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SOLUTION OF THE MAYAN CALENDAR ENIGMA

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The Mayan arithmetical model of astronomy is described. The astronomical origin of the Mayan Calendar (the 260-day Tzolk'in, the 365-day Haab', the 3276-day Kawil-direction-color cycle and the 1872000-day Long Count Calendar) is demonstrated and the position of the Calendar Round at the mythical date of creation 13(0).0.0.0.0 4 Ahau 8 Cumku is calculated. The results are expressed as a function of the Xultun numbers, four enigmatic Long Count numbers deciphered in the Maya ruins of Xultun, dating from the IX century CE. (Saturno 2012) Evidence shows that this model was used in the Maya Classic period (200 to 900 CE) to determine the Palenque lunar equation. This provides evidence of the high proficiency of Mayan naked-eye astronomy and mathematics.

Mayan priests-astronomers were known for their astronomical and mathematical proficiency, as exemplified in the Dresden Codex, a XIV century CE bark-paper book containing accurate astronomical almanacs aiming to correlate ritual practices with astronomical observations. However, due to the zealous role of the Inquisition during the XVI century CE Spanish conquest of Mexico, number of these Codices were destroyed, leaving us with few information on pre-Columbian Mayan culture. Thanks to the work of Mayan archeologists and epigraphists since the early XX century, the few Codices left, along with numerous inscriptions on monuments, were deciphered, underlying the Mayan cyclical concept of time. This is demonstrated by the Mayan Calendar formed by a set of three interlocking cycles: the Calendar Round, the Kawil-direction-color cycle and the Long Count Calendar.

The Calendar Round (CR) represents a day in a non-repeating 18980-day cycle, a period of roughly 52 years, the combination of the 365-day solar year Haab' and the 260-day religious year Tzolk'in. The Tzolk'in comprises 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etnab, Cauac, and Ahau). This forms a list of 260 ordered Tzolk'in dates from 1 Imix, 2 Ik, ... to 13 Ahau. (Aveni 2001: 143) The Haab' comprises 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. This forms a list of 365 ordered Haab' dates from 0 Pop, 1 Pop, ... to 4 Uayeb. (Aveni 2001: 147) The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round, the least common multiple (LCM) $1 \text{ CR} = \text{LCM}(260,365) = 73 \times 260 = 52 \times 365 = 18980$ days. In the Calendar Round, a date is represented by $\alpha X\beta Y$

with the religious month $1 \leq \alpha \leq 13$, X one of the 20 religious days, the civil day $0 \leq \beta \leq 19$, and Y one of the 18 civil months, $0 \leq \beta \leq 4$ for the Uayeb.

For longer period of time, the Maya used the Long Count Calendar (LCC), describing a date D in a 1872000-day Maya Era of 13 Baktun, a religious cycle of roughly 5125 years, counting the number of day elapsed since the Mayan origin of time. This mythical date of creation, carved on Stela 1 of Coba (present-day Mexico), a Maya site from the VII century CE, is 13(0).0.0.0.0 4 Ahau 8 Cumku (Fuls 2007), corresponding to the Gregorian Calendar date 11 August 3114 BC according to the Goodman-Martinez-Thompson (GMT) correlation. (Aveni 2001: 136, Bricker 2011: 71, 93) An interesting example of Long Count number can be found on page 24 of the Dresden Codex in the introduction of the Venus table: the so-called Long Round number $\mathcal{LR} = 9.9.16.0.0 = 1366560$ days $= 9 \times 144000 + 9 \times 7200 + 16 \times 360 + 0 \times 20 + 0 \times 1$ expressed as a function of the Long Count periods (the 1-day Kin, the 20-day Winal, the 360-day Tun, the 7200-day Katun and the 144000-day Baktun). (Aveni 2001: 191) The Long Count periods are commensurate with the Tzolk'in and the Haab': $\{\text{LCM}(260, P_i)/P_i = 13, \text{LCM}(365, P_i)/P_i = 73, P_i = 18 \times 20^i, i > 0\}$. The XXI century saw the passage of a new Maya Era on 21 December 2012 (GMT correlation) or 13(0).0.0.0.0 4 Ahau 3 Kankin, a date carved on Monument 6 of Tortuguero (present-day Mexico), a Maya stone from the VII century CE. (Stuart 2012: 25)

The Kawil-direction-color cycle or 4-Kawil is a 3276-day cycle, the combination of the 819-day Kawil and the 4 directions-colors (East-Red, South-Yellow, West-Black, North-White). (Berlin 1961) The 4-Kawil counts the number of day in four 819-day months (each of them corresponding to one direction-color) in a non-repeating 3276-day cycle. At the

mythical date of creation, the Kawil count is 3 and the direction-color is East-Red. A Kawil date is then defined as $D \equiv \text{mod}(D + 3, 819)$ and the direction-color is given by $n = \text{mod}(\text{int}((D+3)/819), 4)$, $n = \{0, 1, 2, 3\} = \{\text{East-Red, South-Yellow, West-Black, North-White}\}$. Although several myths exist around Mayan religion, the origin of the Mayan Calendar remains unknown.

A complete Maya date contains also a glyph G_i with $i = 1..9$ corresponding to the 9 Lords of the Night and the lunar series: the 29(30)-day Moon age (number of days elapsed in the current lunation) and a lunation count (number of lunation in a series of five or six). The calculation of the Moon age in the lunar series of a mythical date LC is calculated from the new Moon date LC_0 as: $MA = \text{remainder of } (LC - LC_0)/S$ where $S = n/m$ is the Moon ratio corresponding to the lunar equation m lunations = n days. (Fuls 2007) Mayan priests-astronomers used particular lunar equations such as 149 lunations = 4400 days (Copan Moon ratio) and 81 lunations = 2392 days (Palenque formula). The Palenque formula corresponds to a Moon synodic period of 29.530864 days, differing by only 24 seconds from the modern value (29.530588 days). (Aveni 2001: 163, Teeple 1930, Fuls 2007) It is unclear how the Maya determined the Palenque formula.

\mathcal{X}_i	LCC	D [day]	$\mathcal{X}_i/56940$
\mathcal{X}_0	2.7.9.0.0	341640	6
\mathcal{X}_1	8.6.1.9.0	1195740	21
\mathcal{X}_2	12.5.3.3.0	1765140	31
\mathcal{X}_3	17.0.1.3.0	2448420	43

TABLE I. Xultun numbers \mathcal{X}_i . (Saturno 2012) 56940 = LCM(365,780) is their largest common divisor.

In 2012, four Long Count numbers, the Xultun numbers (Table I) and three lunar tables, have been discovered on the walls of a small painted room in the Maya ruins of Xultun (present-day Guatemala), dating from the early IX century CE. (Saturno 2012) These numbers have a potential astronomical meaning. Indeed, \mathcal{X}_0 is a whole multiple of Venus and Mars synodic periods: $341640 = 585 \times 584 = 438 \times 780$. $\mathcal{X}_0 = \mathcal{LR}/4 = \text{LCM}(260, 360, 365)$ is the commensuration of the Tzolk'in, the Tun and the Haab' and $\mathcal{X}_1 = 365 \times 3276$ is the commensuration of the Haab' and the 4-Kawil. However, the meaning of \mathcal{X}_2 and \mathcal{X}_3 is unknown. The greatest common divisor of the \mathcal{X}_i 's is 56940 = LCM(365,780) = 3 CR, the commensuration of the Haab' and Mars synodic period.

The three Xultun lunar tables, corresponding to a time span of 4429 (12.5.9), 4606 (12.14.6) and 4784 (13.5.4) days were attributed to solar/lunar eclipse cycles due to similarities in structure with the Dresden Codex eclipse table. (Saturno 2012) It was noted that $4784 = 2 \times 2392$ days represented 162 lunations, twice that of the Palenque lunar reckoning system 81 lunations = 2392 days. (Teeple 1930) The length of the Dresden Codex eclipse table $11960 = 5 \times 2392$ days = 405 lunations corresponds to five times the Palenque formula. (Bricker 1983) The lengths of the solar/lunar eclipse tables are unexplained.

The level of sophistication displayed in the Dresden Codex suggests the high astronomical proficiency of Mayan priests-astronomers. It is therefore reasonable to assume that the Maya measured the synodic periods of the five planets Mercury, Venus, Mars, Jupiter and Saturn visible by naked-eye observation of the night sky. Their canonic synodic periods are given in Table II. Evidence of their use has been found in different Mayan Codices for Mercury, Venus and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn. (Bricker 2011:163, 367, 847) The three relevant lunar months are the two lunar semesters of 177 and 178 days (6 Moon synodic periods) and the pentalunex of 148 days (5 Moon synodic periods), parameters used for the prediction of solar/lunar eclipses in the Dresden Codex eclipse table. (Bricker 1983) From the prime factorizations of the 9 astronomical input parameters (Table II), we calculate the calendar super-number \mathcal{N} defined as the least common multiple of the P_i 's:

$$\begin{aligned}
 \mathcal{N} &= 768039133778280 & (1) \\
 &= 2^3 \times 3^3 \times 5 \times 7 \times 13 \times 19 \times 29 \times 37 \\
 &\quad \times 59 \times 73 \times 89 \\
 &= 365 \times 3276 \times 2 \times 3 \times 19 \times 29 \times 37 \\
 &\quad \times 59 \times 89 \\
 &= \text{LCM}(360, 365, 3276) \times 3 \times 19 \times 29 \\
 &\quad \times 37 \times 59 \times 89
 \end{aligned}$$

Equ. 1 gives the calendar super-number and its prime factorization. It is expressed as a function of the Tun ($360 = 18 \times 20$), the Haab' ($365 = 5 \times 73$) and the 4-Kawil ($3276 = 2^2 \times 3^2 \times 7 \times 13$). The solar year Haab' and the P_i 's are relatively primes (except Venus and Mars): the $\{\text{LCM}(P_i, 365)/365, i = 1..9\} = \{116, 8, 1, 156, 399, 378, 177, 178, 148\}$ (Table II). The 4-Kawil is defined as the $\{\text{LCM}(P_i, 3276)/3276, i = 1..9\} = \{29, 146, 365, 5, 19, 3, 59, 89, 37\}$. The Haab' and the 4-Kawil are relatively primes: the $\text{LCM}(365, 3276) = 365 \times 3276 =$

Planet	i	P_i [day]	Prime factorization
Mercury	1	116	$2^2 \times 29$
Venus	2	584	$2^3 \times 73$
Earth	3	365	5×73
Mars	4	780	$2^2 \times 3 \times 5 \times 13$
Jupiter	5	399	$3 \times 7 \times 19$
Saturn	6	378	$2 \times 3^3 \times 7$
Lunar	7	177	3×59
senesters	8	178	2×89
Pentalunex	9	148	$2^2 \times 37$

TABLE II. Prime factorization of the planet canonic synodic periods and the three Mayan lunar months. (Bricker 1983)

\mathcal{X}_1 such as $\{\text{LCM}(P_i, \mathcal{X}_1)/\mathcal{X}_1, i = 1..9\} = \{29, 2, 1, 1, 19, 3, 59, 89, 37\}$. 360 is the nearest integer to 365 such that the $\text{LCM}(360, 3276) = 32760$ and the $\{\text{LCM}(P_i, 32760)/32760, i = 1..9\} = \{29, 73, 73, 1, 19, 3, 59, 89\}$. The Tun-Haab'-Kawil cycle is given by $\mathcal{Y} = \text{LCM}(360, 365, 3276) = 7 \times \mathcal{X}_0 = 2391480$ such as $\{\text{LCM}(P_i, \mathcal{Y})/\mathcal{Y}, i = 1..9\} = \{29, 1, 1, 1, 19, 3, 59, 89, 37\}$. The commensuration of the Haab', the 4-Kawil and the Baktun (400 Tun = 144000 days) gives rise to the calendar grand cycle $\mathcal{G}\mathcal{C} = \text{LCM}(365, 3276, 144000) = 400 \times \text{LCM}(360, 365, 3276) = 400 \times 7 \times \mathcal{X}_0 = 956592000$. The Euclidean division of $\mathcal{N}/37$ (LCM of all the astronomical input parameters except the pentalunex) by $\mathcal{G}\mathcal{C}$ gives:

$$\begin{aligned} \mathcal{N}/37 &= \mathcal{G}\mathcal{C} \times \mathcal{Q} + \mathcal{R} & (2) \\ \mathcal{Q} &= 21699 \\ \mathcal{R} &= 724618440 \\ &= 101 \times 126 \times 56940 \\ &= 126 \times \sum_{i=0}^3 \mathcal{X}_i. \end{aligned}$$

If we note the Maya Aeon $\mathcal{A} = 13 \times 73 \times 144000 = 400 \times \mathcal{X}_0 = 100 \times \mathcal{L}\mathcal{R} = 136656000$ such as $\mathcal{G}\mathcal{C} = 7 \times \mathcal{A}$, the Euclidean division of $\mathcal{N}/37$ by \mathcal{A} gives:

$$\begin{aligned} \mathcal{N}/37 &= \mathcal{A} \times \mathcal{Q} + \mathcal{R} & (3) \\ \mathcal{Q} &= 151898 \\ \mathcal{R} &= 41338440 \\ &= 6 \times 121 \times 56940 \\ &= 121 \times \mathcal{X}_0 \end{aligned}$$

The Maya Aeon such that $\mathcal{A} = \text{LCM}(260, 365, 144000) = 7200 \times 18980 = 3600 \times 37960 = 2400 \times 56940$ is commensurate to the 7200-day Katun, the Calendar Round, Venus and Mars synodic periods such as $\text{LCM}(365, 584) = 37960$ and $\text{LCM}(365, 780)$

= 56940. We can rewrite Equ. 2 and 3 as:

$$\begin{aligned} \mathcal{N}/37 - 121 \times \mathcal{X}_0 &= 151898 \times \mathcal{A} & (4) \\ \mathcal{N}/37 - 126 \times \sum_{i=0}^3 \mathcal{X}_i &= 151893 \times \mathcal{A} \end{aligned}$$

The subtraction of the two equations in Equ. 4 can be expressed as a function of the Xultun numbers:

$$\begin{aligned} 5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + 95 \times 126 \times 56940 & (5) \\ 5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + \text{LCM}\left(\sum_{i=1}^3 \mathcal{X}_i, \mathcal{X}_1 + 2\mathcal{X}_2 + \mathcal{X}_3\right) \end{aligned}$$

The four Xultun numbers provides evidence that Mayan priests-astronomers determined the canonic synodic periods of the five planets visible by naked-eye observation of the night sky: Mercury, Venus, Mars, Jupiter and Saturn. At this point, a question arises how the Maya, as early as the IX century CE, were able to compute tedious arithmetical calculations on such large numbers with up to 14 digits in decimal basis. Here is a possible method. They determined the prime factorizations of the canonic synodic periods P_i (Table II) and listed each primes p_i with their maximal order of multiplicity α_i . They determined the Haab', the Tun and the 4-Kawil as described earlier. They calculated the calendar super-number \mathcal{N} (the LCM of the P_i 's) by multiplying each p_i 's α_i time. The Euclidean division of $\mathcal{N}/37$ by $\mathcal{G}\mathcal{C} = 7 \times \mathcal{A} = 400 \times 7 \times \mathcal{X}_0$ (Equ. 2) is equivalent to a simplification of $\mathcal{N}/37$ by $7 \times \mathcal{X}_0 = \text{LCM}(360, 365, 3276) = 2391480$ and the Euclidean division of the product of the 5 left primes ($3 \times 19 \times 29 \times 59 \times 89 = 8679903$) by 400. The Euclidean division of $\mathcal{N}/37$ by $\mathcal{A} = \mathcal{G}\mathcal{C}/7 = 400 \times \mathcal{X}_0$ (Equ. 3) is equivalent to a simplification of $\mathcal{N}/37$ by $\mathcal{X}_0 = \text{LCM}(260, 360, 365) = 341640$ and the Euclidean division of the product of the 6 left primes ($3 \times 7 \times 19 \times 29 \times 59 \times 89 = 60759321$) by 400. It is to be noted that the prime factorization of the calendar super-number only includes prime numbers < 100 which facilitates the operation (there are only 25 prime numbers lower than 100).

The 5 Maya Aeon $5\mathcal{A}$ defined by Equ. 5 is such that $5 \times \mathcal{A} = 5 \times 13 \times 73 \times 144000 = 12000 \times 56940 = 365 \times \mathcal{E}$, multiple of the 13 Baktun Maya Era $\mathcal{E} = 1872000$. $5\mathcal{X}_0$ such that $5 \times \mathcal{A} = 5 \times \mathcal{X}_0 + 570 \times \mathcal{X}_1$ has the same properties as $5\mathcal{A}$ (same Tzolk'in, Haab', Kawil and direction-color). The Maya Era is such that $\mathcal{E} - 5 \times \mathcal{X}_0 = 1872000 - 1708200 = 163800 = 10 \times \text{LCM}(260, 3276)$. The Maya Era \mathcal{E} has the same Tzolk'in, Kawil and direction-color

as $5\mathcal{X}_0$ and $5\mathcal{A}$. The grand cycle is such as $\mathcal{GC} = 7 \times \mathcal{A} = 7 \times 73 \times \mathcal{E} = 511 \times \mathcal{E} = \text{LCM}(260, 365, 3276, \mathcal{E})$.

The initialization of the Calendar Round at the origin of time can be obtained by arithmetical calculations on the calendar super-number. For that purpose, we first create ordered lists of the Haab' and the Tzolk'in, assigning a unique set of 2 numbers for each day of the 18980-day Calendar Round. (Aveni 2001: 143, 147) For the Haab', the first day is 0 Pop (numbered 0) and the last day 4 Uayeb (numbered 364). For the Tzolk'in, the first day is 1 Imix (numbered 0) and the last day 13 Ahau (numbered 259). In this notation, the date of creation 4 Ahau 8 Cumku is equivalent to $\{160; 349\}$ and a date D in the Calendar Round can be written as $D \equiv \{\text{mod}(D + 160, 260); \text{mod}(D + 349, 365)\}$. The calendar super-number is such that: $\text{mod}(\mathcal{N}/13/37/73, 260) = 160$, $\text{mod}(\mathcal{N}/13/37/73, 13) = 4$, $\text{mod}(\mathcal{N}/13/37/73, 20) = 0$ and $\text{mod}(\mathcal{N}/13/37/73, 73) = 49$. The date $\{160; 49\}$ corresponds to 4 Ahau 8 Zip, the day 0 (mod 18980), beginning/completion of a Calendar Round. Starting the CR count at 4 Ahau 8 Zip, the next date in the ordered CR list such as $\text{mod}(D, 4680) = 0$ (completion of a 13 Tun cycle) is the date 4 Ahau 8 Cumku chose as the day 0 of the Long Count Calendar. A date D is then expressed as $\{\text{mod}(D + 4680 + 160, 260); \text{mod}(D + 4680 + 49, 365)\} = \{\text{mod}(D + 160, 260); \text{mod}(D + 349, 365)\}$. The Kawil-direction-color indices can be initialized at the LCC origin of time as $\text{mod}(\mathcal{N}/37/32760, 4) = 3$ East-Red. That defines the position of the Calendar Round and the Kawil-direction-color indices at the mythical date of creation, the LCC date $13(0).0.0.0.0$ 4 Ahau 8 Cumku $\{160; 349\}$, 3 East-Red.

Date D	LCC date	Cyclical date
\mathcal{I}_0	0	$13(0).0.0.0.0$ $\{160, 349, 3, 0\}$
$5\mathcal{X}_0$	1708200	$11.17.5.0.0$ $\{160, 349, 588, 1\}$
\mathcal{E}	1872000	$13(0).0.0.0.0$ $\{160, 264, 588, 1\}$
$5\mathcal{A}$	$365 \times 13(0).0.0.0.0$	$\{160, 349, 588, 1\}$
\mathcal{GC}	$511 \times 13(0).0.0.0.0$	$\{160, 349, 3, 0\}$

TABLE III. Important Mayan cultural date: \mathcal{I}_0 (mythical date of creation), $5\mathcal{X}_0$ (date of the Itza prophecy), \mathcal{E} (end of the 13 Baktun Era), $5\mathcal{A}$ (end of the 5 Maya Aeon) and $\mathcal{GC} = 7\mathcal{A}$ (end of the Maya grand cycle). A date is defined as its linear time day D and its cyclical equivalent given by the LCC date and a set of 4 integers $\{T; H; K; n\}$ where T is the Tzolk'in, H the Haab', K the Kawil and n the direction-color calculated from the definitions given in the text.

We now calculate the full Mayan Calendar

date of $5\mathcal{X}_0$ and \mathcal{E} as compared to the mythical date of creation \mathcal{I}_0 , the 5 Maya Aeon $5\mathcal{A}$ and the grand cycle of 7 Maya Aeon $\mathcal{GC} = 7\mathcal{A}$. Table III gives the results. The previous Maya Era is characterized by two important dates: $5\mathcal{X}_0 = 11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow and $\mathcal{E} = 13(0).0.0.0.0$ 4 Ahau 3 Kankin, 588 South-Yellow, which are defined by their equivalent properties compared to the 5 Maya Aeon cycle $5\mathcal{A}$ (4 Ahau 8 Cumku, 588 South-Yellow). The date $5\mathcal{X}_0$ or 3 July 1564 CE (GMT correlation) may have been related to the Itza prophecy of intense cultural change that occurred concomitantly with the Spanish conquest of Mexico (from February 1519 to 13 August 1521). (Stuart 2012: 19-27) Fig. 1 represents the Mayan cyclical concept of time, with a grand cycle \mathcal{GC} defined as the commensuration of the Tzolk'in, the Haab', the Kawil-direction-color and the Maya Era. The three important dates of the previous Maya Era are represented: the mythical date of creation $13(0).0.0.0.0$ 4 Ahau 8 Cumku, 3 East-Red (11 August 3114 BC), the date corresponding to the Itza prophecy $11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow (3 July 1564) and the end of the Maya Era $13(0).0.0.0.0$ 4 Ahau 3 Kankin, 588 South-Yellow (21 December 2012).

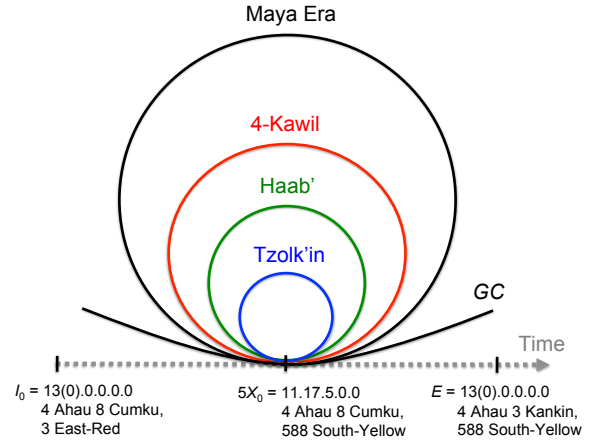


FIG. 1. Mayan/Aztec cyclical *vs* linear concept of time, with the 260-day Tzolk'in, the 365-day Haab', the 3276-day 4-Kawil, the Maya Era of 13 Baktun (5125 years) and the grand cycle \mathcal{GC} of 511 Maya Eras. The mythical date of creation \mathcal{I}_0 (11 August 3114 BC), the date of the Itza prophecy $5\mathcal{X}_0$ (3 July 1564 CE) and the end of the previous Maya Era \mathcal{E} (21 December 2012) are also represented.

The calculation of the Moon age in the lunar series of mythical dates necessitated precise value of the Moon ratio corresponding to a particular lunar equation. To determine the lu-

T [day]	L	S [day]	ε [day]
11960 ^a	405	29.530864	1
4784 ^b	162	29.530864	1
4606 ^b	156	29.525641	8
4429 ^b	150	29.526667	11
4400 ^c	149	29.530201	2
2392 ^d	81	29.530864	1
Modern value		29.530588	4

TABLE IV. Mayan lunar period $S = T/L$ calculated from the length T of the lunar tables and the corresponding number of lunations $L = \text{Rd}(T, 29.53)$ as compared to the modern value of the Moon synodic period. The length of the lunar tables are taken from the Dresden Codex eclipse table^a (Bricker 1983), the Xultun lunar table^b (Satorno 2012), the Copan Moon ratio^c (Aveni 2001: 163) and the Palenque formula.^d (Teeple 1930) The error ε of the Moon ratio is calculated from Equ. 6.

nar equation, Mayan priests-astronomers developed the following method. They correlated the synodic movement of the Moon (using the pentalunex and the two lunar semesters) with the solar year and the five planet synodic periods corresponding to the calendar super-number \mathcal{N} . They recorded the lunar equation L lunations = T days for extended periods of time. They were looking for a Moon ratio $S = T/L$ such as \mathcal{N}/S is an integer with the error $\varepsilon \rightarrow 0$:

$$\varepsilon = |\mathcal{N} - \text{Rd}(\mathcal{N}/S) \times S| \quad (6)$$

where $\text{Rd}()$ is the nearest integer round function. The results are given in Table IV. In Palenque (present-day Mexico), somewhere between the III century BC and the VIII century CE, a careful analysis of the lunar data allowed Mayan priests-astronomers to determine the Palenque formula 81 lunations = 2392 days (1-day error). In Copan (present-day Honduras), somewhere between the V and the IX centuries CE, similar attempts allowed to determine the Copan Moon ratio 149 lunations = 4400 days (2-day error). In Xultun (present-day Guatemala), in the IX century CE, Mayan priests-astronomers tried to find other solutions by considering three different periods close to the Copan value: 150 lunations = 4429 days (11-day error), 156 lunations = 4606 (8-day error) and 162 lunations = 4784 days (1-day error). It seems that from this date, a unified lunar ratio, the Palenque formula, was used up to the XIV century CE as shown in the Dresden Codex eclipse table with a Moon ratio 405 lunations = 11960 days such as $S_0 = 11960/405 = 4784/162 = 2392/81 = 2^3 \times 13 \times 23/3^4 = 29.530864$ days. Indeed, the Palenque for-

mula ($\varepsilon = 1.28$ day) constitutes a slight improvement compared to the Copan Moon ratio ($\varepsilon = 1.88$ day). The Palenque formula corresponds to the equation $81 \times \mathcal{N} + 104 = 26008014145502 \times 2392$ and the error $\varepsilon = 104/81 = 1.28$ (Equ. 6). To perform such tedious calculations, Mayan priests-astronomers may have used a counting device, such as an abacus, (Thompson 1941, 1950) and benefited from the use of the Mayan numerals. The efficiency of Mayan numerals for arithmetical calculations has been noted previously. (Bietenholz 2013, French Anderson 1971) Archaeological evidence from the X century CE shows that the Aztec used the so-called Nephualtitzin, a counting device consisting of a wooden frame on which were mounted strings threaded with kernels of maize. (Sanchez 1961) A calculation of the lunar series from recent excavations has shown that the Palenque formula was also used in Tikal (present-day Guatemala) on 9.16.15.0.0 or 17 February 766 CE. (Fuls 2007) A question arises about the choice of this particular value. To answer this question, we calculate the exact time span $T_i = i \times S_M$ corresponding to the lunar equation i lunations = T_i^0 days where $T_i^0 = \text{Rd}(T_i)$ (nearest integer approximation), the Moon ratio $S_i = T_i^0/i$, the commensuration with the Tzolk'in $\text{LCM}(260, T_i^0)$ and the corresponding error ε_i (Equ. 6) for $i = 1$ to 643 lunations ($T_i^0 = 18988 > 1$ CR), considering the modern value of the Moon synodic period ($S_M = 29.530588$ days). The list contains the Copan Moon ratio $4400/149 = 29.530201$ ($\varepsilon = 1.88$ day) and the Palenque formula $2392/81 = 4784/162 = 11960/405 = 29.530864$ ($\varepsilon = 1.28$). We consider the values minimizing Equ. 6 such as $\text{LCM}(260, T_i^0) < 18980 = 1$ CR. The values are $30/1$, $59/2 = 118/4 = 236/8 = 295/10 = 29.5$, $148/5 = 29.6$ ($\varepsilon = 0$) and the Palenque formula $2392/81 = 11960/405 = 29.530864$ ($\varepsilon = 1.28$), the best approximation to the Moon synodic period. We can now describe the Mayan arithmetical model of astronomy. The Calendar Round describes the canonic solar year (Haab') and the synodic movement of Venus and Mars: $\text{LCM}(260, 365) = 18980 = 1$ CR, $\text{LCM}(260, 584) = 37960 = 2$ CR and $\text{LCM}(365, 780) = 56940 = 3$ CR, the length of the Dresden Codex Venus and Mars tables. (Bricker 2011: 163, 367) The Tun-Haab'-Kawil wheel $\mathcal{Y} = \text{LCM}(360, 365, 3276) = 2391480$ days induces the movement of the wheels describing the synodic movement of Mercury, Jupiter, Saturn and the lunar months $\{\text{LCM}(P_i, \mathcal{Y})/\mathcal{Y}, i =$

1..9} = {29, 1, 1, 1, 19, 3, 59, 89, 37} such as:

$$\begin{aligned} \mathcal{N} &= \mathcal{Y} \times 3 \times 19 \times 29 \times 37 \times 59 \times 89 \quad (7) \\ &\equiv \alpha \times S_0 \end{aligned}$$

where α is an integer number of lunations: $\alpha = 26008014145502$ corresponds to the Palenque formula $S_0 = 2392/81 = 29.530864$ days. The Tzolk'in and the Moon synodic movements are commensurate via the Palenque formula: $11960 = \text{LCM}(260,2392) = 5 \times 2392$, the length of the Dresden Codex eclipse table (Bricker 1983) related to the Calendar Round as $\text{LCM}(11960,18980) = 73 \times 11960 = 365 \times 2392 = 46 \text{ CR}$. The Maya were aware of the imperfection of the model and were constantly improving it. Evidence of a X century CE Mayan astronomical innovation has been found in Chichen Itza (present-day Mexico) and in the Dresden Codex Venus table. (Aldana 2016)

In conclusion, this study presents a complete description of the Mayan theory of time, characterized by a set of calendar cycles derived from an early model of naked-eye astronomy. This purely arithmetical model is based on an integer approximation of the solar year (Haab'), the three lunar months (the pentalunex and the two lunar semesters) and the synodic periods of Mercury, Venus, Mars, Jupiter and Saturn and was used to determine the Moon ratio from astronomical observation. The calendar super-number, defined as the least common multiple of the 9 astronomical input parameters, leads to the Mayan Calendar cycles: the 3276-day 4-Kawil, combination of the 4 directions-colors and the 819-day Kawil, the 18980-day Calendar Round, combination of the 260-day Tzolk'in and the 365-day Haab', and the 1872000-day Long Count Calendar (the 360-day Tun, the 7200-day Katun and the 144000-day Baktun). The results are expressed as a function of the Xultun numbers, four enigmatic Long Count numbers deciphered on the walls of a small room in the extensive Maya ruins of Xultun (present-day Guatemala), dating from the early IX century CE. (Saturno 2012) The Mayan cyclical concept of time is explained, in particular the existence of the 13 Baktun Maya Era and the position of the Calendar Round at the mythical date of creation 13(0).0.0.0.0 4 Ahau 8 Cumku. The Moon ratio are determined from this model by astronomical observation leading to the Palenque formula. The use of the Palenque formula is attested in several Classic period (200 to 900 CE) Maya sites and in Mayan Codices up to the Post-Classic period (1300 to 1521 CE). The Mayan arithmetical model of astronomy is described.

This provides evidence of the high proficiency of Mayan mathematics as applied to astronomy.

BIOGRAPHY

THOMAS CHANIER a native of Reims, France, studied electronic engineering at the Institut Supérieur d'Electronique et du Numérique Toulon and received his Master and Ph.D. in materials science from Aix-Marseille University. His research is on first-principles study of new functional materials, with applications to nanoelectronics and renewable energy. Beyond physics, he is interested in pre-Columbian Mesoamerican astronomy and mathematics.

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