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Adaptability of Parabolic Wave Equation and Gaussian Beam Shooting methods to Electromagnetic Propagation in Urban Configurations

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Abstract—In spite of well-known limitations, ray based methods are the quasi-unique tool for modelling the electromagnetic propagation in urban areas. As an alternative to these methods, two models based on the resolution of the Parabolic Wave Equation, and on Gaussian Beam Shooting, are proposed. Both are specifically adapted to meet the challenges of urban areas propagation: three-dimensional, wide angle approximation and considering the backscatter propagation for the first one; taking into account ground and grazing angles for the second one. A preliminary test case is presented and configurations of interest under processing are detailed.

I. INTRODUCTION

A. Context

The knowledge of electromagnetic propagation is critical to increase the efficiency and need for control of communication systems. It improves detection and discrimination capabilities of radar systems in presence of dense obstacles.

Nowadays cities become a major configuration, since for the first time in the history of mankind the urban population outnumbered rural population in 2007. More, this ratio is expected to reach 60 % by 2030. This change has to be taken into account by the scientific community, regardless of their speciality.

Indeed, the presence of obstructing buildings in an urban areas causes shadowing of the emitter from the receiver and can lead to severe loss of signals. Thus, electromagnetic propagation models need to be adapted to the specific urban configurations, as far as three-dimensional processing becomes mandatory.

B. Configurations of interest

When dealing with electromagnetic propagation, configurations of interest will support the efficiency assessment of modelling tools regarding two objectives:

- 1) Analysis of urban areas and semi-urban areas as electromagnetic scenes;
- 2) Production of enough results to help the deployment of a system (radar or stations).

The interest lies in the theoretical forecast of performances as close as possible to reality: it concerns attenuation between Base Station (BS) and User Equipment (UE) for broadband mobile or it concerns complete monitoring of radar detection loss due to obstacles in civil, military or security domains.

Masking and multiple reflections must be taken into account in three-dimensional electromagnetic scenes. Elementary mechanisms in play could be:

- Obstruction due to erected obstacle(s) close to the emitting antenna or close to the receiving antenna;
- Lateral deviation on large distant walls;
- Influence of relief (at a large scale): beam is guided in a turning valley or dispersed around a hill.

Configurations of interest can be illustrated in the two following cases.

1) *Microcell*: There are a lot of Line-Of-Sight (LOS) and Non-Line-Of-Sight (NLOS) geometrical situations encountered in urban area as shown in Fig. 1. Various multiple propagation mechanisms occur when BS and UE are operating from and close to tall buildings, in particular those relayed by roofs. Table I lists parameters and their ranges when considering typical configurations.

2) *ATC radars*: A Primary Surveillance Radar (PSR) system is deployed on an airport. The beam set is rotated. The forward way is sought as not obstructed, except for certain beam directions where building or tower can affect the bottom of the radar beam and could also reflect backward a part of energy. For monopulse radar, signal processing can be more

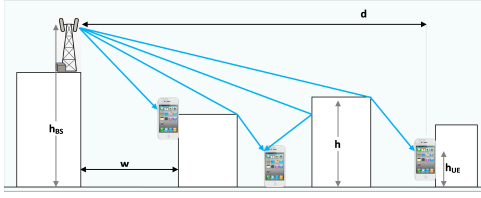


Fig. 1. Configuration of interest no. 1 - Geometry

TABLE I
CONFIGURATION OF INTEREST NO. 1 - PARAMETERS INSPIRED FROM
UIT-R P.1411-6 (02/2012)

	Description	Unit	Range
h	Height of buildings	[m]	4 to 50
w	Street width	[m]	10 to 25
δ	Building density	[%]	20 to 40
h_{BS}	Base Station height	[m]	1 to 100
h_{UE}	User Equipment height	[m]	A few
d	Distance BS to UE	[m]	1000 (up to 5000)
f	frequency	[MHz]	800 to 20000
s	UE speed	[m.s ⁻¹]	0 to 30

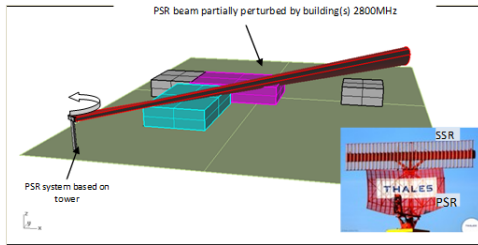


Fig. 2. Configuration of interest no. 2 - Perturbed beam

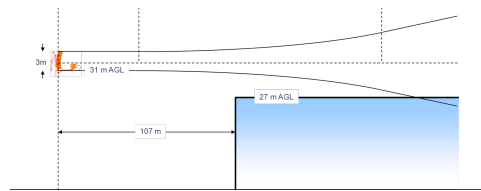


Fig. 3. Configuration of interest no. 2 - Dimensions projected in vertical cut

particularly sensitive to environment through each antenna channel. Accurate calculation requires deep knowledge of real diagrams in presence of the ground and close structure. ATC operation bands are [1215 MHz-1370 MHz] in L-band and [2700 MHz-2900 MHz] in S-band. For secondary Surveillance Radar Systems (SSR), uplink runs at 1030 MHz and downlink at 1090 MHz. All these systems are designed with very narrow beams: azimuthal beam width could be down to 1.5° . Fig. 2 shows a realistic case where a S-band radar is installed near a hangar: dimensions are given for a vertical cut in Fig. 3. The theoretical flanges of the beam at 3dB encounters the flat roof at about 200 m.

C. Literature review

The study of electromagnetic wave propagation in urban areas has led to numerous studies, mostly since the advent of mobile telephony. Thus, in reference to the work done within the COST (European Co-operation in the field of Scientific and Technical research) and more precisely the action 231 (Evolution of Land Mobile Radio Communications), a large number of models have been developed. The following models can be highlighted as the most significant ones: the "COST-Hata-Model" [1], the "COST-Walfish-Ikegami-Model" [2] [3], the "CNET ray launching model" [4], the "ASCOM-ETH micro-cell model" [5] [6], the "Villa Griffone's Lab (VGL) model" [7], the "Uni-Stuttgart 3D micro-cell model" [8] [9] [10] and the "Uni-Karlsruhe model 3D-URBAN-MICRO" [11] [12] [13] [14]. However, these models are either empirical models computing only attenuation in two dimensions and consequently neglecting three-dimensional effects; or models based on ray tracing and thus implying the limitations of this method (*i.e.* very large number of rays and thus unsustainable computation time, presence of caustic(s), far field approximation... specifically in urban configurations).

Our objective is to overcome the limitations of previous models, developing new formulations and new algorithms for modelling the electromagnetic field propagation in urban areas, based on the three-dimensional Parabolic Wave Equation and the three-dimensional Gaussian Beam Shooting.

1) *Parabolic Wave Equation*: Parabolic Wave Equation (PWE) remains for many years one of the most effective method to simulate electromagnetic propagation in complex environments. However, it is based on two restrictions: the paraxial approximation reducing the validity of the method to a cone around the main propagation direction and the ignoring the back-propagation.

In urban or semi-urban areas, we often encounter the scenario of an obstacle (*i.e.* a building) obstructing or interfering with the electromagnetic propagation. This will generate reflections on the sides or diffractions by the edges of this obstacle, or in the worst case a back-propagation of the EM signal. Given the limitations of the Parabolic Wave Equation method, urban or semi-urban configurations are disadvantageous using this method.

Nevertheless, it is possible to enlarge the cone of validity of the Parabolic Wave Equation when using a development of higher order (*i.e.* Feit and Fleck formulation [15]). Recently, a Parabolic Wave Equation resolution taking into account both the forward and backward propagation, called "two-ways", was developed. The idea (behind this resolution) is to consider both directions of propagation by reversing the main direction in the resolution of the Parabolic Wave Equation [16] [17] [18]. In [19], a two-ways finite element resolution is presented for the modelling of ground wave propagation over irregular terrain and in the presence of an inhomogeneous atmosphere.

It is interesting to note that all the two-ways resolutions of the Parabolic Wave Equation currently published are restricted to two-dimensional areas. Our ambition is therefore to implement a three-dimensional wide-angle 2-ways resolution of

the Parabolic Wave Equation in order to overcome its current limitations and to make it adapted to urban configurations.

2) *Gaussian Beam Shooting*: Although the Gaussian Beam Shooting (GBS) method is particularly well suited to 3D field simulations in multi-reflecting contexts, few works are currently devoted to the use of Gaussian Beams (GB) to model electromagnetic propagation in urban or semi-urban areas.

In the millimetric range, "simple" versions of Gaussian Beam Shooting have provided results equivalent to that obtained by ray tracing techniques. In these "simple" algorithms, no specific formulation is developed to account for diffraction. If a beam axis is incident on an obstacle, the beam is considered as if incident on an infinite surface. Conversely, if the beam axis does not intercept the obstacle, the beam is considered as if propagating in free space. This methodology was validated by measurements around 60 GHz in indoor environments [20] [23]. An ultra-wide band version of this kind of methodology has been implemented for building areas at non-millimetre frequencies, but to our knowledge result validation has not been published [26].

More accurate simulations of diffracted fields, yet keeping their representation as a sum of Gaussian beams (formulations sometimes called "B2B", beam to beam), is however a prerequisite for an efficient use of Gaussian Beam Shooting in multipath propagation environments at frequencies of conventional communication systems or radar systems. As mentioned above, only few recent studies have focused on this problem, and the resulting formulations depend on the method used to represent the radiated or scattered fields as discrete sums of Gaussian beams: empirical sampling and normalisation at arrival [25], empirical sampling in the angular domain and frame decomposition along the edge [27], frame decomposition and frame based truncation of fields on obstacle surface [21] [22] [24]. The latter approach is used in this work and yields Physical Optics like representation of diffracted fields.

II. FORMULATION

A. Parabolic Wave Equation

The vectorial wave function $\Psi(x, y, z)$ can be obtained as the solution of the following three-dimensional Helmholtz equation:

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2 n^2(x, y, z) \right\} \Psi(x, y, z) = 0 \quad (1)$$

Where $n(x, y, z)$ is the refractive index, $k_0 = 2\pi/\lambda$ is the free space wavenumber (λ is the wavelength), and x , y and z are the longitudinal (range), the transverse and the height coordinates, respectively. $\Psi(x, y, z)$ corresponds to either the electric and magnetic field components for horizontal and vertical polarizations.

Considering that the direction of wave propagation is predominantly along the $+x$ -axis paraxial direction, the Parabolic Wave Equation is written by separating rapidly varying phase terms ($\Psi(x, y, z) = \exp(ik_0x)u(x, y, z)$). That is, the

Parabolic Wave Equation in terms of the slowly varying amplitude function $u(x, y, z)$ is expressed as follows:

$$\left\{ \frac{\partial}{\partial x} + ik_0(1 - \sqrt{1+T}) \right\} \left\{ \frac{\partial}{\partial x} + ik_0(1 + \sqrt{1+T}) \right\} u(x, y, z) = 0 \quad (2)$$

Where $T = k_0^{-2}(\partial^2/\partial y^2 + \partial^2/\partial z^2) + n^2 - 1$.

The first and second parts of (2) correspond to the forward and backward propagation waves, respectively. If the backward propagation is ignored (2) reduces to:

$$\left\{ A_0 \frac{\partial}{\partial x} + A_1 \frac{\partial}{\partial x} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + A_2 + A_3 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right\} = 0 \quad (3)$$

with the help of $\sqrt{1+T} \approx (a_0 + a_1 t)/(b_0 + b_1 t)$ approximation; therefore, $A_0 = b_0 + b_1(n^2 - 1)$, $A_1 = b_1 k_0^{-2}$, $A_2 = ik_0((b_0 - a_0) + (b_1 - a_1)(n^2 - 1))$, and $A_3 = ik_0^{-1}(b_1 - a_1)$.

If the angle of propagation measured from paraxial direction is less than 15° , the standard Parabolic Wave Equation is obtained with the help of square root approximation $\sqrt{1+t} \approx 1 + t/2$. If the propagation angle is more than 15° , the Claerhout equation is obtained by using the first-order Padé approximation $\sqrt{1+t} \approx (1 + 0.75t)/(1 + 0.25t)$ to satisfy propagation angles up to 40° . Thus, the contribution of a wide-angle resolution can be investigated.

A Finite-Difference methodology is then adopted in order to "introduce" the Leontovich boundaries condition into the propagation algorithm.

A comparison between the mixed-implicit scheme and an alternating method will be performed and the contribution of a 2-ways resolution will be quantified.

B. Gaussian Beam Shooting

In Gaussian Beam Shooting (GBS) algorithms, propagated fields are represented as a superposition of Gaussian beams (GB) launched from the emitting antenna and transformed through successive interactions with obstacles. The formulation developed in this work is based on frame decomposition of source fields:

- The planar distribution of source fields is expressed as a weighted sum of Gaussian windows which form a "frame".
- Each Gaussian window radiates in the form of a Gaussian beam.

1) *Frame of Gaussian windows*: The frame windows $\psi_\mu(y, z)$ used to describe a planar source distribution in the yOz plane are constructed as the product of frame windows $\psi_{m,n}^y(y)$ and $\psi_{p,q}^z(z)$ in $L_2(\mathbb{R})$:

$$\psi_\mu(y, z) = \psi_{m,n}^y(y) \psi_{p,q}^z(z) \quad (4)$$

where $\mu = (m, n, p, q)$ is a composite translation index in \mathbb{Z}^4 . The frame windows $\psi_{m,n}^y(y)$ are obtained by translation of

a Gaussian function $\psi(y)$ along the spatial coordinate y and along its spectral counterpart k_y :

$$\psi_{m,n}^y(y) = \psi(y - m\bar{y})e^{in\bar{k}_y y} \quad (5)$$

- \bar{y} and \bar{k}_y are respectively the spatial and spectral translation steps, m and n the spatial and spectral translation indices.
- $\{\psi_\mu, \mu \in \mathbb{Z}^2\}$ is a frame if and only if $\bar{y}\bar{k}_y = 2\pi\nu$, with $\nu < 1$ (ν : oversampling factor).
- the favorite choice for the translation steps is: $\bar{y} = \sqrt{\nu}L$, $\bar{k}_y = \sqrt{\nu}(2\pi/L)$ ("balanced" frame).

The same construction is used for the frame along z , with p , q the spatial and translation indices along z . In the following, the Gaussian function ψ is taken as:

$$\psi(y) = \left(\sqrt{2}/L\right)^{\frac{1}{2}} e^{-\pi\left(\frac{y}{L}\right)^2} \quad (6)$$

2) *Frame window radiation*: Through paraxial asymptotic evaluation of plane wave spectrum integrals, the fields radiated by the source field distributions $\psi_\mu(y, z)$ (either for y or z component) are put in the form of paraxial Gaussian beams. Let us denote $B_\mu(\mathbf{r})$ the field radiated at point \mathbf{r} . The paraxial expression of B_μ is:

$$B_\mu(\mathbf{r}) = B_0 \sqrt{\frac{\det \Gamma^{-1}(0)}{\det \Gamma^{-1}(x_\mu)}} \exp ik \left[x_\mu + \frac{1}{2} \mathbf{y}_\mu^t \Gamma(x_\mu) \mathbf{y}_\mu \right] \quad (7)$$

with B_0 a vector depending on the source polarization. This expression is analog to that of a geometrical optics ray along the x_μ direction, with Γ the curvature matrix and $\sqrt{\frac{\det \Gamma^{-1}(0)}{\det \Gamma^{-1}(x_\mu)}}$ the divergence factor. $\mathbf{y}_\mu = (y_\mu, z_\mu)$ is the position vector of the point \mathbf{r} in a plane transverse to the x_μ axis.

The difference between geometrical optics rays and Gaussian beams comes from the fact that Γ is a real matrix in the first case, a complex one in the latter. This complex matrix accounts for the Gaussian decay of fields with increasing distance from the x_μ axis in transverse planes. The x_μ axis is then called the beam axis.

3) *Gaussian beam tracking in a built environment*: In the presence of "infinite" smooth interfaces described either as dielectric interfaces or by surface impedance models, GB can be tracked in a way similar to rays, thanks to their paraxial properties. In the case of planar interfaces, reflected or refracted beams can even be expressed as beams originating in "image" source distributions. The presence of ground is easily accounted for in this way. Yet, paraxial beam tracking cannot address problems where beams impinge on discontinuities, as shown in Fig. 4. Specific B2B algorithms are thus developed to simulate propagation in the presence of buildings:

- selection of beams contributing (above a given threshold level) to the incident field distribution on a finite reflecting surface (wall);
- for the selected beams impinging on a building corner, decomposition of their incident fields on a frame of

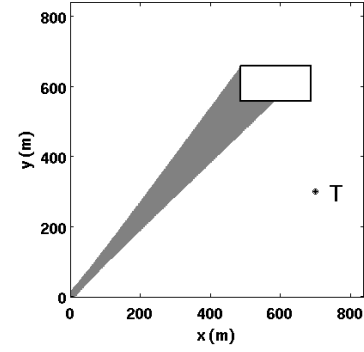


Fig. 4. Gaussian beam impinging on a building corner: 2D cut for a beam propagating in the xOy plane, launched from a frame window source centered at the origin in the yOz plane (7.36m beam width at 430MHz). "T" denotes the target.

narrow windows, which is truncated to the edge of the wall;

- change of frame, yielding the frame coefficients of the field of interest on a set of (spatially) wide windows, which are used to further propagate fields in the form of collimated beams.

Analytical expressions have been established for incident beam field decomposition on a narrow window frame, and for frame change matrix elements [22], [24]. They are under generalization in the case of successive diffractions around the corners of a building.

III. NUMERICAL RESULTS

A. Source modelling

Meaningful comparisons between the Parabolic Wave Equation (PWE) and the Gaussian Beam Shooting (GBS) methods are achievable only when the codes propagate the same source field. For the numerical simulations reported in this article, we define an $Oxyz$ coordinate system, wherein z is the vertical axis, the ground is in the plane xOy defined by $z = 0$, the source is located in the plane yOz defined by $x = 0$.

Vertical ($E_y = 0$) and Horizontal ($E_z = 0$) polarizations are studied separately. The non-zero component of the source field, denoted as $E(y, z)$, is taken as:

$$E(y, z) = B_0 \exp \left[-\pi/L_0^2 \left(y^2 + (z - h)^2 \right) \right] \exp i \left(n\bar{k}_1 y + q\bar{k}_2 z \right) \quad (8)$$

Where:

- $B_0 = \sqrt{2}/L_0$;
- h is the emitter height, defined as $h = p\bar{z}$ with $p \in \mathbb{Z}^2$ and $\bar{z} = \sqrt{\nu}L_0$;
- $\bar{k}_1 = \bar{k}_2 = \sqrt{\nu} 2\pi/L_0$;
- n and q are in \mathbb{Z}^2 and define the beam pointing respectively along the y and z directions.

For the results presented below, the following values have been used:

- the frequency $f = 430MHz$;

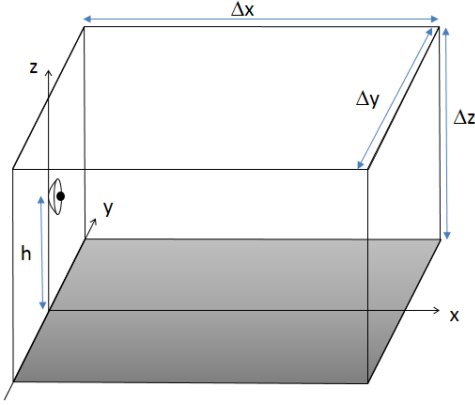


Fig. 5. Test case representation

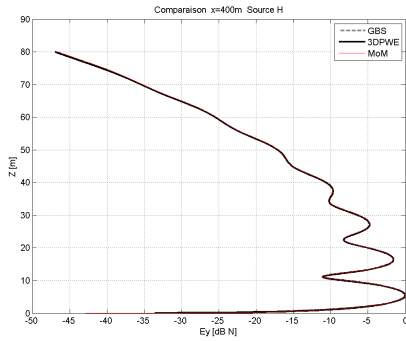


Fig. 6. Comparison between PWE and GBS along z , for $y = 0\text{m}$ and $x = 400\text{m}$. Validation by comparison with the Method of Moments

- $L_0 = 7.5\lambda$;
- $\nu = 0.16$;
- $p = 6$.

With these parameters, the source is centered at a height h approximately equal to 12.5m .

B. Preliminary test case

A preliminary test case is presented in Fig. 5, with the previously defined source. The polarization is horizontal and the ground is considered flat and perfectly conducting.

Simulation results obtained by both PWE and GBS methods are shown in Fig. 6. The field magnitude is represented along the vertical axis defined by $y = 0\text{m}$ (laterally centered) and $x = 400\text{m}$ (distance from the source plane).

We can see an excellent agreement between both methods. Due to the Finite Difference methodology used, PWE results depend on spatial domain sampling (mainly the transverse ones, *i.e.* in y and z dimensions). Thinner the sampling is, better the modelling is. As an example, the presented result has been obtained thanks to a $\lambda/10$ transverse sampling.

Validation by comparison with Method of Moments (MoM) [28] is also presented in Fig. 6. Due to the perfect comparison, both methods implemented can be considered as validated on this test case.

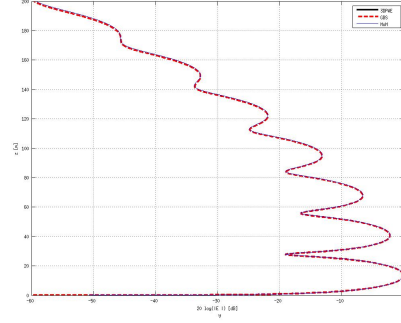


Fig. 7. Comparison between PWE and GBS along z , for $y = 0\text{m}$ and $x = 1000\text{m}$. Validation by comparison with the Method of Moments

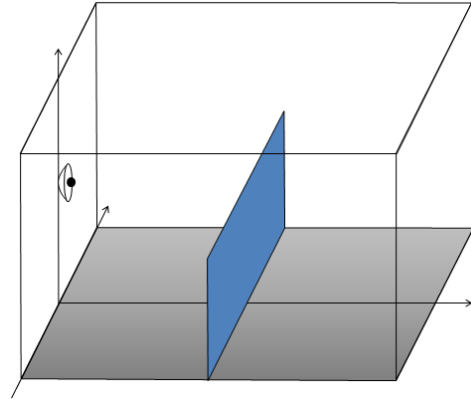


Fig. 8. Screen test case

Comparison between both methods and validation with MoM at a longer distance, *i.e.* 1km , is presented in Fig. 7.

Agreement between all methods can still be considered as excellent.

IV. FURTHER WORK

The first test case was presented, comparing the electromagnetic wave propagation over a perfectly conducting flat ground (at a distance of 400m and 1km), modelled by the Parabolic Wave Equation and the Gaussian Beam Shooting methods.

Comparisons on all components of the electromagnetic field are under progress.

The specific developments needed for each method, in order to cope with the configurations of interest presented above, are under test.

More complex test case results (cf. Fig. 8, 9 and 10), and further applied in configurations of interest (cf. Fig. 1 and 2), will be presented at the conference.

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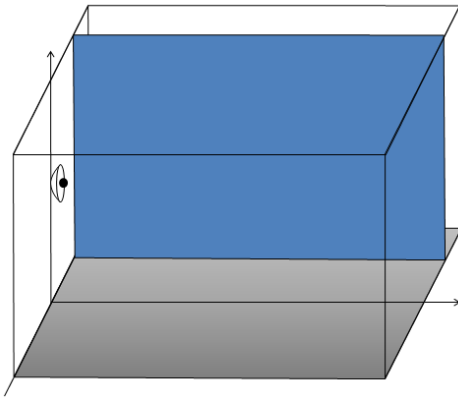


Fig. 9. Lateral wall test case

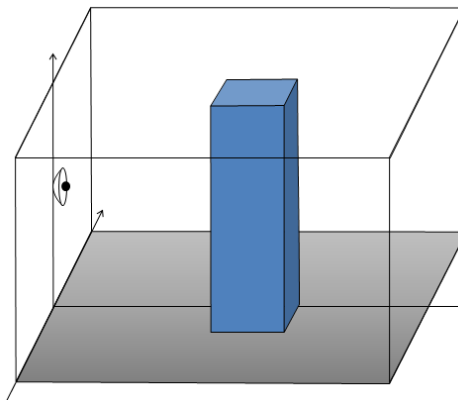


Fig. 10. Isolated building test case

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