



HAL
open science

What if quantum vacuum fluctuations are virtual gravitational dipoles?

Dragan Hajdukovic

► **To cite this version:**

Dragan Hajdukovic. What if quantum vacuum fluctuations are virtual gravitational dipoles?. 3rd International Workshop on Antimatter and Gravity (WAG2015), University College London (UK) Aug 2015, London, United Kingdom. hal-01254678v2

HAL Id: hal-01254678

<https://hal.science/hal-01254678v2>

Submitted on 9 Feb 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

What if quantum vacuum fluctuations are virtual gravitational dipoles?

Dragan Slavkov Hajdukovic

Institute of Physics, Astrophysics and Cosmology. Cetinje, Montenegro

dragan.hajdukovic@physac.me

Abstract. The hypothesis stated in the title might be the basis for a new model of the Universe; we give a pedagogical overview of the new model. According to the new model, the only content of the Universe is the known Standard Model matter immersed in the quantum vacuum “enriched” with virtual gravitational dipoles. Apparently, what we call dark matter and dark energy can be explained as the local and global effects of the gravitational polarization of the quantum vacuum by the immersed baryonic matter. Further, the hypothesis leads to a cyclic model of the Universe with cycles alternatively dominated by matter and antimatter; with each cycle beginning with a macroscopic size and the accelerated expansion. Consequently, there is no singularity, no need for inflation field, and there is an elegant explanation of the matter-antimatter asymmetry in the universe: our universe is dominated by matter because the previous cycle of the Universe was dominated by antimatter. The forthcoming experiments (AEGIS, ALPHA, GBAR ...) will reveal if particles and antiparticles have gravitational charge of the opposite sign, while study of orbits of tiny satellites in trans-Neptunian binaries (e.g. UX25) can be a reasonable test of some astronomical predictions of the theory.

1. Introduction

Contemporary physics has two cornerstones.

The first cornerstone is the Standard Model of Particles and Fields. According to the Standard Model everything is made from apparently structureless fermions (quarks and leptons) which interact through the exchange of gauge bosons (photons for electromagnetic, gluons for strong, and W^+ , W^- and Z for weak interactions).

So far, the Standard Model is the most successful and the best tested theory of all time. The recent LHC experiments at CERN have been a new triumph for the Standard Model contrary to the mainstream conviction that experiments will be a triumph of supersymmetric theories.

The second cornerstone is Einstein’s General Relativity, i.e. our best theory of gravitation.

The Standard Model tell us what is the content of the Universe, while, for a given content, General Relativity can tell us how the Universe evolves in time.

The cosmological principle (i.e. the assumption of the large-scale homogeneity and isotropy of the Universe) determines the Friedman-Lemaitre-Robertson-Walker (FLRW) metric (see for instance [1]):

$$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

where $k=+1$, $k=-1$ and $k=0$ correspond respectively to closed, open and flat Universe.

The dynamics of the above space-time geometry is entirely characterised by the scale factor $R(t)$ which is solution of the Einstein equation

$$G_{\mu\nu} = -(8\pi G/c^4)T_{\mu\nu} \quad (2)$$

Einstein tensor $G_{\mu\nu}$ is determined by FLRW, but in order to solve the Einstein equation we must know Energy-momentum tensor $T_{\mu\nu}$. Key point: Energy-momentum tensor is approximated by energy-momentum tensor of a perfect fluid characterised at each point by its proper density ρ and pressure p .

If cosmological fluid consists of several distinct components denoted by n , the final results are cosmological field equations [1]:

$$\ddot{R} = -\frac{4\pi G}{3}R\sum_n\left(\rho_n + \frac{3p_n}{c^2}\right) \quad (3)$$

$$\dot{R}^2 = \frac{8\pi G}{3}R^2\sum_n\rho_n - kc \quad (4)$$

The cosmological field equations can be solved only if we know the content of the Universe. According to the Standard Model the content of the Universe are three cosmological fluids:

Non-relativistic Standard Model matter (usually called pressureless matter or dust)

$$\rho_m = \rho_{m0}\left(\frac{R_0}{R}\right)^3, \quad p_m = 0 \quad (5)$$

Relativistic Standard Model matter (usually called radiation)

$$\rho_r = \rho_{r0}\left(\frac{R_0}{R}\right)^4, \quad p_r = \frac{1}{3}\rho_r c^2 \quad (6)$$

Quantum vacuum

$$\rho_{qv} = \text{constant}, \quad p_{qv} = -\rho_{qv}c^2 \quad (7)$$

Note that index 0 denotes the present day value. The first two fluids are well established content of the Universe, while the cosmological constant problem (a nice name for the worst prediction in the history of physics) prevents the use of the Standard Model vacuum as the content of the Universe [2].

A Standard Model physicist can calculate the mass-energy density ρ_{ve} of the quantum vacuum. The result of calculations [2] is given by the first of equations (8):

$$\rho_{ve} = \frac{1}{16\pi^2}\left(\frac{c}{\hbar}\right)^2 M_c^4 \equiv \frac{\pi}{2}\frac{M_c}{\lambda_{Mc}^3}; \quad \rho_{qv} = \frac{\pi}{2}\frac{M_c}{\lambda_{Mc}^3}\frac{\lambda_{Mc}}{R} \quad (8)$$

For instance, for vacuum energy density in quantum chromodynamics the cut-off mass M_c is roughly mass of a pion and the density is about $5.6 \times 10^{13} \text{ kg/m}^3$. If we insist that vacuum energy density and vacuum gravitational charge density is the same thing, the quantum vacuum behaves as if it has the mass of nearly 10^{14} kg/m^3 !

It is unjust to blame the Standard Model for the cosmological constant problem; what they estimated is the vacuum energy density, nothing more than that. The cosmological constant problem might be a hint that vacuum energy density and vacuum gravitational charge density are radically different. Have a look at the second equation in (8); in fact it is the first equation multiplied by λ_{Mc}/R which is a necessary correction if instead of gravitational monopoles there are gravitational dipoles. Calculate it!

Instead of nearly 10^{14} kg/m³ you will get about 10^{-27} kg/m³ in agreement with observations! It might be a miraculous numerical coincidence but it might be the true nature of quantum vacuum as well.

At the end of this introduction let us remember that astronomical observations have revealed a series of phenomena which are a complete surprise and mystery for contemporary theoretical physics.

First, in galaxies and clusters of galaxies, the gravitational field is much stronger than it should be according to our theory of gravitation and the existing amount of the Standard Model matter (note that astrophysicists use “baryonic matter” as a synonym for Standard Model matter). The mainstream “solution” is ad hoc introduction of *dark matter* of unknown nature.

Second, in certain periods of its history the expansion of the Universe is accelerated. The mainstream “explanation” is *dark energy* of unknown nature.

Third, our Universe is dominated by matter. Apparently, in the primordial Universe, something has forced the matter-antimatter asymmetry. The mainstream “solution”: A kind of CP violation, but please do not be misled by the known CP violation; it must be an unknown type of CP violation, many orders of magnitude stronger than the known one!

Fourth, the cosmological constant problem already mentioned above.

In addition the Big Bang model has two inherent troubles, the initial singularity and *conflict with observations*. For instance, the old Big-Bang theory predicts the existence of the cosmic microwave background (CMB), but contradicts its major characteristics: high level of homogeneity and isotropy. The current mainstream “solution” is cosmic inflation, within the first 10^{-30} seconds. The first content of the Universe was not matter but a mysterious hypothetical inflation field; the creation of matter has happened after inflation, at a macroscopic size, when the energy concentrated in the inflation field was converted into particle-antiparticle pairs.

We are faced with so many fundamental problems and presumably the fall of supersymmetries; apparently physics goes through the greatest crisis in its history.

2. Quantum vacuum and virtual gravitational dipoles: the common solution to all problems?

Quantum vacuum is an essential part [3] of the Standard Model of Particles and Fields.

“Nothing is plenty” as nicely said by Aitchison in his classical review [3]. Physical vacuum is plenty of quantum vacuum fluctuations, or, in more popular wording, of short-living virtual particle-antiparticle pairs (for instance, the lifetime of a virtual electron-positron pair is only about 10^{-21} s) which in permanence appear and disappear (as is allowed by time-energy uncertainty relation). Quantum vacuum is an omnipresent state of matter-energy apparently as real as the familiar states: gas, liquid, solid, plasma in stars, quark-gluon plasma... And, quantum vacuum is a state with perfect symmetry between matter and antimatter; a particle always appears in pair with its antiparticle.

Let us briefly consider a few illuminating phenomena in quantum electrodynamics (QED).

In QED, quantum vacuum can be considered as an omnipresent “ocean” of virtual electric dipoles with random orientation; the random orientation of virtual dipoles can be perturbed only by a very strong electric field. For example, the electric field of an electron at the distance of its Compton wavelength is of the order of 10^{14} V/m. Such a strong field perturbs the random orientation. Hence, roughly speaking, an electron “immersed” in the quantum vacuum produces around itself *a halo of non-random oriented virtual electric dipoles*, i.e. a halo of the polarized quantum vacuum. The size of a halo is roughly of the order of the Compton wavelength of an electron; as far as we know, there are no sufficiently strong electric fields in nature to produce halos of the macroscopic size. Thus, the observed charge of an electron, or equivalently the fine structure constant should be position dependent; today, this theoretical prediction is a confirmed reality [4].

Quantum vacuum, as “ocean” of virtual electric dipoles has a tiny impact (but impact!) on the “orbits” of electrons in atoms [3]. It is known as the Lamb shift.

In QED, vacuum fluctuations can be converted into real particles. For instance, a virtual electron-positron pair can be converted to a real one by an external field which, during their short lifetime, can separate particle and antiparticle to a distance of about one reduced Compton wavelength; this is known as the Schwinger mechanism [5]. The Schwinger mechanism is not the only way to create

something from the quantum vacuum; for instance virtual photons can be converted into directly observable real photons [6].

If quantum vacuum fluctuations are virtual gravitational dipoles, we may expect the gravitational version of the above phenomena. For instance the gravitational version of the Lamb shift would be if quantum vacuum produces a tiny perturbation of orbits of planets and satellites!

Hence, it is intriguing to study the astrophysical and cosmological consequences of the working hypothesis that, *by their nature, quantum vacuum fluctuations are virtual gravitational dipoles*.

The simplest and the most elegant realization of this hypothesis is, if particles and antiparticles have the gravitational charge of the opposite sign; of course nature may surprise us with a different realization of the gravitational dipoles like behavior of the quantum vacuum.

Preliminary results published in a series of papers [7-16] suggest that what we call dark matter and dark energy may be result of the gravitational polarization of the quantum vacuum by the immersed Standard Model matter. This part of the theory can be understood in a simple intuitive way.

According to our hypothesis we consider a quantum vacuum fluctuation as a system of two gravitational charges of the opposite sign; consequently the total gravitational charge of a vacuum fluctuation is zero but it has a non-zero gravitational dipole moment \mathbf{p}_g

$$\mathbf{p}_g = m_g \mathbf{d}, \quad |\mathbf{p}_g| < \frac{\hbar}{c} \quad (9)$$

Here, m_g denotes the magnitude of the gravitational charge, while, by definition, the vector \mathbf{d} is directed from the antiparticle to the particle, and has a magnitude d equal to the distance between them. The inequality in (9) follows from the fact that the size d of the virtual pair must be smaller than the reduced Compton wavelength (for a larger separation a virtual pair becomes real). Consequently, the gravitational polarization density \mathbf{P}_g , i.e. the gravitational dipole moment per unit volume, may be attributed to the quantum vacuum. It is obvious that the magnitude of the gravitational polarization density $|\mathbf{P}_g|$ satisfies the inequality $0 \leq |\mathbf{P}_g| \leq P_{g \max}$ where 0 corresponds to the random orientations of dipoles, while the maximal magnitude $P_{g \max}$ corresponds to the case of saturation (when all dipoles are aligned with the external field). The most plausible theoretical estimate for $P_{g \max}$ [8, 11, 16] is

$P_{g \max} = [0.06 \pm 0.02] \text{ kg/m}^3$, in calculations we use $P_{g \max} = 0.06 \text{ kg/m}^2 \approx 28.5 M_{\text{Sun}} / \text{pc}^2$. In addition, following the previous results [8, 11, 12, 14, 16], in calculations concerning dipoles, we use the mass of a pion m_π (which is a typical mass in the physical vacuum of quantum chromodynamics).



Figure 1. Randomly oriented gravitational dipoles (in absence of an external gravitational field)

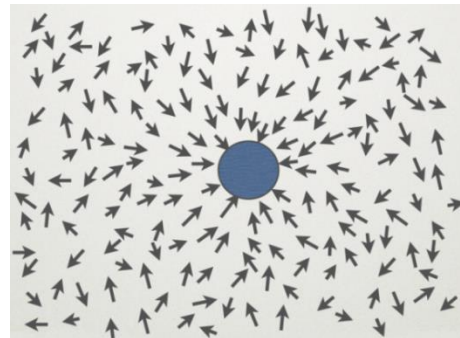


Figure 2. Halo of non-random oriented gravitational dipoles around a body (or a galaxy) with baryonic mass M_b

In the limit of zero external gravitational field quantum vacuum may be considered as a fluid of randomly oriented gravitational dipoles (Figure 1); both the total gravitational charge and \mathbf{P}_g are zero.

Random orientation of virtual dipoles may be broken by the immersed Standard Model matter. Massive bodies (stars, black holes ...) but also multi-body systems as galaxies are surrounded by an invisible *halo* of the gravitationally polarized quantum vacuum, i.e. a region of non-random orientation of virtual gravitational dipoles (Figure 2). This halo of the polarized quantum vacuum acts as an *effective gravitational charge*. Namely, the spatial variation of the gravitational polarization density \mathbf{P}_g , generates a gravitational bound charge density [8, 11, 16] of the quantum vacuum

$$\rho_{qv} = -\nabla \cdot \mathbf{P}_g \quad (10)$$

what may explain phenomena usually attributed to dark matter.

It is obvious that gravitational field can align only quantum vacuum fluctuations which are gravitational dipoles but not electric dipoles; for instance random orientation of electron-positron pairs cannot be broken by a gravitational field, while neutrino-antineutrino pairs and gluon fluctuations might be aligned.

3. Dark matter as the local effect of gravitational vacuum polarization

In the simplest case of spherical symmetry, the equation (10) reduces to

$$\rho_{qv}(r) = \frac{1}{r^2} \frac{d}{dr} [r^2 P_g(r)]; \quad P_g(r) \equiv |\mathbf{P}_g| \quad (11)$$

The most robust solution corresponds to the region of saturation (a sphere with a characteristic radius R_{sat}); the effective gravitational charge of the quantum vacuum is given by [8, 11 and 16]

$$M_{qv}(r) = 4\pi P_{g \max} r^2; \quad r < R_{sat} \approx \lambda_\pi \sqrt{\pi \frac{M_b}{m_\pi}} \quad (12)$$

It is obvious that in the region of saturation, equation (12) *predicts* a universal surface density $M_{qv}(r)/4\pi r^2 = P_{g \max}$. In fact, such a surprising surface density (which is the same for *all* galaxies) was observed [17, 18] and presents a surprise and mystery for dark matter paradigm.

According to equation (12), in the region of saturation, the gravitational polarization of the quantum vacuum produces an additional anomalous constant gravitational field oriented towards the centre. If such acceleration exists it causes a retrograde precession of the perihelion of the satellite in a binary; the anomalous acceleration and precession per orbit in radians are respectively [13]

$$g_{qv} = 4\pi G P_{g \max}, \quad \Delta\omega_{qv} \approx -8\pi P_{g \max} a^2 / \mu \quad (13)$$

where a and μ are respectively the semi-major axis of the orbit and the total mass of the binary. Transneptunian binaries (mini planets with miniscule satellites, as for instance UX25) are the best natural laboratory [13] to test the prediction (13) that quantum vacuum causes an anomalous perihelion precession (in fact a gravitational version of the Lamb shift!). Astronomers have already started [19] the preparation of the future tests (see also the paper of Gai and Vecchiato on the Project QVADIS in the present Proceedings).

Considering quantum vacuum as an ideal system of non-interacting gravitational dipoles in an external gravitational field (analogous to polarization of a dielectric in external electric field, or a paramagnetic in an external magnetic field) leads to the following approximation

$$M_{qv}(r) = 4\pi P_{g \max} r^2 \tanh\left(\frac{R_{sat}}{r}\right) \quad (14)$$

valid also for the region $r > R_{sat}$ which is the most relevant for “dark matter” phenomena.

In the absence of physical understanding of the phenomenon, the distribution of (real or effective) dark matter in a galaxy is described by *empirical laws* (NFW profile, Einasto profile, Burkert profile . . .). Contrary to it, distribution (14) is not empirical one but comes from fundamental principles. It is encouraging that such a simplified model for a galaxy (spherical symmetry and ideal system of non-interacting gravitational dipoles) gives good results. Let us give a few examples. First, our theory of the gravitational effects of the quantum vacuum leads [8, 11, 16] to the Tully-Fisher relation which is one of the most robust empirical results, unexplained by “dark matter”. Second, according to observations [20] the median Milky Way mass within 260kpc is $M_{MW}(260kpc) = 1.6 \times 10^{12} M_{Sun}$

with a 90% confidence interval of $[1.0 - 2.4] \times 10^{12} M_{Sun}$, while our theoretical estimate [15, 16] is $M_{MW}(260kpc) \approx 1.45 \times 10^{12} M_{Sun}$. Third, apparently the best estimate [21] for the local dark matter density (an average over a small volume, typically a few hundred parsecs around the Sun) is $[0.0075 \pm 0.0021] M_{Sun}/pc^3$ while our theoretical estimate is $\approx 0.0069 M_{Sun}/pc^3$.

4. Dark energy as the global effect of gravitational vacuum polarization

Roughly speaking there is a *maximum size of the halo* for each massive body, galaxy or cluster of galaxies. Simply, after a characteristic size the random orientation of dipoles dominates again. However, because of proximity of other bodies, many halos are smaller than the maximum size. A halo of the maximum size can be formed only if other bodies (competing in the polarization of the quantum vacuum) are sufficiently far. This behaviour is schematically presented in Figure 3.

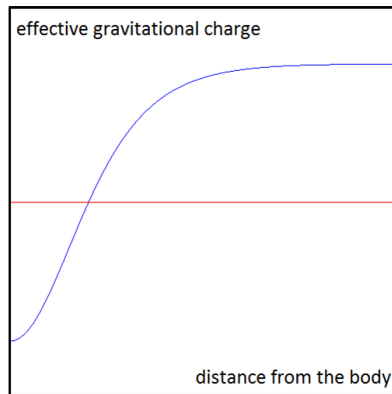


Figure 3. The effective gravitational charge of a body (blue line) increases from the “bare” charge measured at its surface to a constant maximum charge at a large distance from the body. The other bodies can prevent the effective gravitational charge to increase above a limit presented by the red line

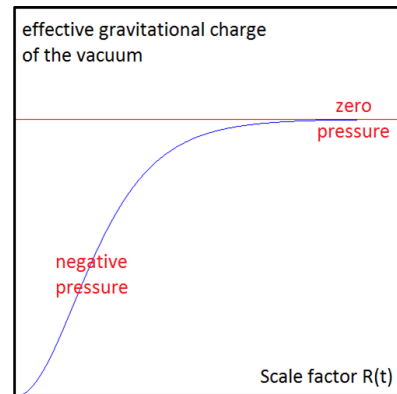


Figure 4. The effective gravitational charge of the quantum vacuum in the Universe depends on the scale factor $R(t)$. Note that with the expansion of the Universe the polarized quantum vacuum converts from a cosmological fluid with negative pressure to a pressureless fluid!

Consequently, with the expansion of the universe, galactic halos must increase in size and the content of the effective gravitational charge caused by the polarization of the quantum vacuum. However, it is not an endless increase; once the halos have reached their maximum size they remain unchanged with the further expansion.

If quantum vacuum fluctuations are virtual gravitational dipole, the quantum vacuum in the Universe is a cosmological fluid with the sum of all gravitational charges equal to zero; however, because of the existence of the halos of non-random oriented dipoles, it is a fluid with a large *effective gravitational charge*. Now, the key point is that there is a period in which the size of the individual galactic halos and the total effective gravitational charge of the quantum vacuum *increase with the*

expansion, which means that the polarized quantum vacuum behaves as a fluid with *negative pressure*, and, according to the cosmological equation (3) it is exactly what is necessary for the accelerated expansion of the Universe. Namely, the cosmological field equation (3) tells us that the expansion of the Universe can be accelerated ($\ddot{R} > 0$) *only if* there is a cosmological fluid with sufficiently big negative pressure, so that the sum in the equation (3) is negative.

5. A cyclic Universe alternatively dominated by matter and antimatter

During the expansion of the Universe quantum vacuum converts from a cosmological fluid with negative pressure to nearly pressureless fluid. According to the cosmological field equation (3) it means that the accelerated expansion converts to the decelerated one.

The eventual collapse of the Universe cannot end in singularity. There is an ultimate mechanism to prevent it: the gravitational version of the Schwinger mechanism [7, 9, 16] i.e. conversion of quantum vacuum fluctuations into real particles, by an extremely strong gravitational field.

For a constant acceleration g the particle creation rate per unit volume and time, can be written as

$$\frac{dN_{m\bar{m}}}{dt dV} = \frac{c}{\tilde{\lambda}_m^4} \left(\frac{g}{g_{cr}} \right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-n \frac{g_{cr}}{g}\right); \quad \tilde{\lambda}_m \equiv \frac{\hbar}{mc}, \quad g_{cr} \equiv \pi \frac{c^2}{\tilde{\lambda}_m} \quad (15)$$

An extremely strong gravitational field (stronger than the critical acceleration g_{cr} in the equation (15)) would create a huge number of particle-antiparticle pairs from the physical vacuum; with the additional feature that matter tends to reach toward the eventual singularity while antimatter is violently ejected farther and farther from singularity. The amount of created antimatter is equal to the decrease in the mass of the collapsing matter Universe. Hence, the quantity of matter decreases while the quantity of antimatter increases by the same amount; the final result might be conversion of nearly all matter into antimatter. If the process of conversion is very fast, it may look like a Big Bang but it is not a Big Bang: it starts with a macroscopic initial size without singularity and without need for inflation field of unknown nature. In addition, there is an elegant explanation of the matter-antimatter asymmetry in the universe: our cycle of the universe is dominated by matter because the previous cycle was dominated by antimatter (and the next cycle would be dominated by antimatter again).

Just for an illustration of the above qualitative picture, let us limit to the radiation dominated Universe. According to the cosmological field equation (3)

$$\ddot{R} = -\frac{8\pi G \rho_{r0}}{3} \frac{R_0^4}{R^3} \quad (16)$$

For $R = 10^9 m$ (i.e. when universe has the size of the Sun), \ddot{R} has tremendous value of the order of $10^{42} m/s^2$, 9 orders of magnitude greater than the critical acceleration needed for creation of let us say neutron-antineutron pairs. Using the equation (15), the corresponding creation rate for neutron-antineutron pairs is of the order of

$$\frac{dN_{m\bar{m}}}{dt dV} \approx 10^{90} \text{ particles} / \text{sm}^3 \approx 10^{33} M_{Sun} / \text{sm}^3 \quad (17)$$

With such an enormous conversion rate the matter of our Universe can be transformed into antimatter of the next cycle of the universe in a tiny fraction of second!

6. Other topics

If our hypothesis is correct a black hole made from matter is in fact a white hole for antimatter [7, 10, 16]. Hawking radiation (and the information loss paradox) based on a different model of the vacuum cannot exist. Supermassive black holes in the centre of Andromeda and Milky Way, through the gravitational version of the Schwinger mechanism, may emit a huge quantity of antineutrinos [10] which might be detectable with a future generation of neutrino telescopes. Also, supermassive black

holes may contribute to the high energy component of antimatter in cosmic rays; namely antiparticles created in collisions of infalling matter deep inside the horizon would be violently ejected.

Astronomical tests would be complemented with experiments in our laboratories; three competing experiment in CERN will reveal if atoms of antihydrogen fall down or fall up...

The 21st century may be the century of antimatter gravity.

Acknowledgments: We thank “The Petrovic Njegos Foundation” for sponsorship of our participation on the 3rd International Workshop on Antimatter and Gravity.

References

- [1] Hobson M P, Efstathiou G and Lasenby A N 2006 *General Relativity an Introduction for Physicists* (Cambridge: Cambridge University Press) 355-381
- [2] Weinberg S 1989 The cosmological constant problem *Rev. Mod. Phys.* **61** 1–23
- [3] Aitchison I J R 2009 Nothing’s plenty - the vacuum in modern quantum field theory *Contemp. Phys.* **50** 261–319
- [4] L3 Collaboration 2000 Measurement of the running of the fine-structure constant *Phys. Lett. B* **476** 40–48
- [5] Schwinger J S 1951 On gauge invariance and vacuum polarization *Phys. Rev.* **82** 664–679
- [6] Wilson C M et al. 2011 Observation of the dynamical Casimir effect in a superconducting circuit *Nature* **479** 376–379
- [7] Hajdukovic D S 2010 What Would Be Outcome of a Big Crunch *International Journal of Theoretical Physics* **49** 1023-1028
- [8] Hajdukovic D S 2011 Is dark matter an illusion created by the gravitational polarization of the quantum vacuum *Astrophysics and Space Science* **334** 215-218
- [9] Hajdukovic D S 2011 Do we live in the universe successively dominated by matter and antimatter *Astrophysics and Space Science* **334** 219-223
- [10] Hajdukovic D S 2011 Can the New Neutrino Telescopes Reveal the Gravitational Properties of Antimatter *Advances in Astronomy* **2011** 196852
- [11] Hajdukovic D S 2012 Quantum vacuum and dark matter *Astrophysics and Space Science* **337** 9-14
- [12] Hajdukovic D S 2012 Quantum vacuum and virtual gravitational dipoles: the solution to the dark energy problem? *Astrophysics and Space Science* **339** 1-5
- [13] Hajdukovic D S 2013 Can observations inside the Solar System reveal the gravitational properties of the quantum vacuum? *Astrophysics and Space Science* **343** 505-509
- Hajdukovic D S 2014 Testing the gravitational properties of the quantum vacuum within the Solar System *Preprint* hal-00908554v3
- [14] Hajdukovic D S 2013 The signatures of new physics, astrophysics and cosmology? *Modern Physics Letters A* **28** 1350124
- [15] Hajdukovic D S 2014 A new model of dark matter distribution in galaxies *Astrophysics and Space Science* **349** 1-4
- [16] Hajdukovic D S 2014 Virtual gravitational dipoles: The key for the understanding of the Universe *Physics of the Dark Universe* **3** 34-40
- Hajdukovic D S 2014 The basis for a new model of the universe *Preprint* hal-01078947v2
- [17] Kormendy J and Freeman K C 2004 Scaling laws for dark matter halos in late-type and dwarf spheroidal galaxies *IAU Symp. Astron. Soc. Pacif.* **220** 377–395
- [18] Donato F et al. 2009 A constant dark matter halo surface density in galaxies *Mon. Not. R. Astron. Soc.* **397** 1169–1176
- [19] Gai M and Vecchiato A 2014 Astrometric detection feasibility of gravitational effects of quantum vacuum <http://arxiv.org/abs/1406.3611v2>
- [20] Boylan-Kolchin M 2013 *The Astrophysical Journal* **768** 140
- [21] Zhang L et.al. 2013 *The Astrophysical Journal* **772** 1-14