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Solution of the Mayan Calendar Enigma

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The Mayan Codices, bark-paper books from the Late Postclassic period (1300 to 1521 CE) contain many tables correlating ritual practices to astronomical cycles. This demonstrates the achievements of Mayan mathematics as applied to time measurements characterized by the use of a unique time reckoning system: the Mayan Calendar. Despite many myths, the origin of the Mayan Calendar and its connection to Mayan religion remains unknown, due to the few clues left for us by the Maya. In this study, we propose a solution to this enigma. A pseudo-astronomical calendar supernumber is calculated by computing the least common multiple of 8 canonical astronomical periods. The three major calendar cycles are shown to derive from this supernumber: the Calendar Round (the 260-day religious year Tzolk'in and the 365-day civil year Haab'), the Long Count Calendar (the 360-day Tun, the 7200-day Katun and the 144000-day Baktun) and the 3276-day Kawil-direction-color cycle. My calculations explain certain aspect of Mayan religion notably the existence of the Maya Era, a cycle of 13 Baktun. Modular arithmetic considerations on the calendar supernumber give the position of the Calendar Round at the Mayan origin of time, the Long Count Calendar date 0.0.0.0.0 4 Ahau 8 Cumku. Various Long Count numbers present on Mayan Codices and monuments are explained by my approach. In particular, the results provide the meaning of the Xultun numbers, four enigmatic Long Count numbers deciphered in 2012 by Saturno *et al.* in the Mayan ruins of Xultun, Guatemala, dating from the IX century CE.¹

Mayan priests-astronomers were known for their astronomical and mathematical proficiency, as exemplified in the Dresden Codex, a XIV century CE bark-paper book containing many astronomical tables correlated to ritual cycles. However, due to the zealous role of the Inquisition during the XVI century CE Spanish conquest of Mexico, number of these Codices were burnt, leaving us with few information on Pre-Columbian Mayan culture. Thanks to the work of Mayan archeologists and epigraphists since the early XX century, the few Codices left, along with numerous inscriptions on monuments, were deciphered, underlying the importance of the concept of time in Maya civilization. This is reflected by the three major Mayan Calendars, reminiscent of the Mayan cyclical conception of time: the Calendar Round, the Long Count Calendar and the Kawil-direction-color cycle.

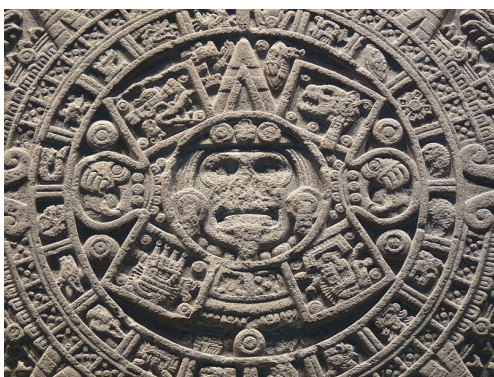


FIG. 1. Mayan/Aztec Calendar Stone representing the Five Suns, discovered in 1790 at El Zócalo, Mexico City, Mexico.

The Calendar Round (CR) represents a day in a non-repeating 18980-day cycle, a period of roughly 52 years, the combination of the 365-day civil year Haab' and the

260-day religious year Tzolk'in. The Tzolk'in comprises 13 months (numerated from 1 to 13) containing 20 named days (Imix, Ik, Akbal, Kan, Chicchan, Cimi, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Etnab, Cauac, and Ahau). This forms a list of 260 ordered Tzolk'in dates from 1 Imix, 2 Ik, ... to 13 Ahau.² The Haab' comprises 18 named months (Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumku) with 20 days each (Winal) plus 1 extra month (Uayeb) with 5 nameless days. This forms a list of 365 ordered Haab' dates from 0 Pop, 1 Pop, ... to 4 Uayeb.³ The Tzolk'in and the Haab' coincide every 73 Tzolk'in or 52 Haab' or a Calendar Round, the least common multiple (LCM) of 260 and 365: $LCM(260,365) = 73 \times 260 = 52 \times 365 = 18980$ days. In the Calendar Round, a date is represented by $\alpha X \beta Y$ with the religious month $1 \leq \alpha \leq 13$, X one of the 20 religious days, the civil day $0 \leq \beta \leq 19$, and Y one of the 18 civil months, $0 \leq \beta \leq 4$ for the Uayeb. To reckon time in a period longer than 52 years, the Maya used the Long Count Calendar (LCC), describing a date D in a 1872000-day Maya Era of 13 Baktun, a period of roughly 5125 years, counting the number of day elapsed from the Mayan origin of time, the mythical date of creation 0.0.0.0.0 4 Ahau 8 Cumku, corresponding to the Gregorian Calendar date 11 August 3114 BC according to the Goodman-Martinez-Thompson (GMT) correlation. The XXI century saw the passage of the new Era on 21 December 2012, a date related to several apocalyptic or world renewal New Age theories, relayed by mass-media. Whereas Mayan mathematics are based on a vigesimal basis, the LCC is a 18-20-mixed radix basis representation of time: a date D is defined by a set of 5 numbers $(C_4, C_3, C_2, C_1, C_0)$ such as $D \equiv \text{mod}(D, 13 \times 144000) = C_0 + C_1 \times 20 + C_2 \times 360 + C_3 \times 7200 + C_4 \times 144000$ where C_4 is the number of Baktun (144000 days) in

the 13 Baktun Era ($0 \leq C_4 < 13$), C_3 the number of Katun (7200 days) in the current Baktun ($0 \leq C_3 < 20$), C_2 the number of Tun (360 days) in the current Katun ($0 \leq C_2 < 18$), C_1 the number of Winal (20 days) in the current Tun ($0 \leq C_1 < 20$) and C_0 the number of Kin (days) in the current Winal ($0 \leq C_0 < 20$). According to various myths around the Mayan Calendar, there are 5 Maya Eras or Suns of 13 Baktun as represented by the Mayan/Aztec Calendar Stone (Fig. 1). The Kawil-direction-color cycle or 4-Kawil is a 3276-day cycle, the combination of the 819-day Kawil and the 4 directions-colors (East-Red, South-Yellow, West-Black, North-White).⁴ The Kawil counts the number of day in a 819-day month (4 months corresponding to the directions-colors) in a non-repeating 3276-day 4-Kawil cycle. At the mythical date of creation, the Kawil count is 3 and the direction-color is East-Red. A Kawil date is then defined as $D \equiv \text{mod}(D + 3, 819)$ and the direction-color is given by $n = \text{mod}(\text{int}((D+3)/819); 4)$, $n = \{0, 1, 2, 3\} = \{\text{East-Red, South-Yellow, West-Black, North-White}\}$. Table I gives the different calendar cycles with their prime factorizations.

Cycle	P [day]	Prime factorization
Haab'	365	5×73
Tzolk'in	260	$2^2 \times 5 \times 13$
Winal	20	$2^2 \times 5$
Tun	360	$2^3 \times 3^2 \times 5$
Katun	7200	$2^5 \times 3^2 \times 5$
Baktun	144000	$2^7 \times 3^2 \times 5$
Kawil	819	$3^2 \times 7 \times 13$
4-Kawil	3276	$2^2 \times 3^2 \times 7 \times 13$

TABLE I. Calendar cycles and their prime factorizations.

The origin of the Long Count Periods is unknown. A common assumption is the desire of the calendar keeper to maintain the Tun in close agreement with the tropical/solar year of approximately 365.24 days.⁵ There is no consensus concerning the origin of the Tzolk'in, which has been associated with various astronomical cycles. 3 Tzolk'in correspond to Mars synodic period, 16 Tzolk'in equal 11 of Saturn synodic periods (+2 days), and 23 Tzolk'in are equivalent to 15 Jupiter synodic periods (-5 days).⁶ It has been tentatively connected to the eclipse half-year (173.31 days) because 2 Tzolk'in are very close to 3 eclipse half-years.⁷ Finally, it has been noted that the Tzolk'in approximates the length of time Venus is visible as a morning or evening star.⁸ The Kawil cycle has been attributed to the observation of Jupiter and Saturn^{9,10} because 19 (6) Kawil correspond to 39 (13) Jupiter (Saturn) synodic period. However, these interpretations fail to link the Tzolk'in and the Kawil to the Long Count Periods. The reason why the initial state of the Calendar Round at the LCC origin of time is 0.0.0.0.0 is 4 Ahau 8 Cumku remains unexplained up to now.

In 2012, four LCC numbers, the Xultun numbers (Table II), have been discovered on the walls of a

small painted room in the Mayan ruins of Xultun, dating from the early IX century CE.¹ These numbers have a potential astronomical meaning. Indeed, $\mathcal{X}_0 = \text{LCM}(260, 360, 365)$ is a whole multiple of the Tzolk'in, the Haab', the Tun, Venus and Mars synodic periods: $341640 = 1314 \times 260 = 936 \times 365 = 949 \times 360 = 585 \times 584 = 438 \times 780$, $\mathcal{X}_1 = 365 \times 3276$ is the commensuration of the Haab' and the 4-Kawil cycle. However, the meaning of \mathcal{X}_2 and \mathcal{X}_3 , related to \mathcal{X}_0 by the equation $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$ is unknown. The greatest common divisor of the \mathcal{X}_i 's is $56940 = \text{LCM}(365, 780) = 3 \text{ CR}$ corresponding to the commensuration of the Haab' and Mars synodic period of 780 days.

\mathcal{X}_i	LCC	D [day]	$\mathcal{X}_i/56940$
\mathcal{X}_0	2.7.9.0.0	341640	6
\mathcal{X}_1	8.6.1.9.0	1195740	21
\mathcal{X}_2	12.5.3.3.0	1765140	31
\mathcal{X}_3	17.0.1.3.0	2448420	43

TABLE II. Xultun numbers \mathcal{X}_i .¹ $56940 = \text{LCM}(365, 780) = 2^2 \times 3 \times 5 \times 13 \times 73$ is their largest common divisor and $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$.

Mayan priests-astronomers, known for their astronomical proficiency, may have observed with a naked eye the periodic movements of the five planets visible in the night sky: the moon, Mercury, Venus, Earth (solar year), Mars, Jupiter, and Saturn. Their respective canonical cycles are given in Table III. Evidences have been found in different Mayan Codices for Mercury, Venus, and Mars, but it is unclear whether they tracked the movements of Jupiter and Saturn.¹¹ In particular, on page 24 of the Dresden Codex is written the so-called Long Round number noted 9.9.16.0.0 or 1366560 days, a whole multiple of the Tzolk'in, the Haab', the Tun, Venus and Mars synodic periods, the Calendar Round and the Xultun number \mathcal{X}_0 : $\mathcal{LR} = 1366560 = 5256 \times 260 = 3744 \times 365 = 3796 \times 360 = 2340 \times 584 = 1752 \times 780 = 72 \times 18980 = 4 \times 341640$. The relevant periods for the prediction of solar/lunar eclipses are the lunar semesters of 177 or 178 days (6 Moon synodic periods), which are the time intervals between subsequent eclipse warning stations present in the Eclipse Table in the Dresden Codex and the lunar

Planet	i	P_i [day]	Prime factorization
Mercury	1	116	$2^2 \times 29$
Venus	2	584	$2^3 \times 73$
Earth	3	365	5×73
Mars	4	780	$2^2 \times 3 \times 5 \times 13$
Jupiter	5	399	$3 \times 7 \times 19$
Saturn	6	378	$2 \times 3^3 \times 7$
Lunar	7	177	3×59
senesters	8	178	2×89

TABLE III. Planet canonical cycles^{1,12} and their prime factorizations.

tables inscribed on the Xultun walls.^{1,12} From their prime factorizations (Table III), we calculate the calendar supernumber \mathcal{N} defined as the least common multiple of the P_i 's:

$$\begin{aligned}\mathcal{N} &= 20757814426440 & (1) \\ &= 2^2 \times 3^2 \times 7 \times 13 \times 19 \times 29 \times 59 \times 73 \times 89 \\ &= 365 \times 3276 \times 2 \times 3 \times 19 \times 29 \times 59 \times 89 \\ &= \text{LCM}(360, 365, 3276) \times 3 \times 19 \times 29 \times 59 \times 89\end{aligned}$$

Equ. 1 gives the calendar supernumber and its prime factorization. It is expressed as a function of the Haab', the 4-Kawil and the Tun. The Haab', canonical solar year, is such that the Haab' and the P_i 's are relatively primes (except Venus and Mars): the $\{\text{LCM}(P_i, 365)/365, i = 1..8\} = \{116, 8, 1, 156, 399, 378, 177, 178\}$ (Table III). The 4-Kawil and the Haab' are relatively primes: the $\text{LCM}(365, 3276) = 365 \times 3276$ and their largest common divisor is 1. The 4-Kawil has the following properties: the $\{\text{LCM}(P_i, 3276)/3276, i = 1..8\} = \{29, 146, 365, 5, 19, 3, 59, 89\}$. That defines the 4-Kawil. The commensuration of the 4-Kawil and the Haab', corresponding to the Xultun number $\mathcal{X}_1 = 365 \times 3276 = 1195740$, gives: $\{\text{LCM}(P_i, \mathcal{X}_1)/\mathcal{X}_1, i = 1..8\} = \{29, 2, 1, 1, 19, 3, 59, 89\}$. 360 is the integer closest to 365 such that the $\text{LCM}(360, 3276) = 32760$ and the $\{\text{LCM}(P_i, 32760)/32760, i = 1..8\} = \{29, 73, 73, 1, 19, 3, 59, 89\}$. The number 32760 or 4.11.0.0 has been derived from inscriptions on the Temple of the Cross in Palenque, Chiapas, Mexico.¹³ The Tun has the following properties: $\mathcal{Y} = \text{LCM}(360, 365, 3276) = 7 \times 341640 = 2391480$ such as $\{\text{LCM}(P_i, \mathcal{Y})/\mathcal{Y}, i = 1..8\} = \{29, 1, 1, 1, 19, 3, 59, 89\}$. The commensuration of the Haab', the 4-Kawil and the Baktun (400 Tun = 144000 days) gives rise to a calendar grand cycle $\mathcal{GC} = \text{LCM}(365, 3276, 144000) = 400 \times \text{LCM}(360, 365, 3276) = 400 \times 7 \times \mathcal{X}_0 = 956592000$. The Euclidean division of the calendar supernumber \mathcal{N} by \mathcal{GC} gives:

$$\begin{aligned}\mathcal{N} &= \mathcal{GC} \times \mathcal{Q} + \mathcal{R} & (2) \\ \mathcal{Q} &= 21699 \\ \mathcal{R} &= 724618440 \\ &= 101 \times 126 \times 56940 \\ &= 126 \times \sum_{i=0}^3 \mathcal{X}_i.\end{aligned}$$

If we pose $\mathcal{A} = 13 \times 73 \times 144000 = 400 \times \mathcal{X}_0 = 136656000$ such as $\mathcal{GC} = 7 \times \mathcal{A}$, the Euclidean division of \mathcal{N} by \mathcal{A} gives:

$$\begin{aligned}\mathcal{N} &= \mathcal{A} \times \mathcal{Q} + \mathcal{R} & (3) \\ \mathcal{Q} &= 151898 \\ \mathcal{R} &= 41338440 = 121 \times 341640 \\ &= 121 \times \mathcal{X}_0\end{aligned}$$

The only P_i 's commensurate to $\mathcal{A} = 13 \times 73 \times 144000$ are the Haab', Venus and Mars canonical periods such as

the $\text{LCM}(584, 365) = 37960$ and $\text{LCM}(780, 365) = 56940$, the length of the Venus and Mars Table in the Dresden Codex. We have: $\mathcal{A} = \text{LCM}(260, 365, 144000) = 100 \times \mathcal{LR} = 7200 \times 18980 = 3600 \times 37960 = 2400 \times 56940$. We can rewrite Equ. 2 and 3 as:

$$\begin{aligned}\mathcal{N} - 121 \times \mathcal{X}_0 &= 151898 \times \mathcal{A} & (4) \\ \mathcal{N} - 126 \times \sum_{i=0}^3 \mathcal{X}_i &= 151893 \times \mathcal{A}\end{aligned}$$

The Long Count Periods appear in Equ. 4: $151898 = 338 + 360 + 7200 + 144000 = 1.1.1.16.18$ and $151893 = 333 + 360 + 7200 + 144000 = 1.1.1.16.13$. The Haab' and the Long Count Periods are such that $\{\text{LCM}(365, P_i)/P_i = 73, P_i = 18 \times 20^i, i > 0\}$. The Tzolk'in is defined by the $\text{LCM}(13, 20)$ such as $\text{LCM}(260, 365) = 73 \times 260 = 52 \times 365 = 18980$ days or a Calendar Round. The subtraction of the two equations in 4 leads to the main religious cycle $5\mathcal{A}$:

$$\begin{aligned}5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + 95 \times 126 \times 56940 & (5) \\ 5 \times \mathcal{A} &= 5 \times \mathcal{X}_0 + \text{LCM}\left(\sum_{i=1}^3 \mathcal{X}_i, \mathcal{X}_1 + \mathcal{X}_3 + 2\mathcal{X}_2\right)\end{aligned}$$

Since \mathcal{X}_1 is known (the commensuration of the Haab' and the 4-Kawil), that defines \mathcal{X}_2 and \mathcal{X}_3 . The relationship $\mathcal{X}_3 = \mathcal{X}_2 + 2\mathcal{X}_0$ may be a mnemonic tool to calculate $\mathcal{Y} = \mathcal{X}_3 + 2\mathcal{X}_2 = 105 \times 56940$. The four Xultun numbers provides a proof that Mayan priests-astronomers determined the canonical synodic periods of the five planets visible with a naked eye: Mercury, Venus, Mars, Jupiter and Saturn. A question arises at this point to know how the Maya, as early as the IX century CE, were able to compute tedious arithmetical calculations on such large numbers with up to 14 digits in decimal basis. Here is a possible method. They determined the prime factorizations of the canonical astronomical periods P_i (Table III) and listed each primes p_i with their maximal order of multiplicity α_i . They determined the Haab', the 4-Kawil and the Tun by their definitions given earlier. They calculated the calendar supernumber \mathcal{N} (the LCM of the P_i 's) by multiplying each p_i 's α_i time. The Euclidean division of \mathcal{N} by $\mathcal{GC} = 7 \times \mathcal{A} = 400 \times 7 \times \mathcal{X}_0$ (Equ. 2) is equivalent to a simplification of \mathcal{N} by $7 \times \mathcal{X}_0 = 2391480$ and the Euclidean division of the product of the 5 left primes ($3 \times 19 \times 29 \times 59 \times 89 = 8679903$) by 400. The Euclidean division of \mathcal{N} by $\mathcal{A} = \mathcal{GC}/7 = 13 \times 73 \times 144000 = 400 \times \mathcal{X}_0$ (Equ. 3) is equivalent to a simplification of \mathcal{N} by $\mathcal{X}_0 = 341640$ and the Euclidean division of the product of the 6 left primes ($3 \times 7 \times 19 \times 29 \times 59 \times 89 = 60759321$) by 400. It is to be noted that the prime factorization of the calendar supernumber only includes prime numbers < 100 which facilitates the operation (there are only 25 prime numbers lower than 100).

The Mayan Calendar is then constituted by 3 grand

cycles such as:

$$\begin{aligned}
7 \times \mathcal{A} &= 400 \times 7 \times \mathcal{X}_0 & (6) \\
&= 7 \times \mathcal{X}_0 + 399 \times 7 \times \mathcal{X}_0 \\
5 \times \mathcal{A} &= 400 \times 5 \times \mathcal{X}_0 \\
&= 5 \times \mathcal{X}_0 + 285 \times 7 \times \mathcal{X}_0 \\
\mathcal{A} &= 400 \times \mathcal{X}_0 \\
&= \mathcal{X}_0 + 57 \times 7 \times \mathcal{X}_0
\end{aligned}$$

The grand cycle $\mathcal{GC} = 7 \times \mathcal{A}$ is defined by a period $7 \times \mathcal{X}_0 = \text{LCM}(360, 365, 3276) = \text{LCM}(260, 360, 365, 3276) = 2391480$, the commensuration of the Tzolk'in, the Tun, the Haab' and the 4-Kawil. The main religious cycle $5 \times \mathcal{A}$ is connected to the date $5 \times \mathcal{X}_0 = 5 \times \text{LCM}(260, 360, 365) = 1708200$, which has the same properties (same Tzolk'in date, Haab' date, Kawil and direction-color) as the end of the main religious cycle $5 \times \mathcal{A}$. The cycle \mathcal{A} is divided by a smaller cycle, the Xultun number $\mathcal{X}_0 = \text{LCM}(260, 360, 365) = 341640$, the commensuration of the Tzolk'in, the Tun and the Haab'. It is such that $5 \times \mathcal{A} = 5 \times 13 \times 73 \times 144000 = 12000 \times 56940 = 5 \times 73 \times \mathcal{E}$ with the 13 Baktun Era $\mathcal{E} = 13 \times 144000$ defined as:

$$\begin{aligned}
\mathcal{E} &= 13 \times 360 + 114 \times \text{LCM}(260, 3276) & (7) \\
&= 400 \times 13 \times 360
\end{aligned}$$

The $\text{LCM}(260, 3276) = 16380$ is the commensuration of the Tzolk'in and the 4-Kawil. The date 13 Tun has the same Tzolk'in date, Kawil and direction-color as the end of the 13 Baktun Era. Since $\mathcal{E} - 5 \times \mathcal{X}_0 = 1872000 - 1708200 = 163800 = 10 \times \text{LCM}(260, 3276)$, \mathcal{E} has the same Tzolk'in, Kawil and direction-color as the end of the main religious cycle $5 \times \mathcal{A}$. The two dates $5 \times \mathcal{X}_0$ and \mathcal{E} are important due to their relationship with the main religious cycle $5 \times \mathcal{A}$ defined by Equ. 5.

Modular arithmetic considerations on the calendar supernumber allow to calculate the position of the Calendar Round at the Mayan mythical date of creation. We have: $\mathcal{A} = 400 \times \mathcal{X}_0 = \mathcal{X}_0 + 114 \times \text{LCM}(260, 360, 365, 3276)$ i.e. \mathcal{A} and \mathcal{X}_0 have the same Tzolk'in date, Haab' date, Kawil and direction-color. It is then sufficient to initialize \mathcal{X}_0 . For that purpose, we first create ordered lists of the Haab' and the Tzolk'in, assigning a unique set of 2 numbers for each day of the 18980-day Calendar Round.^{2,3} For the Haab', the first day is 0 Pop (numbered 0) and the last day 4 Uayeb (numbered 364). For the Tzolk'in, the first day is 1 Imix (numbered 0) and the last day 13 Ahau (numbered 259). In this notation, the date of creation 4 Ahau 8 Cumku is equivalent to $\{160; 349\}$ and a date D in the Calendar Round can be written as $D \equiv \{\text{mod}(D + 160, 260); \text{mod}(D + 349, 365)\}$. We have: $18980 = \text{LCM}(260, 365) = 13 \times 20 \times 73$. The calendar supernumber is such that: $\text{mod}(\mathcal{N}/13/73, 260) = 160$, $\text{mod}(\mathcal{N}/13/73, 13) = 4$, $\text{mod}(\mathcal{N}/13/73, 20) = 0$ and $\text{mod}(\mathcal{N}/13/73, 73) = 49$. The date $\{160; 49\}$ corresponds to 4 Ahau 8 Zip, the day 0 (mod 18980), beginning/completion of a Calendar Round. We now consider the Xultun number $\mathcal{X}_0 = \text{LCM}(260, 365, 360) = 13 \times 73$

$\times 360 = 18 \times 18980 = 341640$ which corresponds to the completion of a 13 Tun cycle, a period of 4680 days. The completion of a Calendar Round corresponds to 18980 days elapsed such as $\text{mod}(18980, 4680) = 260$. Starting the CR count at 4 Ahau 8 Zip, the next date in the ordered CR list such as $\text{mod}(D, 4680) = 0$ is the date 4 Ahau 8 Cumku, 4680 days later. A date D is then expressed as $\{\text{mod}(D + 4680 + 160, 260); \text{mod}(D + 4680 + 49, 365)\} = \{\text{mod}(D + 160, 260); \text{mod}(D + 349, 365)\}$. We have also: $\text{mod}(\mathcal{N}/32760, 4) = 3$. That defines the position of the Calendar Round and the Kawil at the mythical date of creation, the LCC date 0.0.0.0 4 Ahau 8 Cumku $\{160; 349\}$, 3 East-Red, the beginning of the previous 13 Baktun cycle or 400 cycles of 13 Tun.

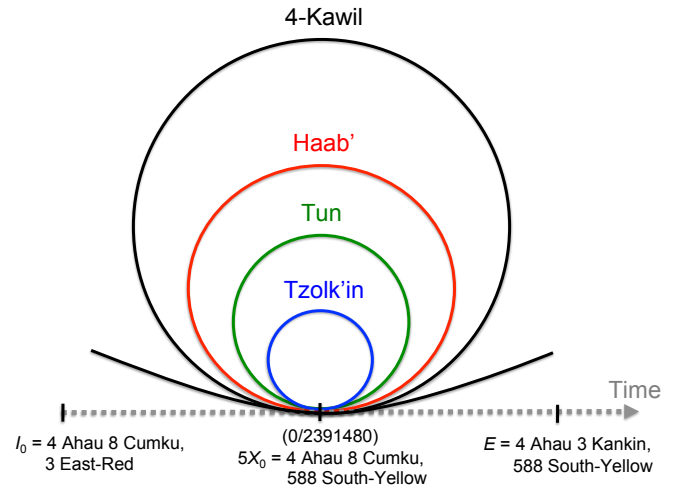


FIG. 2. Mayan/Aztec cyclical *vs* linear conception of time, with the 2391480-day Mayan cycle $\mathcal{C} = \text{LCM}(260, 360, 365, 3276)$, the commensuration of the Tzolk'in, the Tun, the Haab' and the 4-Kawil. The mythical creation date I_0 (11 August -3114 BC), the date of the return of Kukulcan $5\mathcal{X}_0$ (3 July 1564 CE) and the end of the 13 Baktun Era \mathcal{E} (21 December 2012) are also represented.

Fig. 2 gives the interpretation of the results. The prime factorization of 8 canonical astronomical periods defines a grand cycle $\mathcal{GC} = 400 \times \text{LCM}(360, 365, 3276)$ and a smaller cycle $\mathcal{A} = \mathcal{GC}/7 = 400 \times \text{LCM}(260, 360, 365) = 400 \times 341640 = 400 \times \mathcal{X}_0$. \mathcal{GC} is divided by the periodic cycle $7 \times \mathcal{X}_0 = \text{LCM}(260, 360, 365, 3276) = 2391480$, the time interval between two dates having the same properties (same Tzolk'in date, Haab' date, Kawil and direction-color at the end of a Tun). Arithmetical calculations on the pseudo-astronomical calendar supernumber \mathcal{N} , defined as the least common multiple of 8 astronomical input parameters, lead to the main religious cycle $5 \times \mathcal{A}$. It defines the date $5\mathcal{X}_0 = 5 \times 341640 = 1708200 = 11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow or 3 July 1564 CE according to the GMT correlation, a date having the same properties as the end of the main religious cycle $5\mathcal{A}$ (same Tzolk'in date, Haab' date, Kawil and direction-color). The 13 Baktun Era \mathcal{E}

is a smaller cycle originating from the cycle $\mathcal{A} = \mathcal{GC}/7$. The end date of the previous Era 13(0).0.0.0.0 4 Ahau 3 Kankin, 588 South-Yellow or 21 December 2012 has the same Tzolk'in date, Kawil and direction-color as the end date of the main religious cycle $5\mathcal{A}$, underlying the importance of the 13 Baktun Era in Mayan religion. The Mayan concept of time is then cyclical, with a cycle of 2391480 day or approximately 6547 years. This cycle is characterized by two most important dates: $5\mathcal{X}_0 = 11.17.5.0.0$ 4 Ahau 8 Cumku, 588 South-Yellow and $\mathcal{E} = 13(0).0.0.0.0$ 4 Ahau 3 Kankin, 588 South-Yellow, which are defined by their equivalent properties compared to the main religious cycle $5\mathcal{A}$ (4 Ahau 8 Cumku, 588 South-Yellow). The date $5\mathcal{X}_0$ might correspond to a series of omens prophesying the Spanish conquest of Mexico which occurred from February 1519 to 13 August 1521, misinterpreted as the return of the main Aztec (Mayan) God Quetzalcoatl (Kukulcan), the feathered serpent.¹⁵ This prophesy might have originated from the calculations realized by Mayan priests-astronomers in Xultun in the IX century CE.

In conclusion, this study presents a unified description of the Mayan Calendar based on an early model of naked-eye astronomy. A calendar supernumber \mathcal{N} is calculated by taking the least common multiple of 8 canonical astronomical periods describing the planet synodic movements and the apparition of solar/lunar eclipse. This calendar supernumber defines the three major Mayan Calendar cycles: the 3276-day 4-Kawil,

combination of the 4 directions-colors and the 819-day Kawil, the 18980-day Calendar Round, combination of the Haab' and the Tzolk'in, and the 1872000-day Long Count Calendar. The 360-day Tun, the 365-day Haab' and the 4-Kawil are issued from the prime factorization of the 8 canonical astronomical input parameters. The 260-day Tzolk'in and the Long Count Periods (the 360-day Tun, the 7200-day Katun and the 144000-day Baktun) are obtained from arithmetical calculations on \mathcal{N} . The correlation of the three major calendar cycles represents a calendar grand cycle \mathcal{GC} . The two Euclidean divisions of \mathcal{N} by \mathcal{GC} and $\mathcal{GC}/7 = 13 \times 73 \times 144000$ lead to the Xultun numbers, deciphered by Saturno *et al.* in 2012 inside a small room of the extensive Mayan ruins of Xultun, Guatemala and dating from the early IX century CE.¹ Various myths around Mayan religion are explained by this approach, in particular the existence of the Maya Era of 13 Baktun. Modular arithmetic considerations on the calendar supernumber determine the position of the Calendar Round at the Mayan mythical date of creation 0.0.0.0.0 4 Ahau 8 Cumku, reflecting the Mayan cyclical concept of time. The combination of astronomy and mathematics in Mayan religion is a form of celestial divination for which Mayan priests-astronomers used arithmetics to correlate ritual cycles with astronomical events in order to rhythm political life and religious practices, embedding Maya/Aztec civilization in the endless course of time.

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