



HAL
open science

Solution of the Mayan Calendar Enigma

Thomas Chanier

► **To cite this version:**

| Thomas Chanier. Solution of the Mayan Calendar Enigma. 2016. hal-01254966v2

HAL Id: hal-01254966

<https://hal.science/hal-01254966v2>

Preprint submitted on 16 Jan 2016 (v2), last revised 13 Nov 2018 (v7)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Solution of the Mayan Calendar Enigma

Thomas Chanier*¹

¹ *Université de Bretagne Occidentale, 6 avenue Victor le Gorgeu, F-29285 Brest Cedex, France*

The Mayan Codices, bark-paper books from the Late Postclassic period (1300 to 1521 CE) contain many tables correlating ritual practices to astronomical cycles. This demonstrates the achievements of Mayan mathematics as applied to the measure of time characterized by the use of a unique time reckoning system: the Mayan Calendar. Although several myths exist around Mayan religion, the origin of the Mayan Calendar and its connection to Mayan religion remain unknown. Because of the few clues left for us by the Maya, the Mayan cyclical concept of time is unexplained up to now. In this study, I propose a solution to this enigma. A calendar supernumber is calculated by computing the least common multiple of 8 canonical astronomical periods. The three major calendar cycles are shown to derive from this supernumber: the Calendar Round (the 260-day religious year Tzolk'in and the 365-day civil year Haab'), the Long Count Calendar (the 360-day Tun, the 7200-day Katun and the 144000-day Baktun) and the 3276-day Kawil-direction-color cycle. My calculations demonstrate the Mayan cyclical concept of time, notably the existence of the Maya Era, a cycle of 13 Baktun. Modular arithmetic considerations on the calendar supernumber give the position of the Calendar Round at the Mayan origin of time, the Long Count Calendar date 0.0.0.0.0 4 Ahau 8 Cumku, 3 East-Red. Various Long Count numbers present on Mayan Codices and monuments are explained by my approach. In particular, the results provide the meaning of the Xultun numbers, four enigmatic Long Count numbers deciphered in 2012 by Saturno *et al.* in the Mayan ruins of Xultun, Guatemala, dating from the IX century CE.¹

Mayan priests-astronomers were known for their astronomical and mathematical proficiency, as exemplified in the Dresden Codex, a XIV century CE bark-paper book containing many astronomical tables correlated to ritual cycles. However, due to the zealous role of the Inquisition during the XVI century CE Spanish conquest of Mexico, number of these Codices were burnt, leaving us with few information on Pre-Columbian Mayan culture. Thanks to the work of Mayan archeologists and epigraphists since the early XX century, the few Codices left, along with numerous inscriptions on monuments, were deciphered, underlying the importance of the concept of time in Maya civilization. This is reflected by the three major Mayan Calendars, reminiscent of the Mayan cyclical conception of time: the Calendar Round, the Long Count Calendar and the Kawil-direction-color cycle.



FIG. 1. Mayan/Aztec Calendar Stone representing the Five Suns, discovered in 1790 at El Zócalo, Mexico City, Mexico.

The Calendar Round (CR) represents a day in a non-repeating 18980-day cycle, a period of roughly 52 years,

the combination of the 365-day civil year Haab' and the 260-day religious year Tzolk'in. The Tzolk'in comprises 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etnab, Cauac, and Ahau). This forms a list of 260 ordered Tzolk'in dates from 1 Imix, 2 Ik, ... to 13 Ahau.² The Haab' comprises 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. This forms a list of 365 ordered Haab' dates from 0 Pop, 1 Pop, ... to 4 Uayeb.³ The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round, the least common multiple (LCM) of 260 and 365: $LCM(260,365) = 73 \times 260 = 52 \times 365 = 18980$ days. In the Calendar Round, a date is represented by $\alpha X\beta Y$ with the religious month $1 \leq \alpha \leq 13$, X one of the 20 religious days, the civil day $0 \leq \beta \leq 19$, and Y one of the 18 civil months, $0 \leq \beta \leq 4$ for the Uayeb. To reckon time in a period longer than 52 years, the Maya used the Long Count Calendar (LCC), describing a date D in a 1872000-day Maya Era of 13 Baktun, a period of roughly 5125 years, counting the number of day elapsed from the Mayan origin of time, the mythical date of creation 0.0.0.0.0 4 Ahau 8 Cumku, corresponding to the Gregorian Calendar date 11 August 3114 BC according to the Goodman-Martinez-Thompson (GMT) correlation. The XXI century saw the passage of the new Era on 21 December 2012, a date related to several apocalyptic or world renewal New Age theories, relayed by mass-media. Whereas Mayan mathematics are based on a vigesimal basis, the LCC is a 18-20-mixed radix basis representation of time: a date D is defined by a set of 5 numbers $(C_4.C_3.C_2.C_1.C_0)$ such as $D \equiv \text{mod}(D, 13 \times 144000) =$

$C_0 + C_1 \times 20 + C_2 \times 360 + C_3 \times 7200 + C_4 \times 144000$ where C_4 is the number of Baktun (144000 days) in the 13 Baktun Era ($0 \leq C_4 < 13$), C_3 the number of Katun (7200 days) in the current Baktun ($0 \leq C_3 < 20$), C_2 the number of Tun (360 days) in the current Katun ($0 \leq C_2 < 18$), C_1 the number of Winal (20 days) in the current Tun ($0 \leq C_1 < 20$) and C_0 the number of Kin (days) in the current Winal ($0 \leq C_0 < 20$). According to various myths around the Mayan Calendar, there are 5 Maya Eras or Suns of 13 Baktun as represented by the Mayan/Aztec Calendar Stone (Fig. 1). The Kawil-direction-color cycle or 4-Kawil is a 3276-day cycle, the combination of the 819-day Kawil and the 4 directions-colors (East-Red, South-Yellow, West-Black, North-White).⁴ The Kawil counts the number of day in a 819-day month (4 months corresponding to the directions-colors) in a non-repeating 3276-day 4-Kawil cycle. At the mythical date of creation, the Kawil count is 3 and the direction-color is East-Red. A Kawil date is then defined as $D \equiv \text{mod}(D + 3, 819)$ and the direction-color is given by $n = \text{mod}(\text{int}((D+3)/819), 4)$, $n = \{0, 1, 2, 3\} = \{\text{East-Red, South-Yellow, West-Black, North-White}\}$. Table I gives the different calendar cycles with their prime factorizations.

Cycle	P [day]	Prime factorization
Haab'	365	5×73
Tzolk'in	260	$2^2 \times 5 \times 13$
Winal	20	$2^2 \times 5$
Tun	360	$2^3 \times 3^2 \times 5$
Katun	7200	$2^5 \times 3^2 \times 5$
Baktun	144000	$2^7 \times 3^2 \times 5$
Kawil	819	$3^2 \times 7 \times 13$
4-Kawil	3276	$2^2 \times 3^2 \times 7 \times 13$

TABLE I. Calendar cycles and their prime factorizations.

The origin of the Long Count Periods is unknown. A common assumption is the desire of the calendar keeper to maintain the Tun in close agreement with the tropical/solar year of approximately 365.24 days.⁵ There is no consensus concerning the origin of the Tzolk'in, which has been associated with various astronomical cycles. 3 Tzolk'in correspond to Mars synodic period, 16 Tzolk'in equal 11 of Saturn synodic periods (+2 days), and 23 Tzolk'in are equivalent to 15 Jupiter synodic periods (-5 days).⁶ It has been tentatively connected to the eclipse half-year (173.31 days) because 2 Tzolk'in are very close to 3 eclipse half-years.⁷ Finally, it has been noted that the Tzolk'in approximates the length of time Venus is visible as a morning or evening star.⁸ The Kawil cycle has been attributed to the observation of Jupiter and Saturn^{9,10} because 19 (6) Kawil correspond to 39 (13) Jupiter (Saturn) synodic period. However, these interpretations fail to link the Tzolk'in and the Kawil to the Long Count Periods. The reason why the initial state of the Calendar Round at the LCC origin of time is 0.0.0.0.0 is 4 Ahau 8 Cumku remains unexplained up to now.

In 2012, four LCC numbers, the Xultun numbers (Table II), have been discovered on the walls of a small painted room in the Mayan ruins of Xultun, dating from the early IX century CE.¹ These numbers have a potential astronomical meaning. Indeed, $\mathcal{X}_0 = \text{LCM}(260, 360, 365)$ is a whole multiple of the Tzolk'in, the Haab', the Tun, Venus and Mars synodic periods: $341640 = 1314 \times 260 = 936 \times 365 = 949 \times 360 = 585 \times 584 = 438 \times 780$, $\mathcal{X}_1 = 365 \times 3276$ is the commensuration of the Haab' and the 4-Kawil cycle. However, the meaning of \mathcal{X}_2 and \mathcal{X}_3 , related to \mathcal{X}_0 by the equation $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$ is unknown. The greatest common divisor of the \mathcal{X}_i 's is $56940 = \text{LCM}(365, 780) = 3 \text{ CR}$ corresponding to the commensuration of the Haab' and Mars synodic period of 780 days.

\mathcal{X}_i LCC	D [day]	$\mathcal{X}_i/56940$
\mathcal{X}_0 2.7.9.0.0	341640	6
\mathcal{X}_1 8.6.1.9.0	1195740	21
\mathcal{X}_2 12.5.3.3.0	1765140	31
\mathcal{X}_3 17.0.1.3.0	2448420	43

TABLE II. Xultun numbers \mathcal{X}_i .¹ $56940 = \text{LCM}(365, 780) = 2^2 \times 3 \times 5 \times 13 \times 73$ is their largest common divisor and $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$.

Mayan priests-astronomers, known for their astronomical proficiency, may have observed with a naked eye the periodic movements of the five planets visible in the night sky: the moon, Mercury, Venus, Earth (solar year), Mars, Jupiter, and Saturn. Their respective canonical cycles are given in Table III. Evidences have been found in different Mayan Codices for Mercury, Venus, and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn.¹¹ In particular, on page 24 of the Dresden Codex is written the so-called Long Round number noted 9.9.16.0.0 or 1366560 days, a whole multiple of the Tzolk'in, the Haab', the Tun, Venus and Mars synodic periods, the Calendar Round and the Xultun number \mathcal{X}_0 : $\mathcal{LR} = 1366560 = 5256 \times 260 = 3744 \times 365 = 3796 \times 360 = 2340 \times 584 = 1752 \times 780 = 72 \times 18980 = 4 \times 341640$. The relevant periods for the prediction of solar/lunar eclipses are the lunar semesters of 177 or 178 days (6 Moon synodic periods), which are the time intervals between subsequent eclipse warning stations present

Planet	i	P_i [day]	Prime factorization
Mercury	1	116	$2^2 \times 29$
Venus	2	584	$2^3 \times 73$
Earth	3	365	5×73
Mars	4	780	$2^2 \times 3 \times 5 \times 13$
Jupiter	5	399	$3 \times 7 \times 19$
Saturn	6	378	$2 \times 3^3 \times 7$
Lunar	7	177	3×59
senesters	8	178	2×89

TABLE III. Planet canonical cycles^{1,12} and their prime factorizations.

in the Eclipse Table in the Dresden Codex and the lunar tables inscribed on the Xultun walls.^{1,12} From their prime factorizations (Table III), we calculate the calendar supernumber \mathcal{N} defined as the least common multiple of the P_i 's:

$$\begin{aligned}\mathcal{N} &= 20757814426440 & (1) \\ &= 2^2 \times 3^2 \times 7 \times 13 \times 19 \times 29 \times 59 \times 73 \times 89 \\ &= 365 \times 3276 \times 2 \times 3 \times 19 \times 29 \times 59 \times 89 \\ &= \text{LCM}(360, 365, 3276) \times 3 \times 19 \times 29 \times 59 \times 89\end{aligned}$$

Equ. 1 gives the calendar supernumber and its prime factorization. It is expressed as a function of the Haab', the 4-Kawil and the Tun. The Haab', canonical solar year, is such that the Haab' and the P_i 's are relatively primes (except Venus and Mars): the $\{\text{LCM}(P_i, 365)/365, i = 1..8\} = \{116, 8, 1, 156, 399, 378, 177, 178\}$ (Table III). The 4-Kawil and the Haab' are relatively primes: the $\text{LCM}(365, 3276) = 365 \times 3276$ and their largest common divisor is 1. The 4-Kawil has the following properties: the $\{\text{LCM}(P_i, 3276)/3276, i = 1..8\} = \{29, 146, 365, 5, 19, 3, 59, 89\}$. That defines the 4-Kawil. The commensuration of the 4-Kawil and the Haab', corresponding to the Xultun number $\mathcal{X}_1 = 365 \times 3276 = 1195740$, gives: $\{\text{LCM}(P_i, \mathcal{X}_1)/\mathcal{X}_1, i = 1..8\} = \{29, 2, 1, 1, 19, 3, 59, 89\}$. 360 is the integer closest to 365 such that the $\text{LCM}(360, 3276) = 32760$ and the $\{\text{LCM}(P_i, 32760)/32760, i = 1..8\} = \{29, 73, 73, 1, 19, 3, 59, 89\}$. The number 32760 or 4.11.0.0 has been derived from inscriptions on the Temple of the Cross in Palenque, Chiapas, Mexico.¹³ The Tun has the following properties: $\mathcal{Y} = \text{LCM}(360, 365, 3276) = 7 \times 341640 = 2391480$ such as $\{\text{LCM}(P_i, \mathcal{Y})/\mathcal{Y}, i = 1..8\} = \{29, 1, 1, 1, 19, 3, 59, 89\}$. The commensuration of the Haab', the 4-Kawil and the Baktun (400 Tun = 144000 days) gives rise to a calendar grand cycle $\mathcal{G}\mathcal{C} = \text{LCM}(365, 3276, 144000) = 400 \times \text{LCM}(360, 365, 3276) = 400 \times 7 \times \mathcal{X}_0 = 956592000$. The Euclidean division of the calendar supernumber \mathcal{N} by $\mathcal{G}\mathcal{C}$ gives:

$$\begin{aligned}\mathcal{N} &= \mathcal{G}\mathcal{C} \times \mathcal{Q} + \mathcal{R} & (2) \\ \mathcal{Q} &= 21699 \\ \mathcal{R} &= 724618440 \\ &= 101 \times 126 \times 56940 \\ &= 126 \times \sum_{i=0}^3 \mathcal{X}_i.\end{aligned}$$

If we pose $\mathcal{A} = 13 \times 73 \times 144000 = 400 \times \mathcal{X}_0 = 136656000$ such as $\mathcal{G}\mathcal{C} = 7 \times \mathcal{A}$, the Euclidean division of \mathcal{N} by \mathcal{A} gives:

$$\begin{aligned}\mathcal{N} &= \mathcal{A} \times \mathcal{Q} + \mathcal{R} & (3) \\ \mathcal{Q} &= 151898 \\ \mathcal{R} &= 41338440 = 121 \times 341640 \\ &= 121 \times \mathcal{X}_0\end{aligned}$$

We have: $\mathcal{A} = \text{LCM}(260, 365, 144000) = 100 \times \mathcal{L}\mathcal{R} = 7200 \times 18980 = 3600 \times 37960 = 2400 \times 56940$. The

only P_i 's commensurate to $\mathcal{A} = 13 \times 73 \times 144000$ are the Haab', Venus and Mars canonical periods such as the $\text{LCM}(584, 365) = 37960$ and $\text{LCM}(780, 365) = 56940$, the length of the Venus and Mars Table in the Dresden Codex. We can rewrite Equ. 2 and 3 as:

$$\begin{aligned}\mathcal{N} - 121 \times \mathcal{X}_0 &= 151898 \times \mathcal{A} & (4) \\ \mathcal{N} - 126 \times \sum_{i=0}^3 \mathcal{X}_i &= 151893 \times \mathcal{A}\end{aligned}$$

The Long Count Periods appear in Equ. 4: $151898 = 338 + 360 + 7200 + 144000 = 1.1.1.16.18$ and $151893 = 333 + 360 + 7200 + 144000 = 1.1.1.16.13$. The Haab' and the Long Count Periods are such that $\{\text{LCM}(365, P_i)/P_i = 73, P_i = 18 \times 20^i, i > 0\}$. The Tzolk'in is defined by the $\text{LCM}(13, 20)$ such as $\text{LCM}(260, 365) = 73 \times 260 = 52 \times 365 = 18980$ days or a Calendar Round. The subtraction of the two equations in 4 can be expressed as a function of the Xultun numbers:

$$\begin{aligned}5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + 95 \times 126 \times 56940 & (5) \\ 5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + \text{LCM}\left(\sum_{i=1}^3 \mathcal{X}_i, \mathcal{X}_1 + \mathcal{X}_3 + 2\mathcal{X}_2\right)\end{aligned}$$

Since \mathcal{X}_1 is known (the commensuration of the Haab' and the 4-Kawil), that defines \mathcal{X}_2 and \mathcal{X}_3 . The relationship $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$ may be a mnemonic tool to calculate $\mathcal{Y} = \mathcal{X}_3 + 2\mathcal{X}_2 = 105 \times 56940$. The four Xultun numbers provides a proof that Mayan priests-astronomers determined the canonical synodic periods of the five planets visible with a naked eye: Mercury, Venus, Mars, Jupiter and Saturn. A question arises at this point to know how the Maya, as early as the IX century CE, were able to compute tedious arithmetical calculations on such large numbers with up to 14 digits in decimal basis. Here is a possible method. They determined the prime factorizations of the canonical astronomical periods P_i (Table III) and listed each primes p_i with their maximal order of multiplicity α_i . They determined the Haab', the 4-Kawil and the Tun by their definitions given earlier. They calculated the calendar supernumber \mathcal{N} (the LCM of the P_i 's) by multiplying each p_i 's α_i time. The Euclidean division of \mathcal{N} by $\mathcal{G}\mathcal{C} = 7 \times \mathcal{A} = 400 \times 7 \times \mathcal{X}_0$ (Equ. 2) is equivalent to a simplification of \mathcal{N} by $7 \times \mathcal{X}_0 = 2391480$ and the Euclidean division of the product of the 5 left primes ($3 \times 19 \times 29 \times 59 \times 89 = 8679903$) by 400. The Euclidean division of \mathcal{N} by $\mathcal{A} = \mathcal{G}\mathcal{C}/7 = 13 \times 73 \times 144000 = 400 \times \mathcal{X}_0$ (Equ. 3) is equivalent to a simplification of \mathcal{N} by $\mathcal{X}_0 = 341640$ and the Euclidean division of the product of the 6 left primes ($3 \times 7 \times 19 \times 29 \times 59 \times 89 = 60759321$) by 400. It is to be noted that the prime factorization of the calendar supernumber only includes prime numbers < 100 which facilitates the operation (there are only 25 prime numbers lower than 100).

Equ. 5 defines the main religious cycle $5\mathcal{A}$ such as:

$$\begin{aligned}5 \times \mathcal{A} &= 400 \times 5 \times \mathcal{X}_0 & (6) \\ &= 5 \times \mathcal{X}_0 + 285 \times 7 \times \mathcal{X}_0\end{aligned}$$

The grand cycle \mathcal{GC} is defined by a period $\mathcal{C} = 7 \times \mathcal{X}_0 = \text{LCM}(360,365,3276) = \text{LCM}(260,360,365,3276) = 2391480$ (commensuration of the Tzolk'in, the Tun, the Haab' and the 4-Kawil). The main religious cycle $5\mathcal{A}$ gives the religious date $5\mathcal{X}_0$: $5 \times \mathcal{X}_0 = 5 \times 341640 = \text{mod}(5 \times \mathcal{A}, 2391480) = 1708200 = 11.17.5.0.0$. $5\mathcal{X}_0$ has the same properties (same Tzolk'in date, Haab' date, Kawil and direction-color) as the end of the main religious cycle $5\mathcal{A}$. The main religious cycle $5\mathcal{A}$ is such that $5 \times \mathcal{A} = 5 \times 13 \times 73 \times 144000 = 12000 \times 56940 = 365 \times \mathcal{E}$ with the 13 Baktun Era $\mathcal{E} = 13 \times 144000 = 1872000$ defined as:

$$\begin{aligned} \mathcal{E} &= 13 \times 360 + 114 \times \text{LCM}(260, 3276) & (7) \\ &= 400 \times 13 \times 360 \end{aligned}$$

The $\text{LCM}(260,3276) = 16380$ is the commensuration of the Tzolk'in and the 4-Kawil. The date 13 Tun has the same Tzolk'in date, Kawil and direction-color as the end of the 13 Baktun Era: $\text{mod}(1872000,16380) = 4680$. Since $\mathcal{E} - 5 \times \mathcal{X}_0 = 1872000 - 1708200 = 163800 = 10 \times \text{LCM}(260,3276)$, \mathcal{E} has the same Tzolk'in, Kawil and direction-color as the end of the main religious cycle $5\mathcal{A}$. The three major calendar cycles give rise to the following commensuration: 1 Maya Era = $\text{LCM}(260,144000) = 1872000$, 7 Maya Eras = $\text{LCM}(3276,144000) = 7 \times 1872000$ and 73 Maya Eras = $\text{LCM}(365,1872000) = 73 \times 1872000$. Finally we have:

$$\begin{aligned} \mathcal{GC} &= 7 \times \mathcal{A} = 73 \times 7 \times \mathcal{E} & (8) \\ &= \text{LCM}(260, 365, 3276, \mathcal{E}) \end{aligned}$$

The day 0 and \mathcal{GC} have therefore the same properties: 13(0).0.0.0.0 4 Ahau 8 Cumku, 3 East-Red. That defines the Mayan cyclical concept of time. The two dates $5\mathcal{X}_0$ and \mathcal{E} are important due to their relationships with the main religious cycle $5\mathcal{A}$ defined by Equ. 5.

Since the Mayan time is cyclical, the initialization of the grand cycle is arbitrary. Here is a possible method to explain the initial cyclical date (Mayan mythical date of creation), on the date 0.0.0.0.0 4 Ahau 8 Cumku, 3 East-Red. We consider modular arithmetic relations between the calendar supernumber and the three major calendar cycles. Since $\mathcal{GC} = 2800 \times \mathcal{X}_0$, it is sufficient to initialize \mathcal{X}_0 . For that purpose, we first create ordered lists of the Haab' and the Tzolk'in, assigning a unique set of 2 numbers for each day of the 18980-day Calendar Round.^{2,3} For the Haab', the first day is 0 Pop (numbered 0) and the last day 4 Uayeb (numbered 364). For the Tzolk'in, the first day is 1 Imix (numbered 0) and the last day 13 Ahau (numbered 259). In this notation, the date of creation 4 Ahau 8 Cumku is equivalent to {160;349} and a date D in the Calendar Round can be written as $D \equiv \{\text{mod}(D + 160,260); \text{mod}(D + 349,365)\}$. We have: $18980 = \text{LCM}(260,365) = 13 \times 20 \times 73$. The calendar supernumber is such that: $\text{mod}(\mathcal{N}/13/73,260) = 160$, $\text{mod}(\mathcal{N}/13/73,13) = 4$, $\text{mod}(\mathcal{N}/13/73,20) = 0$ and $\text{mod}(\mathcal{N}/13/73,73) = 49$. The date {160;49} corresponds to 4 Ahau 8 Zip, the day 0 (mod 18980), beginning/completion of a Calendar Round. We now consider

the Xultun number $\mathcal{X}_0 = \text{LCM}(260,365,360) = 13 \times 73 \times 360 = 18 \times 18980 = 341640$ which corresponds to the completion of a 13 Tun cycle, a period of 4680 days. The completion of a Calendar Round corresponds to 18980 days elapsed such as $\text{mod}(18980,4680) = 260$. Starting the CR count at 4 Ahau 8 Zip, the next date in the ordered CR list such as $\text{mod}(D,4680) = 0$ is the date 4 Ahau 8 Cumku, 4680 days later. A date D is then expressed as $\{\text{mod}(D + 4680 + 160,260); \text{mod}(D + 4680 + 49,365)\} = \{\text{mod}(D + 160,260); \text{mod}(D + 349,365)\}$. Since $\mathcal{GC} = 29200 \times 32760$, the Kawil and direction-color are initialized as $\text{mod}(\mathcal{N}/32760,4) = 3$ East-Red. That defines the position of the Calendar Round and the Kawil at the mythical date of creation, the LCC date 0.0.0.0.0 4 Ahau 8 Cumku {160;349}, 3 East-Red, the beginning of the previous 13 Baktun cycle or 400 cycles of 13 Tun.

Date D	LCC date	Cyclical date
\mathcal{I}_0	0	0.0.0.0.0 {160,349,3,0}
$5\mathcal{X}_0$	1708200	11.17.5.0.0 {160,349,588,1}
\mathcal{E}	1872000	13(0).0.0.0.0 {160,264,588,1}
$5\mathcal{A}$	$365 \times 13(0)$	0.0.0.0.0 {160,349,588,1}
$7\mathcal{A}$	$511 \times 13(0)$	0.0.0.0.0 {160,349,3,0}

TABLE IV. Important Mayan religious date: \mathcal{I}_0 (mythical date of creation), $5\mathcal{X}_0$ (return of Kukulcan), \mathcal{E} (end of the 13 Baktun Era) and $5\mathcal{A}$ (end of the main religious cycle) and $7\mathcal{A}$ (end of the Maya grand cycle). A date is defined as its linear time day D and its cyclical equivalent given by the LCC date and a set of 4 integers $\{T;H;K;n\}$ where T is the Tzolk'in index, H is the Haab' index, K is the Kawil index and n is the direction-color index. The 4 indices are calculated according to their definitions: $\{T;H;K;n\} = \{\text{mod}(D+160,260); \text{mod}(D+349,365); \text{mod}(D+3,819); \text{mod}(\text{int}((D+3)/819),4)\}$.

We now calculate the 4 indices corresponding to the Tzolk'in, the Haab', the Kawil and the direction-color. Table IV gives the results: a date is defined as its linear time day D and its cyclical equivalent. Arithmetical calculations on the pseudo-astronomical calendar supernumber \mathcal{N} , defined as the least common multiple of 8 astronomical input parameters, lead to the main religious cycle $5\mathcal{A}$: $5 \times \mathcal{A} = 365 \times 13 \times 144000$. It defines the religious date $5\mathcal{X}_0 = 5 \times 341640 = 1708200 = 11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow or 3 July 1564 CE according to the GMT correlation, a date having the same properties as the end of the main religious cycle $5\mathcal{A}$ (same Tzolk'in date, Haab' date, Kawil and direction-color). The Maya Era \mathcal{E} of 13 Baktun is a smaller cycle originating from the calendar grand cycle $\mathcal{GC} = 7 \times \mathcal{A} = 511 \times \mathcal{E}$. The end date of the previous Era 13(0).0.0.0.0 4 Ahau 3 Kankin, 588 South-Yellow or 21 December 2012 has the same Tzolk'in date, Kawil and direction-color as the end date of the main religious cycle $5\mathcal{A}$. The 365th Maya Era corresponds to the main religious cycle $5\mathcal{A}$ or the date $365 \times 13(0)$.0.0.0.0 4 Ahau 8 Cumku, 588 South-Yellow. The Mayan Calendar starts/ends on the day 0/956592000 defining the grand cycle $\mathcal{GC} = 7 \times 13 \times 73 \times 144000$. \mathcal{GC} corresponds to $7 \times 73 = 511$ Maya Eras, the begin-

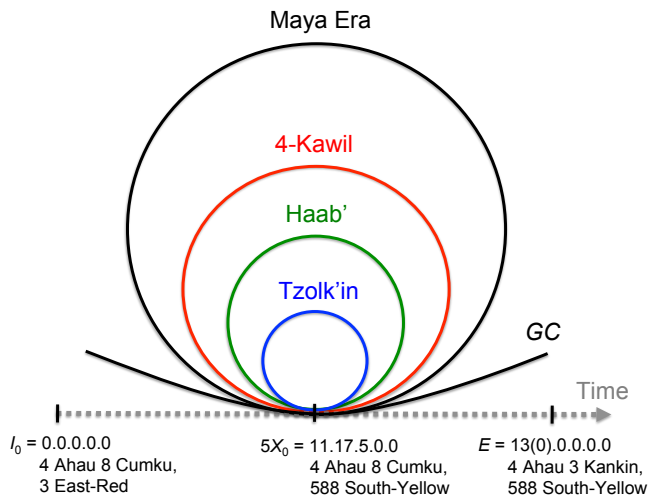


FIG. 2. Mayan/Aztec cyclical *vs* linear concept of time, with the 260-day Tzolk'in, the 365-day Haab', the 3276-day 4-Kawil, the Maya Era of 13 Baktun (5125 years) and the grand cycle \mathcal{GC} of 511 Maya Eras. The mythical date of creation I_0 (11 August 3114 BC), the date of the return of Kukulcan $5X_0$ (3 July 1564 CE) and the end of the previous Maya Era \mathcal{E} (21 December 2012) are also represented.

ning/ending date $511 \times 13(0).0.0.0.0$ 4 Ahau 8 Cumku, 3 East-Red. The Mayan concept of time is therefore cyclical, with a grand cycle $\mathcal{GC} = \text{LCM}(260, 365, 3276, \mathcal{E})$. The previous Maya Era is characterized by two important dates: $5X_0 = 11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow and $\mathcal{E} = 13(0).0.0.0.0$ 4 Ahau 3 Kankin, 588 South-Yellow, which are defined by their equivalent properties compared to the main religious cycle $5A$ (4 Ahau 8 Cumku, 588 South-Yellow). The date $5X_0$ or 3 July 1564 CE may have been related to a series of omens prophesying the Spanish conquest of Mexico which occurred from February 1519 to 13 August 1521, misinterpreted as the return of the main Aztec (Mayan) God Quetzalcoatl (Kukulcan), the feathered serpent.¹⁵ This prophesy might have originated from the calculations realized by Mayan priests-astronomers in Xultun in the IX century CE. To perform such tedious calendrical calculations, Mayan priests-astronomers might have used a counting device, such as an abacus,¹⁶ and benefited from

the use of the Mayan numerals. Indeed, the efficiency of Mayan numerals for arithmetical calculations have been noted previously.^{17,18} Fig. 2 gives the summary of the results. The Mayan time is cyclical, with a grand cycle \mathcal{GC} defined as the commensuration of the Tzolk'in, the Haab', the 4-Kawil and the Maya Era. There are three important dates in the previous Maya Era: the mythical date of creation $0.0.0.0.0$ 4 Ahau 8 Cumku, 3 East-Red, the date of the return of Kukulcan $11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow and the end of the Maya Era $13(0).0.0.0.0$ 4 Ahau 3 Kankin, 588 South-Yellow.

In conclusion, this study presents a unified description of the Mayan Calendar based on an early model of naked-eye astronomy. A calendar supernumber \mathcal{N} is calculated by taking the least common multiple of 8 canonical astronomical periods describing the planet synodic movements and the apparition of solar/lunar eclipse. This calendar supernumber defines the three major Mayan Calendar cycles: the 3276-day 4-Kawil, combination of the 4 directions-colors and the 819-day Kawil, the 18980-day Calendar Round, combination of the Haab' and the Tzolk'in, and the 1872000-day Long Count Calendar. The 360-day Tun, the 365-day Haab' and the 4-Kawil are issued from the prime factorization of the 8 canonical astronomical input parameters. The 260-day Tzolk'in and the Long Count Periods (the 360-day Tun, the 7200-day Katun and the 144000-day Baktun) are obtained from arithmetical calculations on \mathcal{N} . The correlation of the three major calendar cycles represents a calendar grand cycle \mathcal{GC} . The two Euclidean divisions of \mathcal{N} by \mathcal{GC} and $\mathcal{GC}/7 = 13 \times 73 \times 144000$ lead to the Xultun numbers, deciphered by Saturno *et al.* in 2012 inside a small room of the extensive Mayan ruins of Xultun, Guatemala and dating from the early IX century CE.¹ The Mayan cyclical concept of time is demonstrated, in particular the existence of the Maya Era of 13 Baktun. Modular arithmetic considerations on the calendar supernumber determine the position of the Calendar Round at the Mayan mythical date of creation $0.0.0.0.0$ 4 Ahau 8 Cumku, reflecting the Mayan cyclical concept of time. The combination of astronomy and mathematics in Mayan religion is a form of celestial divination. Mayan priests-astronomers used arithmetics to correlate ritual cycles with astronomical events, rythming political life and religious practices, in order to embed Maya civilization in the endless course of time.

* e-mail: thomas.chanier@gmail.com

¹ W. A. Saturno, D. Stuart, A. F. Aveni and F. Rossi, *Science* **336**, 714 (2012).

² A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 143.

³ A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 147.

⁴ H. Berlin and D. H. Kelley, *The 819-day Count and Color-Direction Symbolism among the Classic Maya*. Middle American Research Institute Publication 26 (1961).

⁵ A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 136.

⁶ J. S. Justeson, *The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources*, in *World Archaeoas-*

- tromy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy, edited by A. F. Aveni, (Cambridge University Press, 1989), p. 82.
- ⁷ A. F. Aveni, in *Skywatchers: A Revised and Updated Version of Skywatchers of Ancient Mexico* (University of Texas Press, 2001), p. 144.
- ⁸ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 69.
- ⁹ F. G. Lounsbury, in *Maya Numeration, Computation, and calendar Astronomy. Dictionary of Scientific Biography*, vol. 15, suppl. 1, edited by Charles Coulston-Gillispie, (New York: Charles Scriber's Sons, 1978), p. 811
- ¹⁰ J. S. Justeson, *The ancient Maya Ethnoastronomy: An Overview of Hieroglyphic Sources*, in *World Archaeoastronomy: Selected Papers from the Second Oxford International Conference on Archaeoastronomy*, edited by A. F. Aveni, (Cambridge University Press, 1989), p.103.
- ¹¹ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), pp. 163, 367, 847.
- ¹² H. M. Bricker and V. R. Bricker, *Current Anthropology* **24**, 1-23 (1983).
- ¹³ F. G. Lounsbury, *A Rationale for the Initial Date of the Temple of the Cross at Palenque. Second Palenque Roundtable, 1974. The Art, Iconography & Dynastic History of Palenque, Part III*, edited by Merle Greene Robertson. Pebble Beach, California: Pre-Columbian Art Research, The Robert Louis Stevenson School (1976), p. 217.
- ¹⁴ H. M. Bricker and V. R. Bricker, in *Astronomy in the Maya Codices* (American Philosophical Society, Philadelphia, 2011), p. 235.
- ¹⁵ H. Thomas, in *Conquest: Cortes, Montezuma, and the Fall of Old Mexico* (Simon and Schusters, 1995), p. 185.
- ¹⁶ J. E. S. Thompson, *Carnegie Institution of Washington publication* **528**, 37-62 (1947).
- ¹⁷ W. Bietenholz, *Springer Science+Business Media New York*, **35**, 21 (2013).
- ¹⁸ W. French Anderson, *American Antiquity*, **36**, 54 (1971).