speeds, temperatures. e_k are sources of energy that can be included in the functions. These equations are established from graphs. When **various physics** are involved, each graph is usually associated with variables notations like I_n for electrical currents, T_i for temperatures, etc. All are grouped in an unique vector leading to system(1).

II) Basic mechanism to go from Kron's relations to xTAN

Parametrized hypersurface

We can see system (1) as a **parametrized surface**. A jacobian matrix can be construct based on :

$$J = [J_{k\mu}] = \left[\frac{\partial \psi_k}{\partial x^\mu} \right] \quad (2)$$

The jacobian matrix can be seen as a covector of the basic vector associated with the plane $T_p S$ locally tangent to the hypersurface J .

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C. Writing ψ with J

In order to obtain equations of ψ using J , we must add at least derivative components . More, J can include components that doesn't appear directly in ψ . If ψ can be written :

$$R_{\mu\nu} x^\nu + L_{\mu\nu} x^\nu = e_\mu \quad (3)$$

Let $J = R + \zeta$, we can obtain ψ through :

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This is the fundamental result giving the key to write ? with G (4) become :

$$G_{\sigma\nu} x^m u = J_{\sigma\nu}^T - (L_{\mu\nu} - \zeta_{\mu\nu}) x^\nu \quad (8)$$

L is the metric in the Kron's formalism and G the one in the last evolution of xTAN formalism. This last one is always symmetric and in compliance with Riemannian's concept. To study theoretically the network means to study G and its variation depending on time parameter and x values.

Manifold representation

Each ψ **can be seen as a manifold** (or sub-manifold) if associated with domains giving the limit values for x . If G is fixed whatever x values, it means that the abstract space of variables (including currents) is flat and to study its frontiers, it's only necessary to study the extreme values. If the abstract space of variables is curved, the situation becomes more complex.

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If the abstract space of variables x is flat, the tangent plane T_pS stills the same whatever x values. So if we look to some vector projection on T_pS given by :

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As there is a single plane for all the variables space, it means that we can consider it as an image of the space. The **scalar product** :

$$\langle a, b_k \rangle = a^k G_{kq} \quad (10)$$

is the source covector

Flat space

Our problem in system reliability is to **detect if the system states x can reach values** where **the system can break down** (or is simply be disturbed in electromagnetic compatibility). Major one comes from the fact that to external excitation, the system has its own inertia given by matrix L for which **amplitudes depends on time variation**. To evaluate the domains covered by the system functions, **a solution** can be to consider **particular waveforms** : the most severe that the system can accept. These are both fast rise time waveform and long time durations to combine high dynamic and high power signals. **Another method** can be to **make a statistic on all the signals** used by the system. At each time step, the mean value can be assign to each variable x or the mean value plus the deviation one. Finally **a third technique** previously submit could be to **give each x a value** taken from a set of possible ones recorded in **all the functional life of the system**.

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We have Christoffel's coefficients given by the metric and with : $\Gamma_{11,1} = 4x^1$
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$$\frac{\partial}{\partial x^1} G_{11} = 2\Gamma_{11,1} \quad (13)$$

which can be generalized. This allows to compute to study the system variation for various domains of x variables.

IV) Some applications :!

covariant derivative, riemannian connection

To better explain our approaches we use a simple circuit made of three branches. Whatever the real components of the circuit considered, the graph presented figure 1 stills the same This **single network** (R) has **two nodes** (N), **three branches** (B) and the number of meshes (M) is given by : $M = B - N + R$. In this case, the number of meshes is 2.

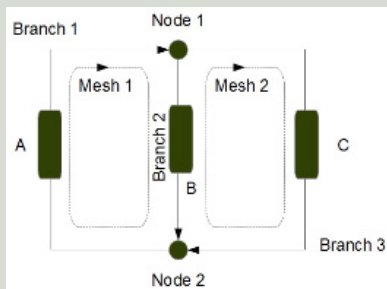


FIGURE: Simple example

IV) Some applications : I

A. Kron method

The Krons method consists in **solving** the circuit **in the mesh space**, using his formalism. Using the circuit described figure 1, we can construct through Krons formalism [2] the impedance matrix in the mesh space given by :

$$Z = \begin{pmatrix} A + B & -B \\ -B & B + C \end{pmatrix} \quad (14)$$

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Staying without current source (no use of the spanning tree) but only with a mesh source on mesh one, the circuit is now represented by the system of equations with $k = 1, 2$:

$$E_k = \sum_q Z_{iq} i_q \quad (15)$$

To complete these two equations we can define a transfer function through $V_3 = B_{i2}$. The parametrized surface can be link with the function (E_1, E_2, V_3) , each function depending on i_1, i_2 . The base vectors can be defined by :

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$$\begin{aligned} b_1 &= (A + B, -B, 0) \\ b_2 &= (-B, B + C, BC) \end{aligned} \quad (16)$$

With this, the metric G_{ij} is given by :

$$Z = \begin{pmatrix} A^2 + 2B^2 + 2AB & -2B^2 + AB + BC \\ -(2B^2 + AB + BC) & 3B^2 + C^2 + BC \end{pmatrix} \quad (17)$$

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Second fundamental form

The two vectors b_1 and b_2 allow to define a **normal vector** to the mobile tangential surface $T_p S$. Its defined by :

$$n = \frac{b_1 \times b_2}{\|b_1 \times b_2\|} \quad (18)$$
$$n = \frac{(-B^2, B(A+B), (A+B)(B+C) - B^2)}{B^4 + (B(A+B))^2 + ((A+B)(A+C) - B^2)^2}$$

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With n we can compute the second fundamental form. First step is to compute the vectors b_{qk} with :

$$b_{qk} = \frac{\partial b_q}{\partial i_k} \quad (19)$$

but until b_1 or b_2 depends on i_1 or i_2 , all b_{qk} are equal to zero. It means that only circuits using the Krons formalism for the mesh space where the currents are involved in the impedance function can create a second fundamental form and as a consequence, a **curvature** of the space.

Some applications : II

Current depending impedance functions

If the impedance depends on the currents, something like $z = a_i$, the product z_i has for derivative $2a_i$, and so, the b_{11} vector may have for value a which is different from zero. To generalize the approach we start from a simple graph made of one mesh but with two sources : one electromotive force on the mesh and one current source applied on the two nodes of the mesh. By this graph, we study a complete space as KRON consider it [2]. Figure 2 shows this circuit and both mesh and nodes pair currents.

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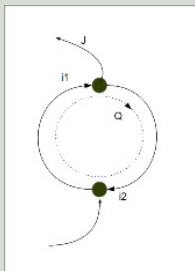


FIGURE: Mesh

Some applications : II

example

The connexion between edges 1 and 2 and the mesh and nodes pair currents Q and J is defined by the **direction choosen** for the currents. It gives the connection :

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In the mesh space we can define :

$$U = C^T Z C = \begin{pmatrix} a + b & a \\ a & a \end{pmatrix} \quad (22)$$

Some applications : II

Mesh

It stills to define $z(Q, J)$. We can define a general function $z(Q, J) = f(Q, J)$: In this case we can define a new basis for **tangent plane** given by :

$$\begin{aligned} b_1 &= \left(a + b, a, \frac{\partial f}{\partial Q} \right) \\ b_2 &= \left(a + b, a, \frac{\partial f}{\partial J} \right) \end{aligned} \tag{23}$$

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The metric is immediately given by :

$$G = \begin{pmatrix} (a + b)^2 + a^2 + \left(\frac{\partial f}{\partial Q}\right)^2 & a(a + b) + a^2 + \frac{\partial f}{\partial Q} \frac{\partial f}{\partial J} \\ a(a + b) + a^2 + \frac{\partial f}{\partial Q} \frac{\partial f}{\partial J} & 2a^2 + \left(\frac{\partial f}{\partial J}\right)^2 \end{pmatrix} \quad (24)$$

by this way **we can introduce the curvature** by the non vanishing component of the second fundamental form :

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$$\begin{aligned}b_{11} &= \left(0, 0, \frac{\partial^2 f}{\partial Q^2}\right) \\b_{12} &= \left(0, 0, \frac{\partial^2 f}{\partial Q \partial J}\right) \\b_{21} &= \left(0, 0, \frac{\partial^2 f}{\partial Q \partial J}\right) \\b_{22} &= \left(0, 0, \frac{\partial^2 f}{\partial J^2}\right)\end{aligned}\tag{25}$$

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Due to the b_{qk} components, the second fundamental form h is defined by :

$$h_{qk} = \langle b_{qk}, n \rangle = b^2 \sqrt{\langle b_{qk}, b_{qk} \rangle}\tag{26}$$

Conclusion

Future works will consist in studying the various manifolds obtained under each kind of system and associated networks. **Understanding their geometry**, we may find global approach to demonstrate in which cases they are stable or unstable, when the risk is raised, etc. About the evolution aspect, the work consists in understanding how the manifolds change depending on the **human factors**, and how the risk can appear after some steps of evolution. Global idea and approach is to go further in geometrization compare to previous works.

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