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MPC framework for system reliability optimization

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Abstract. This paper presents a general framework that takes into account system and components reliability in a Model Predictive Control (MPC) algorithm. The objective is to deal from an availability point of view with a closed-loop system combining a deterministic part related to the system dynamics and a stochastic part related to the system reliability. The main contribution of this work consists in integrating the reliability assessment computed on-line using a Dynamic Bayesian Network (DBN) to compute the weights of the multiobjective cost function into the MPC algorithm. Also, a comparison between a method based on the components reliability (local approach) and a method focused on the system reliability sensitivity analysis (global approach) is considered. The effectiveness and benefits of the proposed control framework are shown by its application on a Drinking Water Network (DWN).

Keywords: reliability, dynamic Bayesian Network, Model Predictive Control

1 Introduction

To improve the system reliability and minimize operational costs, component health monitoring should be considered in the controlled system [10]. After failure the control effort can be redistributed among the available actuators to alleviate the work load and the stress factors on equipments with worst conditions avoiding in this manner their break down [2, 3]. For this purpose, an appropriate policy should be developed to redistribute this control effort until maintenance actions can be taken. Over-actuated systems implies an actuator redundancy, this means that the number of control inputs is larger than the

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number of inputs necessary to satisfy the control objective. This characteristic allows several combinations of control inputs that produce the same desired output which provide the same system performance [9].

Model Predictive Control (MPC) seems to be an efficient technique to manage the actuator redundancy. The MPC algorithm allows to include other criteria in the optimization problem, for example in [7] the authors present an application of MPC for a Drinking Water Network (DWN) taking into account economic, service level and degradation criteria. In [12] MPC formulation includes as a criteria the accumulated actuator usage, its objective is to maintain the accumulated usage under a safe level at the end of the mission. This approach is exported to several applications, i.e. aeronautics [13], where a reliable MPC is applied in a helicopter of 2 degrees of freedom and the weighting parameters are used to maintain the degradation of actuators below a safe level.

Reliability is the ability of a system to operate successfully long enough to complete its assigned mission under stated conditions. The use of Bayesian Networks (BN) as a modelling method for reliability computing, taking into account observations (evidences) about the state of the components, has been recently considered in some works [1, 3, 14, 15].

The purpose of this paper is to present a general framework that takes into account system and component reliability in a Model Predictive Control (MPC) algorithm as a part of a Prognosis and Health Management (PHM) strategy. The proposed PHM scheme uses the reliability information of the system obtained in real-time from a Dynamic Bayesian Network (DBN).

The main contribution is the integration of the reliability assessment that is computed on-line using a DBN in the MPC algorithm. The resulting scheme provides control performance and preserves system reliability. The information related to reliability of the system components and the system itself obtained using the DBN through the inference is used in the MPC algorithm. From an availability point of view, the objective is to deal with a closed-loop system combining a deterministic part related to the system dynamics and a stochastic part related to the actuators and system reliability. A comparison between a local and global approach of reliability computation is proposed. The effectiveness and benefits of the proposed control framework are shown by its application on a Drinking Water Network (DWN).

2 Reliability Modelling

2.1 Bayesian Network framework

Performing inferences in BN is particularly useful in control systems working on real-time, in that case, evidences acquired about a state variable must be propagated to update the state of the rest of the model. Other advantage of BN's is the possibility of model complex systems relatively easy. Basically, the BN's compute the distribution probabilities in a set of variables according to the prior knowledge of some variables and the observation of others [8]. For instance

let \mathbf{A} and \mathbf{B} be two nodes with two possible states (\mathbf{S}_1 and \mathbf{S}_2) as is shown in Fig. 1. A probability is associated to each state of the node and this probability is defined *a priori* for a root node and computed by inference for the others. The *a priori* probabilities of node \mathbf{A} are $P(\mathbf{A}=\mathbf{S}_{A1})$ and $P(\mathbf{A}=\mathbf{S}_{A2})$.



Fig. 1: Basic Bayesian Network.

A Conditional Probability Table (CPT) is associated to node \mathbf{B} and defines the probability $P(\mathbf{B}|\mathbf{A})$ of each state of \mathbf{B} given the states of \mathbf{A} . Thus, the BN inference computes the marginal distribution $P(\mathbf{B}=\mathbf{S}_{B1})$:

$$P(\mathbf{B} = \mathbf{S}_{B1}) = P(\mathbf{B} = \mathbf{S}_{B1} | \mathbf{A} = \mathbf{S}_{A1})P(\mathbf{A} = \mathbf{S}_{A1}) + P(\mathbf{B} = \mathbf{S}_{B1} | \mathbf{A} = \mathbf{S}_{A2})P(\mathbf{A} = \mathbf{S}_{A2}) \quad (1)$$

It is possible then to compute the probability distribution for each variable conditioned to the values of the others variables in the graph.

2.2 System reliability

The system reliability can be used as an indicator of the system health and can be computed from its components reliability through a BN. For this purpose it is necessary to identify all the minimal success paths present in the system.

A minimal success path is a minimum set of components whose functioning (i.e. being up) ensures that the system is up (if all elements of the minimal success path are "Up" then the system is up). The minimal success path cannot be reduced.

For example, consider the system reliability block diagram shown in Fig. 2a, it is composed by three components and it is clear that with a minimum of two components (i.e. components 1 and 3 or components 2 and 3) it can perform its function satisfactorily. With the minimal success paths identified, it is possible then to build the BN of the system. Fig. 2b shows the BN for the example system of three components. The components reliability are represented by nodes e_i , those nodes are connected to their corresponding minimal success path represented by nodes P_i . Finally the reliability of the system is represented by node S [15].

Table 1 shows the CPT of node P_1 , it depends on the states of the components e_1 and e_3 and its behaviour corresponds to an AND gate. The CPT of node S is also shown in Table 1. It depends on the state of success path nodes P_1 and P_2 and has the behaviour of an OR gate. Consequently, the reliability of the system is expressed as:

$$R_s = P(S = \text{Up}) \quad (2)$$

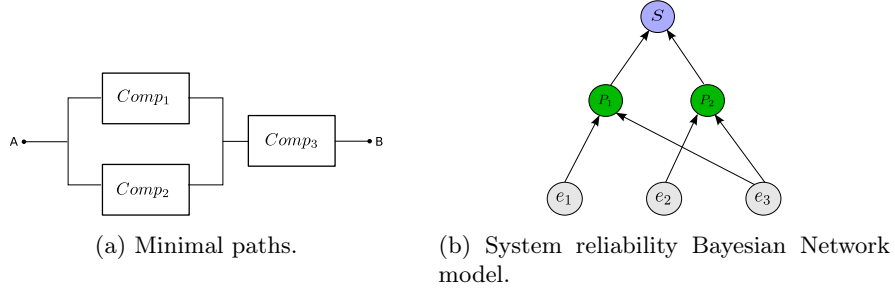


Fig. 2: Three components system example.

P_1				S			
e_1	e_3	Up	Dn	P_2	P_3	Up	Dn
Up	Up	1	0	Up	Up	1	0
Up	Dn	0	1	Up	Dn	1	0
Dn	Up	0	1	Dn	Up	1	0
Dn	Dn	0	1	Dn	Dn	0	1

Table 1: CPTs for nodes P_4 and S .

2.3 Component Reliability

A exponential model for the reliability is used in this work:

$$R_i(t) = e^{-\lambda_i t} \quad (3)$$

where λ_i is the failure rate for the i th component under. Several mathematical models have been proposed to define the failure level in order to estimate the failure rate λ . In [5], the failure rate is modeled as:

$$\lambda_i = \lambda_i^0 \times g(\ell, \vartheta) \quad (4)$$

where λ_i^0 represents the baseline failure rate (nominal failure rate) for the i th component and $g(\ell, \vartheta)$ is a load function (independent of time) also known as covariate, that represents the effect of stress on the component failure rate. ℓ represents an image of the load applied and ϑ is a component parameter.

Different definitions of function $g(\ell, \vartheta)$ exists in the literature. However, the exponential form is the most commonly used. In [10] the authors propose the load function based on the root-mean-square of the applied control input (u_i) until the end of the mission (t_M), and an actuator parameter defined from the upper and lower saturation bounds of u_i . This load function is used to distribute the control efforts between the redundant actuators, and the control action is calculated using a reliable state feedback. In this paper, it is assumed that the failure rate is provided by the following equation (5):

$$\lambda_i = \lambda_i^0 g_i(u_i) \quad (5)$$

where $g_i(u_i)$ corresponds to the following normalized control action:

$$g_i(u_i) = \frac{u_i(k) - \underline{u}_i}{\bar{u}_i - \underline{u}_i} \quad (6)$$

$u_i(k)$ is the control effort at time k , \underline{u}_i and \bar{u}_i are the minimum and maximum control efforts allowed for the i th actuator and represents the amount of load on

the actuator. The major actuator load corresponds to $u_i(k) = \bar{u}_i$, which leads to the worst failure rate $\lambda_i = \lambda_i^0$, through equation (5).

This behaviour can be modeled using a DBN taking advantage in the fact that the knowledge of the distribution probabilities and the CPT allows the calculation of the distribution probabilities at time $k + 1$ which is conditional independent of the past given the present k , and it can be exploited in an iterative process by using the information of time $k + 1$ to compute the distribution probabilities of time $k + 2$ and so on as shown in Fig. 3.

Remark that with the inclusion of the amount of load in the failure rate, the DBN models become in a 1/2 MC [1]. The CPT showed in Table 2 defines the discretized stochastic process of (3) in the DBN model.

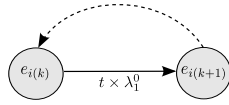


Fig. 3: DBN for component reliability.

Table 2: CPT for node $e_{i,k+1}$.

	$e_{i,k+1}$	
$e_{i,k}$	Up	Dn
Up	$1 - (\lambda_i^0 T_s g_i(u_i))$	$\lambda_i^0 T_s g_i(u_i)$
Dn	0	1

Therefore, the component reliability can be expressed as:

$$R_i(k + 1) = P(e_i(k + 1) = \text{Up}) \quad (7)$$

2.4 Importance factors

In order to take into account the global reliability of the system, the use of an importance factor is proposed. The importance factors are criteria used as an evaluation of components impact over the system. One of them involves determining the system reliability sensitivity against changes in the reliability of its components, this is also known as the Marginal Importance Factor (MIF), introduced by [4]. The sensitivity can be computed using the DBN [2, 3], as:

$$\text{MIF}_i = \frac{\partial R_s}{\partial R_i} = P(S = \text{Up} | e_i = \text{Up}) - P(S = \text{Up} | e_i = \text{Dn}) \quad (8)$$

where S is a binary random variable that represents the system state $\{\text{Up}, \text{Down}\}$.

Other factors like the Diagnostic Importance Factor (DIF) [6] can be used. This factor represents the probability that the functioning of component i contributes to the functioning of the system given that the system is not faulty. The DIF can be computed using the DBN [2, 3], as:

$$\text{DIF}_i = P(e_i = \text{Up} | S = \text{Up}) \quad (9)$$

This is, if component i has a DIF equal to 1, it means that component i becomes critical for the functioning of the system, and if it fails the system will fail. DIF _{i} criteria is intended to produce that components with more importance in the system structure are relieved to mitigate the system reliability decrease.

3 Reliability-aware MPC

3.1 MPC formulation

Consider the following linear discrete-time model described in state-space form of an over actuated system with p actuators:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + E\varepsilon(k) \\ y(k) &= Cx(k) \end{aligned} \quad (10)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^p$ is the control input vector with $u(k) \geq 0 \forall k$, $y(k) \in \mathbb{R}^q$ is the measured output vector, $\varepsilon(k) \in \mathbb{R}^m$ is the disturbance vector, $A \in \mathbb{R}^{n \times n}$ is the state matrix, $B \in \mathbb{R}^{q \times p}$ input matrix, $E \in \mathbb{R}^{n \times m}$ is the disturbance matrix, and $C \in \mathbb{R}^{q \times n}$ is the output matrix.

MPC technique is considered in order to distribute the control effort among the actuators. In this paper, the multiobjective optimization problem [11] is formulated as follows:

$$\begin{aligned} \min_{\substack{(\Delta \hat{u}(k|k), \dots, \\ \Delta \hat{u}(k+H_c-1|k))}} J(k) &= \sum_{j=0}^{H_p-1} \sum_{i=1}^q \alpha_i(k) [\hat{y}_i(k+j|k) - y_i^{ref}(k+j|k)]^2 \\ &+ \sum_{j=0}^{H_c-1} \sum_{i=1}^p \Delta \hat{u}_i(k+j|k)^2 + \sum_{j=0}^{H_c-1} \sum_{i=1}^p \rho_i(k) \hat{u}_i(k+j|k)^2 \\ \text{subject to } \underline{u} &\leq \hat{u}(k+j|k) \leq \bar{u} \quad j = 0, \dots, H_c-1 \\ \underline{x} &\leq \hat{x}(k+i|k) \leq \bar{x} \quad i = 1, \dots, H_p \end{aligned} \quad (11)$$

where \hat{y}_i and y_i^{ref} are the predicted output and the set-point for a H_p horizon respectively, α_i and $\rho_i(k)$ are weighting parameters, \underline{u} and \bar{u} denote the minimum and maximum actuator capacities, and \underline{x} and \bar{x} denote the minimum and maximum state values. The notation $k+i|k$ allows a future time instant $k+i$ to be referred at current time instant k , and the optimization problem consists in minimizing $\Delta \hat{u}_i(k)$ defined as $\hat{u}_i(k) - \hat{u}_i(k-1)$ over a control horizon H_c .

3.2 Enhancing MPC with reliability information

An approach to reliability-aware control will be provided. The aim of this approach is to preserve as much as possible the system reliability and is focused on the $\rho_i(k)$ weights given in the cost function (11), those weights are used then to build the weighting matrix D_u , that distributes the control effort among the actuators, this is:

$$Du_{i,k} = f(\mathbf{I}_{i,p,k}) := \text{diag}(\rho_1, \rho_2, \dots, \rho_i) \quad (12)$$

where \mathbf{I} is the criterion by which the control action is distributed among the available actuators. Different definition for \mathbf{I} criteria are proposed.

The local approach focuses on component reliability $R_i(k)$ provided by the DBN using inference at each time instant. This definition tries to preserve system reliability through preserving components reliability, setting the weights as follows:

$$\rho_i(k) = 1 - R_i(k) \quad (13)$$

With this criteria the aim is to find the optimal control actions and distribute it among the available actuators in a way that actuators with low reliability level are relieved. Hence, the use of highly reliable components is prioritized.

The local approach assumes an equivalent contribution of component reliability to system reliability. However, this is hardly ever true. In fact, the DBN reliability model can intrinsically explain this relation. A global approach using MIF_i criteria is proposed, it is expected that components with more contribution to the system reliability are used less with respect to the others. In others words, its aim is to preserve system reliability by setting the weights as follows:

$$\rho_i(k) = MIF_i(k) \quad (14)$$

Hence, the use of those components with a smaller sensitivity is prioritized. Components with a bigger sensitivity are expected to greatly penalize system reliability, so they are assigned a higher cost. Another possibility consists in combining the MIF and DIF to take advantage of both of them as follows:

$$\rho_i(k) = MIF_i(k) \times DIF_i(k) \quad (15)$$

4 Case study: Drinking Water Network

4.1 DWN description

The proposed MPC framework is applied to a part of a Drinking Water Network (DWN) system [15]. A DWN is a network which is composed by sources (water supplies), sinks (water demand sectors) and pipelines that link sources to sinks. It also contains active elements like pumps and valves. Concerning the DWN reliability study, sources, sinks, tanks and pipelines are considered perfectly reliable whereas active elements are not. In this paper, a subsystem of the Barcelona water transport network is used as a case study (see Fig. 4a).

It is assumed that the demand forecast at the sink is known (see Fig. 4b), and that any single source can satisfy this required water demand.

Different definition of Γ criterion computed using the Dynamic Bayesian Network are used to set the MPC weights.

4.2 System reliability using a Dynamic Bayesian Network model

The system has 10 pumps, 5 sources, 4 tanks and several pipes, Sources, tanks and pipes are considered perfectly reliable and are not subject to loss of reliability. Then, it is necessary to determine the quantity and composition of each minimal success path (Table 3).

Table 3: Components and success paths relationship.

	s_1	s_2	s_3	s_4	s_5	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
P_1	x					x					x				
P_2		x					x				x				
P_3			x					x			x				
P_4				x					x		x				x
P_5				x					x			x			x
P_6				x					x		x		x		x
P_7					x					x	x			x	
P_8					x					x		x			x
P_9					x					x	x		x		x

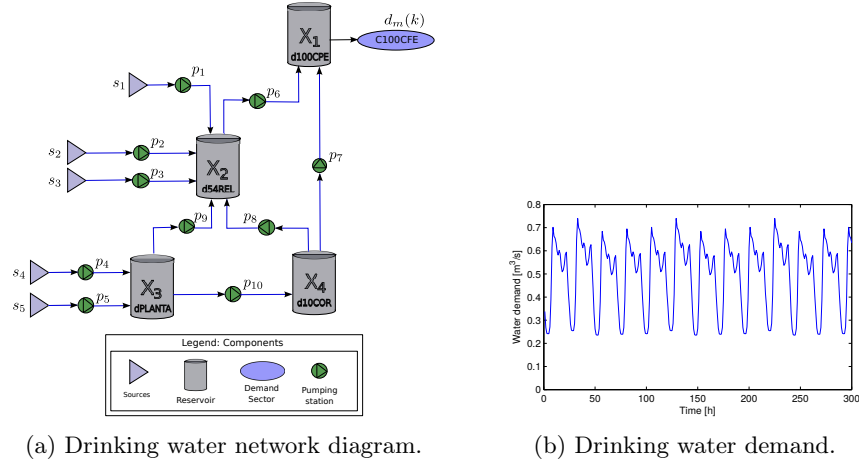


Fig. 4: Drinking Water Network application.

The DBN is built following the procedure described in Sections 2.3 and 2.2. The resulting DBN of the DWN is shown in Fig. 5 where nodes e_i and A_i are drawn for each component (i.e. pump p_i and source s_i , respectively). These nodes are interconnected to their corresponding minimal success path nodes P_i using arcs. Each minimal success path is linked to the corresponding source availability node A'_i . This layer provides the availability of each source with respect to the availability of their corresponding minimal success paths considering that sources are assumed perfectly reliable, this is $P(A_i=Up)=1$. Finally, the source availability nodes are interconnected to the system reliability node S [15].

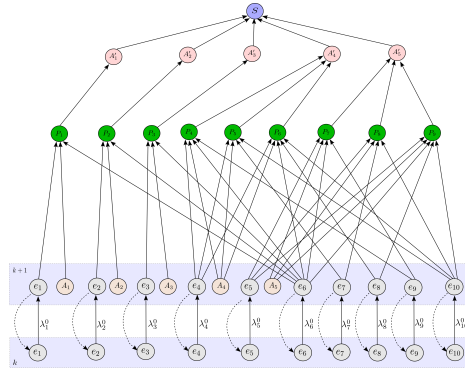


Fig. 5: Bayesian network model of the DWN.

Initially, at instant $k = 0$, the pumps and the system are assumed to be fully reliable i.e. their reliability is 1. Then, the probability of each node is computed using its CPT. In the case of minimal success paths nodes (P_i) their CPT follows an AND gate behaviour, the availability nodes (A'_i) CPT behaviour corresponds

to an OR gate as well as the behaviour of the system node S . At each sample time, the reliability R_i of each pump is computed according to its failure rate using the λ_i values of Table 4 in a MC (shaded blue layers in Fig. 5) as discussed in Section 2.3.

Table 4: Failure rate for pumps.

Failure rate λ^0 [$hours^{-1} \times 10^{-4}$]									
9.85	10.70	10.50	1.40	0.85	0.80	11.70	0.60	0.74	0.78

5 Results

A hierarchical control structure is assumed where the MPC formulation proposed in Section 3 produces every sample time a set of set-points for the lower level flow controllers according to the forecast water demand (Fig. 4b) that exhibits a daily profile. The initial tank volumes have been set to x_0 . Table 5 provides the simulation parameters used.

Table 5: Simulation parameters

parameter	value	parameter	value
H_p / H_c	24 / 8	\bar{u} [m^3/s]	0.75 0.75 0.75 1.2 0.85 1.6 1.7 0.85 1.7 1.6
T_s [h]	1	\underline{u} [m^3/s]	0 0 0 0 0 0 0 0 0 0
T_{sim} [h]	2000	\bar{x} [m^3]	65200 3100 14450 11745
ρ_i	$\{1, 1 - R_i, MIF_i, MIF_i \times DIF_i\}$	\underline{x} [m^3]	25000 2200 5200 3500
α_i	0	x_0 [m^3]	45100 2650 9825 7622

Fig. 6 and 7 present the weights evolution for the studied criteria. As expected in the scenario with $1 - R_i$ criteria the pumps with higher failure rate i.e. 1, 2, 3 and 7 (see Table 4) are greatly penalized in order to decrease its reliability loss as can be seen in Fig. 6.

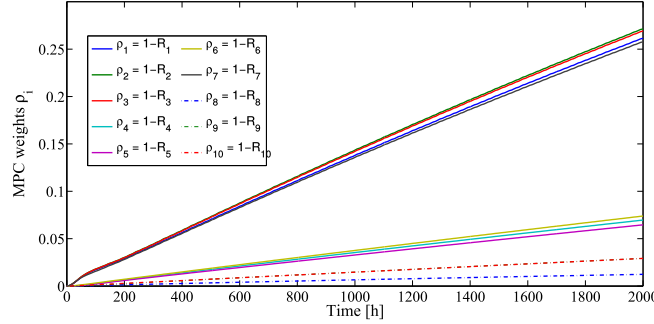


Fig. 6: MPC weights for the case $\rho_i = 1 - R_i$.

Fig. 7 shows the MPC weights evolution for scenario with MIF_i criteria. It is clear that weights of pump 6 is elevated compared to the weights of the others pumps. This is due to the importance of pump 6 in the operation of the system. In the case of $MIF_i \times DIF_i$ criteria the weights evolution presents a similar behaviour as the case MIF_i .

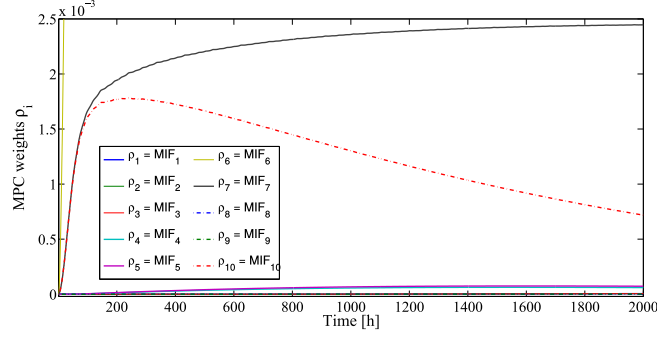


Fig. 7: MPC weights for the case $\rho_i = \text{MIF}_i$.

Fig. 8 shows the evolution of the control effort of the system pumps for scenarios where ρ_i criteria is 1, $1 - R_i$, MIF_i and $\text{MIF}_I \times \text{DIF}_i$ (the evolutions in the case of MIF_i and $\text{MIF}_i \times \text{DIF}_i$ are overlapped). In these scenarios the control maintains the effort between the min and max limits.

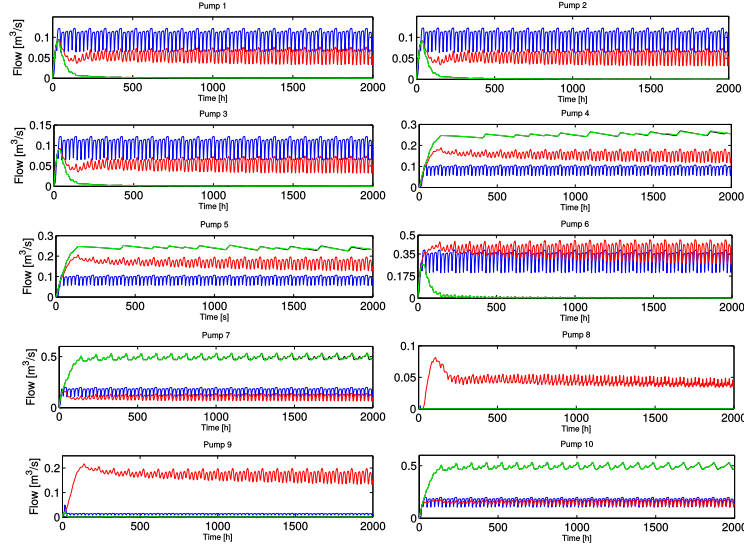


Fig. 8: Pumping commands corresponding to $\rho_i = 1$ (blue), $\rho_i = 1 - R_i$ (red), and $\rho_i = \text{MIF}$ (black), $\rho_i = \text{MIF} \times \text{DIF}_i$ (green).

In order to compare the results, the following indexes are defined: the system reliability R_s at the end of the simulation, the cumulative pump usage U_{cum} defined as a measure of the pump energy consumption (16) and the joint pump reliability index JPR as a measure of the overall pump set reliability (17). The results of these indexes are shown in Table 6.

$$U_{\text{cum}} = T_s \sum_{k=0}^{T_{\text{sim}}/T_s} [u(k)^T u(k)] \quad (16)$$

$$JPR = \prod_{i=1}^p R_i \quad (17)$$

Table 6: Results summary.

ρ_i	R_s at T_{sim}	U_{cum}	JPR at T_{sim}
1	0.97530	1.55685×10^6	0.09492
$1 - R_i$	0.97903	2.02009×10^6	0.21895
MIF_i	0.99794	4.20501×10^6	0.18376
$MIF_i \times DIF_i$	0.99811	4.22680×10^6	0.18408

Figure 9 shows system reliability (R_s) evolution for scenarios $\rho_i = 1$, $\rho_i = 1 - R_i$, $\rho_i = MIF_i$, and $\rho_i = MIF \times DIF$.

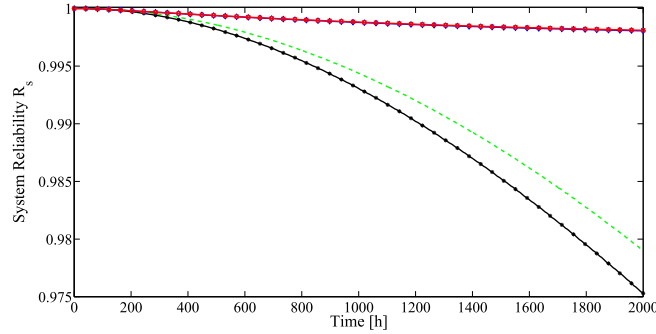


Fig. 9: System reliability: $\rho_i = 1$ (asterisk black line), $\rho_i = 1 - R_i$ (dashed green line), $\rho_i = MIF_i$ (diamond blue line), $\rho_i = MIF \times DIF$ (circle red line).

For $\rho_i = MIF_i$ the system reliability is improved compared to the others criteria, but there is a small improvement using $MIF_i \times DIF_i$. It can also be seen that an improvement of system reliability in the scenarios MIF_i and $MIF_i \times DIF_i$ involves an increment of the energy consumption in the system, this also occurs in the scenario of $1 - R_i$ where besides the reliability improvement is not significant. This certifies that, improving system reliability can lead to an increase of energy consumption and the fact that focusing on the reliability of components is not the best strategy to preserve system reliability.

6 Conclusions

A PHM scheme was proposed using the reliability information of the system obtained in real-time from a DBN and tested in a DWN case study. A DBN model of DWN is used to define the weights of the multiobjective cost function. Three weights assignments have been proposed. In the first approach, component reliability is targeted, whereas in the second and third approach system reliability is focused using MIF and DIF criteria. It has been shown that focusing on the reliability of components does not guarantee the best system reliability, which was the ultimate goal. In order to preserve system reliability as much as possible, its sensitivity to component reliability must be preferably accounted for.

Analytical computation of this sensitivity is not trivial but a DBN model can provide it easily. The analytical computation needs a model of the system and the DBN offers the possibility to compute it through inference.

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