

# COMPLEX ELECTRONIC SYSTEMS MODELLING

Electromagnetic game modelling through  
Tensor Analysis of Networks and Game Theory

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1. Introduction
2. Problem statement
  - Physical issue
  - Generalization
3. Human factor
  - Evolution Tree
  - EM game modelling
4. Numerical application
  - Description
  - Results & Discussion
5. Conclusion & Prospects

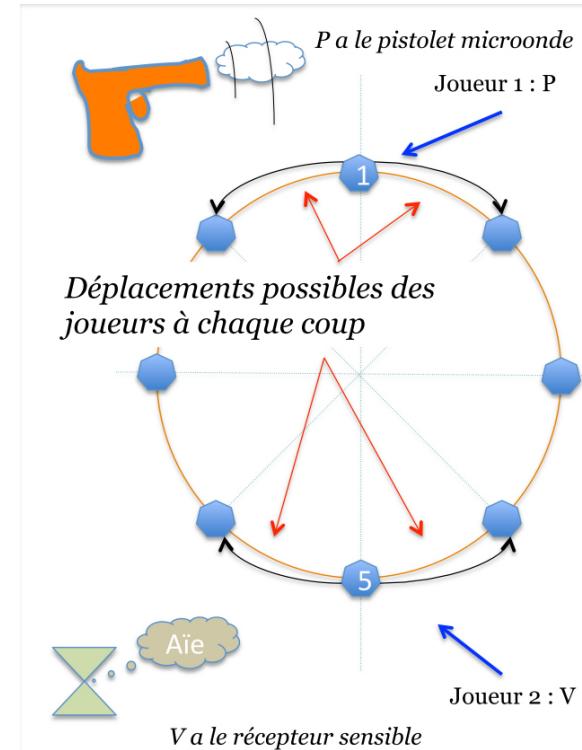
## EM threats in future E-city developments

- Technological development, design of high-power EMI sources
- Dependability of modern society on IT-technology
- Rise of criminal and terrorist threats
- Mobile phone as EM weapon for hi-fi!!!

### Use of EM sources to generate Intentional EM Interference (IEMI)

F. Sabath, *Threat of electromagnetic terrorism – lessons learned from documented IEMI attacks*, EUROEM 2012, Toulouse, France, 2012

⇒ Attempting to jointly model complexity and human factors

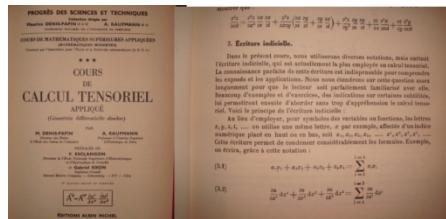


O. Maurice, *Introduction d'une théorie des jeux dans des topologies dynamiques*, PhD Thesis, University of Limoges, September 2013

# THEORETICAL FOUNDATIONS

- Tensorial Analysis of Networks (TAN)  
Ancient formalism → Topical problems
  - Complex systems analysis with multi-physics, multi-scales purposes

G. Kron, *A short course in tensor analysis*, John Wiley and Sons, New York, 1942



- Game Theory → « Psychology »
  - Prisoner's dilemma

2 strategies:

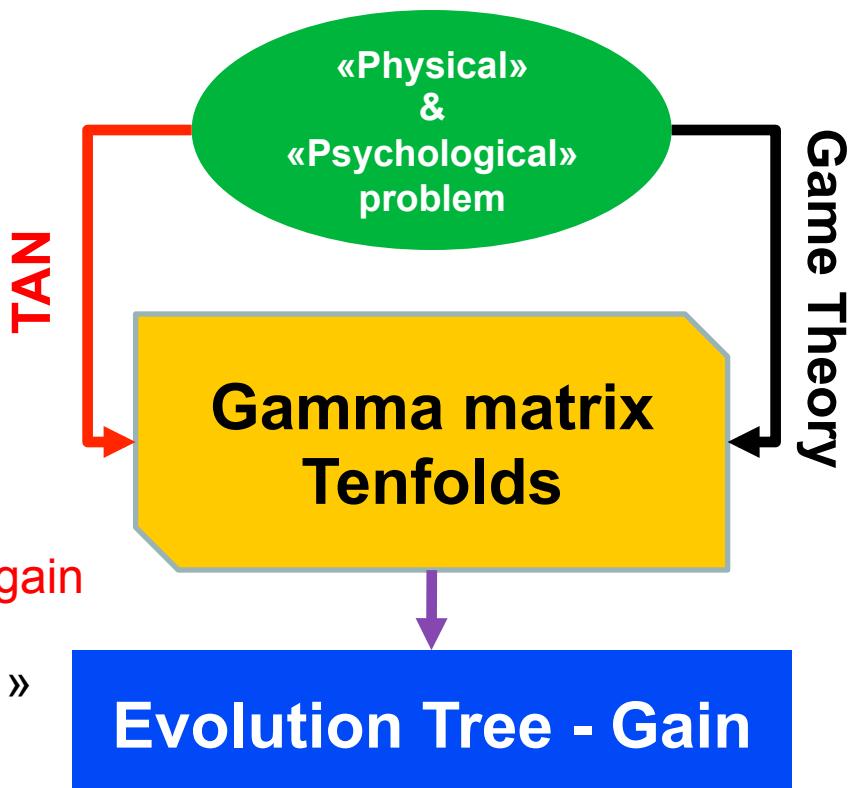
C = Confess

D = Deny

		Player 2	
		C	D
Player 1	Years	(15, 15)	(5, 30)
	C	(30, 5)	(0, 0)

Player 1's gain      Player 2's gain

## Proposed « Methodology »



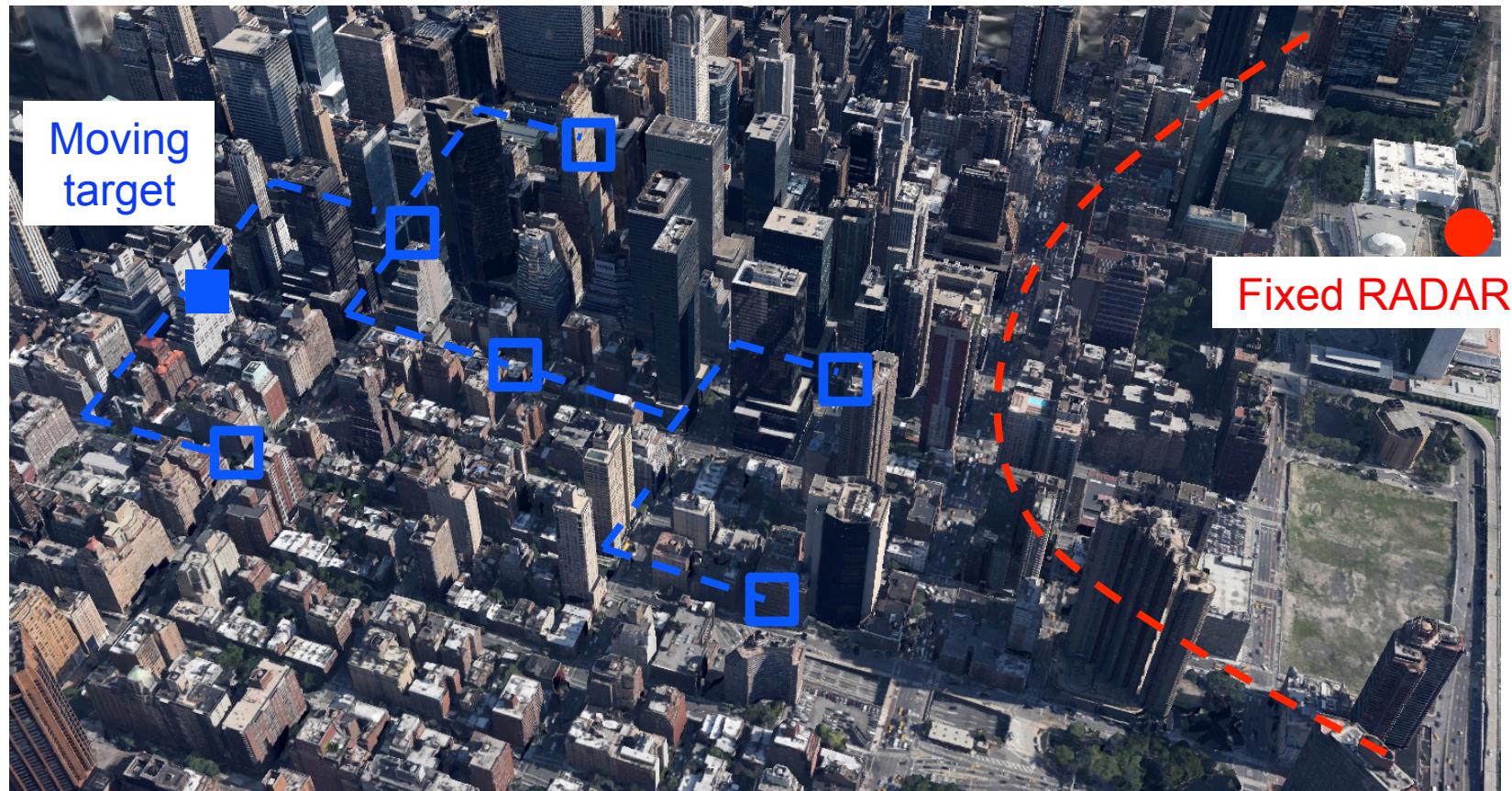
## Conclusions

- « Neutral » players ⇒ Optimal strategy = « CC »
- « External » view ⇒ « DD » choice ...

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- System description: « Moving target detection »

- Fixed RADAR (emitting function)
- Target model (receiving function)
- Model for RADAR / target interactions

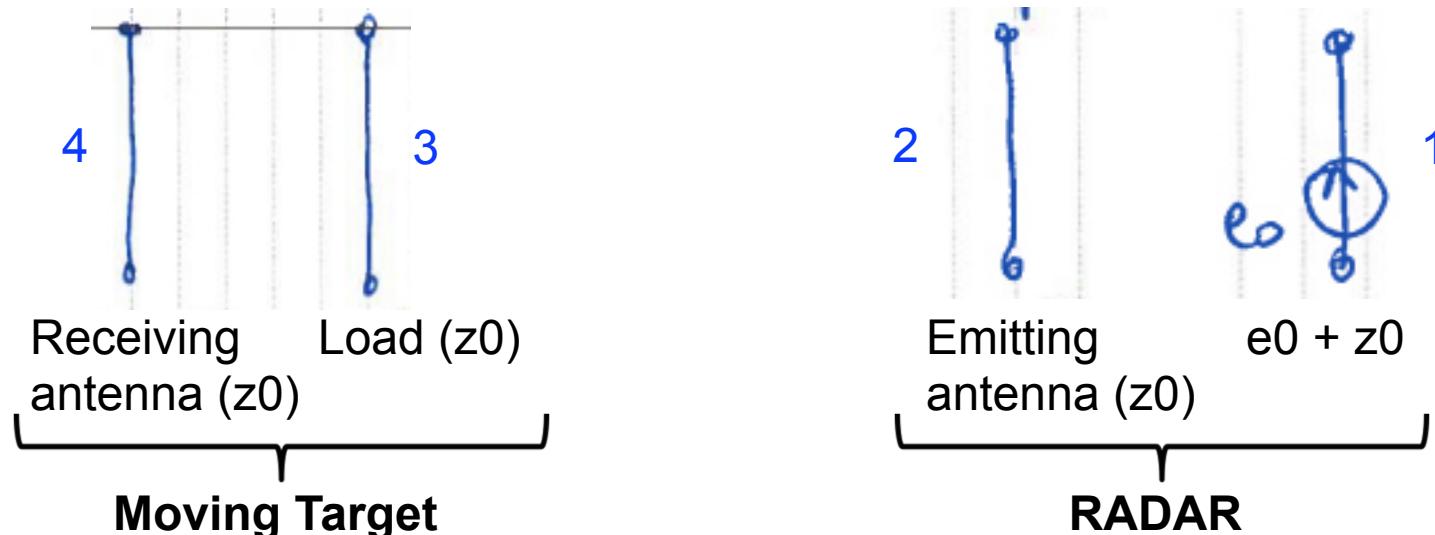


- RADAR: « amplifier + driving electronics »
  - Hyperfrequency source + internal impedance ( $50 \Omega$ )
  - Matched Radiation Resistance ( $50 \Omega$ )
- Target: « antenna + electronics »
  - Receiving antenna (Radiation Resistance,  $50 \Omega$ )
  - Matched load ( $50 \Omega$ )
- System finalization
  - Definition of networks coordinates
  - EM interaction between networks (antennas)

# GENERALIZATION WITH TAN (1/4)

## ■ Definition of Primitive Elements (PE)

- Assuming  $z_i (I, V, f, \dots)$  an impedance function linking the radioelectrical behavior of a given object with outputs and an energy source  $e_i$
- PE = branch or network modelling a given object functionning
- $PE = \{(z_1, e_1), (z_2, e_2), \dots, (z_n, e_n)\}$  in  $R$ ,  $R$ : set of networks



# GENERALIZATION WITH TAN (2/4)

- PE assembly  $\Leftrightarrow$  « Transformation »

- Direct sum of PE impedances
- Addition of coupling interactions

$$z = \begin{bmatrix} z_0 & 0 & 0 & 0 \\ 0 & z_0 & 0 & 0 \\ 0 & 0 & z_0 & 0 \\ 0 & 0 & 0 & z_0 \end{bmatrix}$$

⇒ Metric definition:

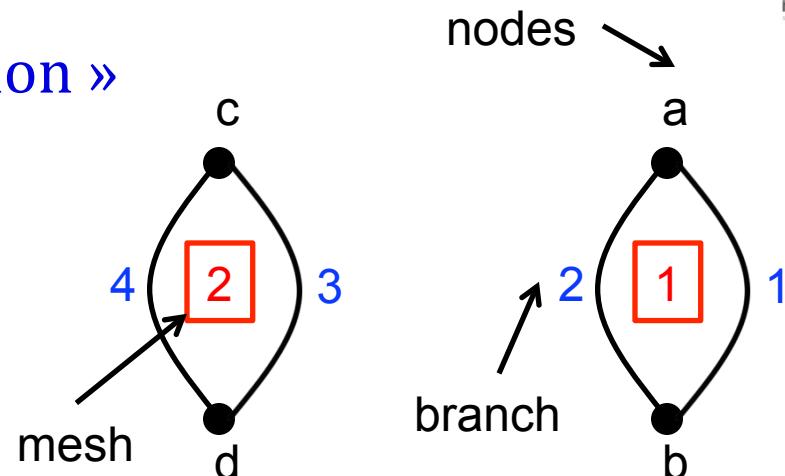
$$e = z i$$

- Definition of « connectivity » matrix

mesh

	1	2
1	1	
2	1	
3		1
4		1

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$



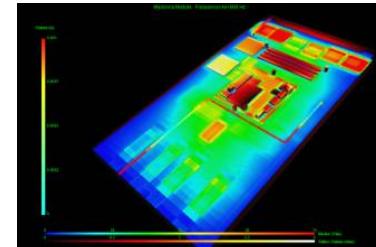
« Transformation »  
 (from branch space to mesh space)

$$z' = \Lambda^T z \Lambda$$

$$z' = \begin{bmatrix} 2z_0 & 0 \\ 0 & 2z_0 \end{bmatrix}$$

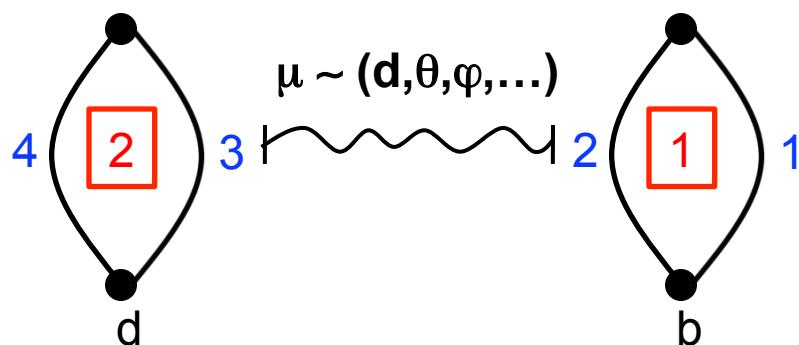
# GENERALIZATION WITH TAN (3/4)

- Addition of « branch » coupling (branch space)
  - Near Field (NF) coupling, welding with resistance, parasitic capacity, ...
  - Direct sum of connectivity matrices



No NF coupling here

- Addition of « cord » coupling (branch space)
  - Far field coupling  $\leftrightarrow$  Matrix  $\mu$  (extra diagonal terms)
  - Antennas reciprocity:  $z_{32} = z_{23}$



$$z_a = z + \mu = \begin{bmatrix} z_0 & 0 & 0 & 0 \\ 0 & z_0 & z_{23} & 0 \\ 0 & z_{32} & z_0 & 0 \\ 0 & 0 & 0 & z_0 \end{bmatrix}$$

⇒ Final « Transformation »

$$z'_{\alpha} = \begin{bmatrix} 2z_0 & z_{32} \\ z_{32} & 2z_0 \end{bmatrix}$$

# GENERALIZATION WITH TAN (4/4)

- Final computation

- Solving system:

$$\begin{bmatrix} e_0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2z_0 & z_{32} \\ z_{32} & 2z_0 \end{bmatrix} \cdot \begin{bmatrix} i^1 \\ i^2 \end{bmatrix}$$

↑ unknowns

- « Tenfold » definition

- Gathering previous descriptions in 1 unique « list »  $\leftrightarrow$  Tenfold
- System evolution requires new « Transformations »
- « Transformations » weighted by probability depicting « human » factor

- « Tenfold » characteristics

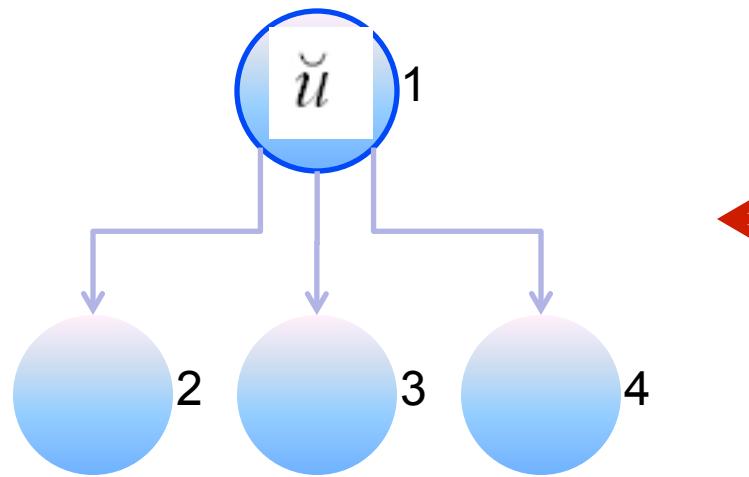
- Topology (connectivity, incidence, ...) T
- Metric (« Z » branch and/or « Z' » mesh)
- Sources (internal) E

$$\check{u} = (T, Z, E)$$

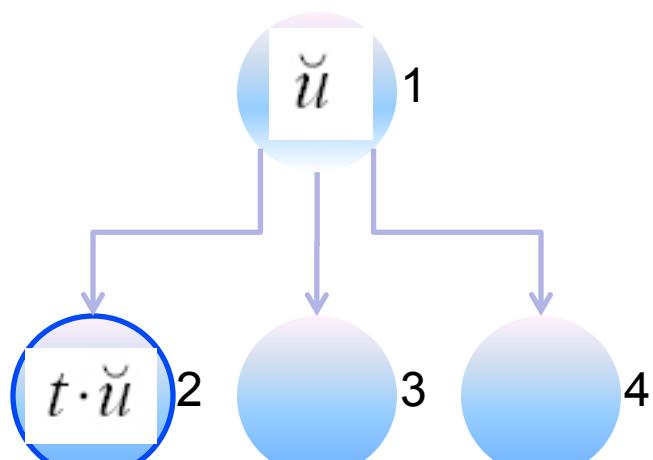
$$\check{u} = (T, \begin{bmatrix} 2z_0 & z_{32} \\ z_{32} & 2z_0 \end{bmatrix}, \begin{bmatrix} e_0 \\ 0 \end{bmatrix})$$

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# EVOLUTION TREE (1/2)



$$I = \begin{bmatrix} \check{u} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Information vector}$$



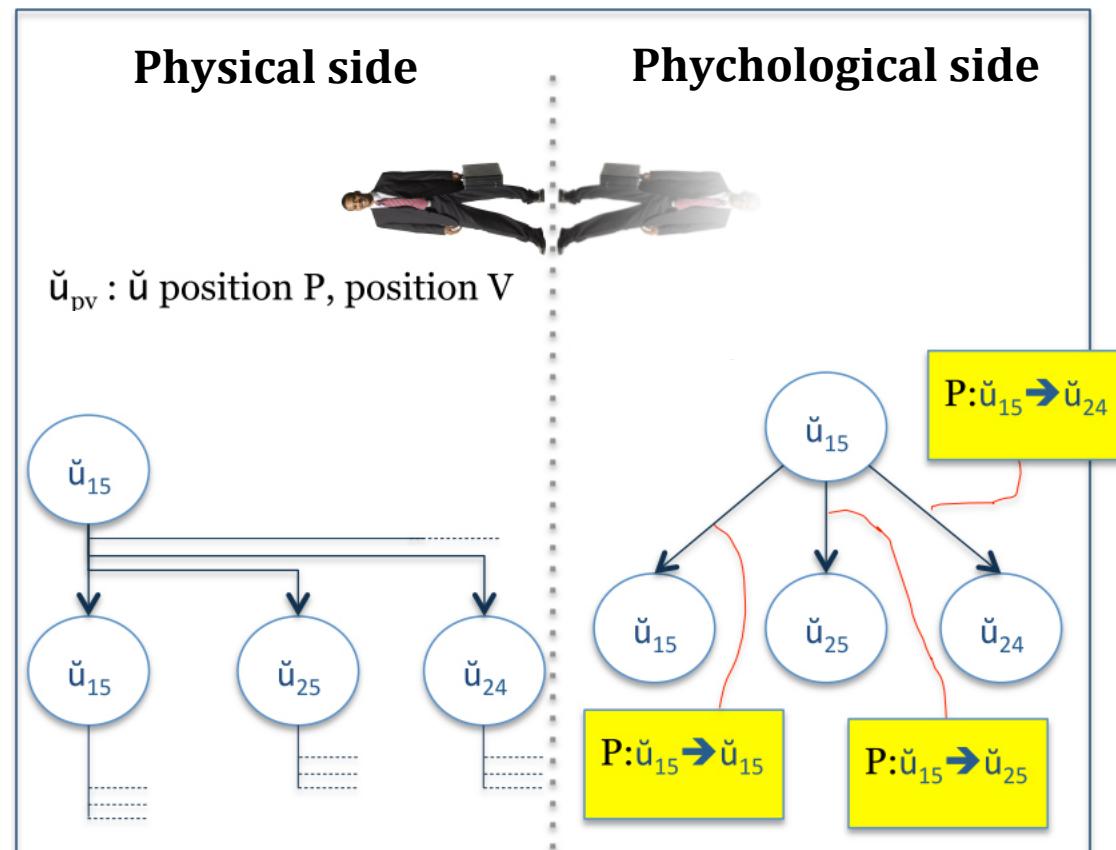
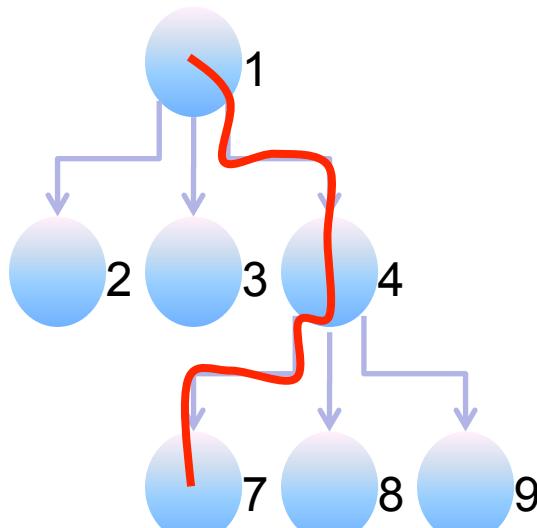
$$\gamma = \begin{bmatrix} 0 & 0 & 0 & 0 \\ t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Propagator}$$

$$\gamma \cdot I = \begin{bmatrix} 0 \\ t \cdot \check{u} \\ 0 \\ 0 \end{bmatrix}$$

# EVOLUTION TREE (2/2)

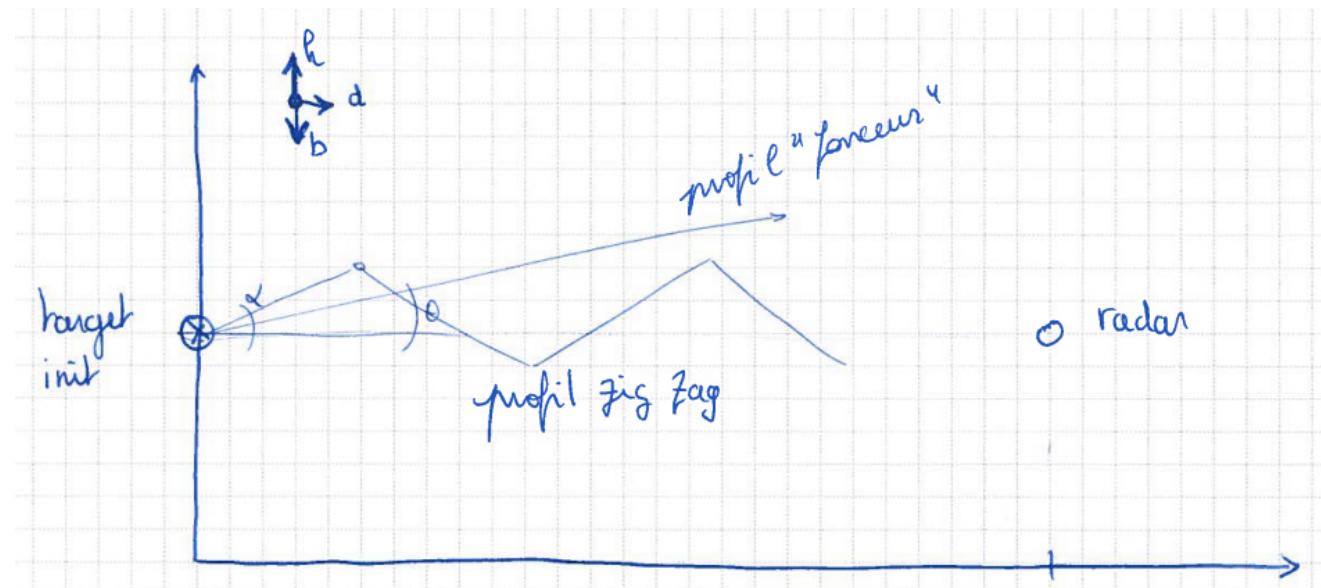
- Paths deduced from « gamma » matrix

- Segments given by  $\{ \text{diag}(\gamma \cdot \gamma^T) \}$
- Entire probability of a given path  $\rightarrow$  Product of segments probability
- Path  $C_i$  (red colored)  
 $= (1:4) \cdot (4:7)$   
 $P(C_i) = P_{14} P_{47}$



# DESCRIPTION OF EM GAME

- Objective: « *Moving target should come closer to RADAR without disturbances* »
- Elements: 1 RADAR, 1 target, 1 playground
- Rules
  - Displacement: 1 step (sampling) by game-turn
  - End of game in case of « Target » disturbance (susceptibility threshold)
  - Objects (RADAR, electronics) used in initial state  
→ Ban: increasing power of emitting antenna, shielding, changing antennas factors, ...)

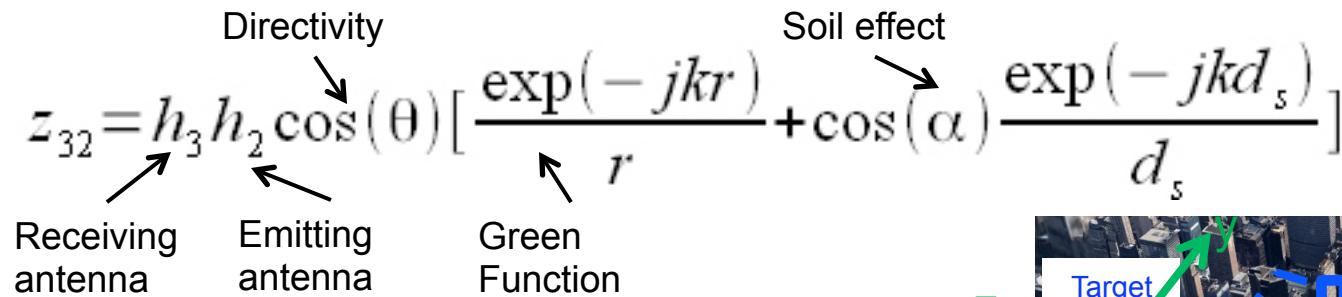


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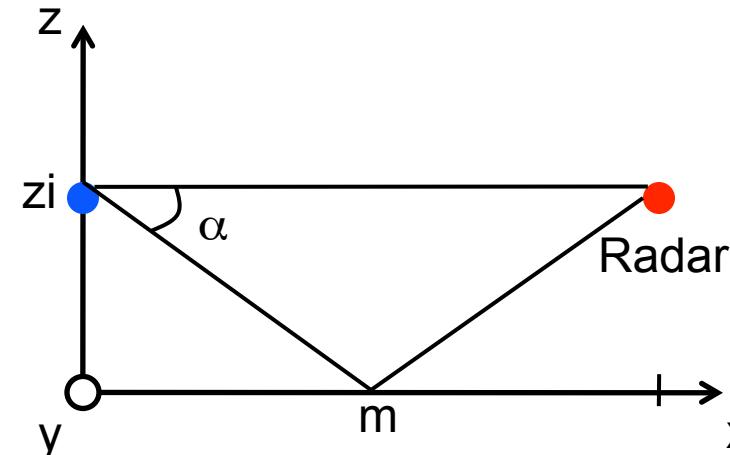
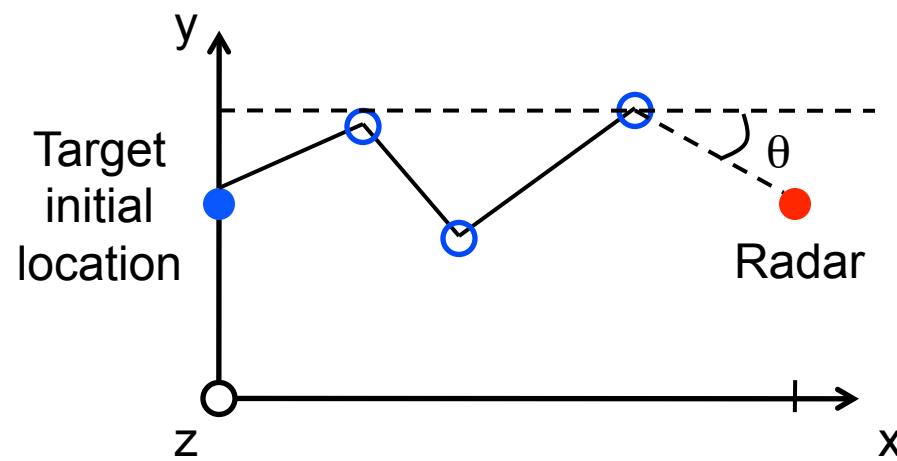
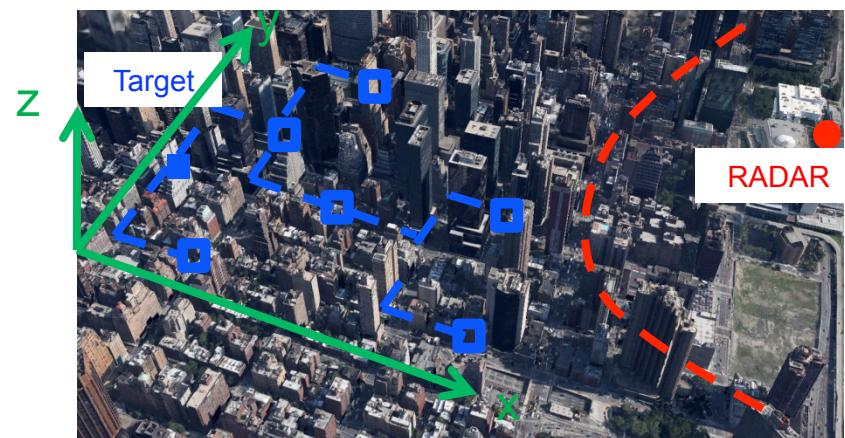
# NUMERICAL DESCRIPTION (1/2)

- RADAR / Moving target interactions

$$z_{32} = h_3 h_2 \cos(\theta) \left[ \frac{\exp(-jkr)}{r} + \cos(\alpha) \frac{\exp(-jkd_s)}{d_s} \right]$$

Directivity  

  
 Receiving antenna      Emitting antenna      Green Function

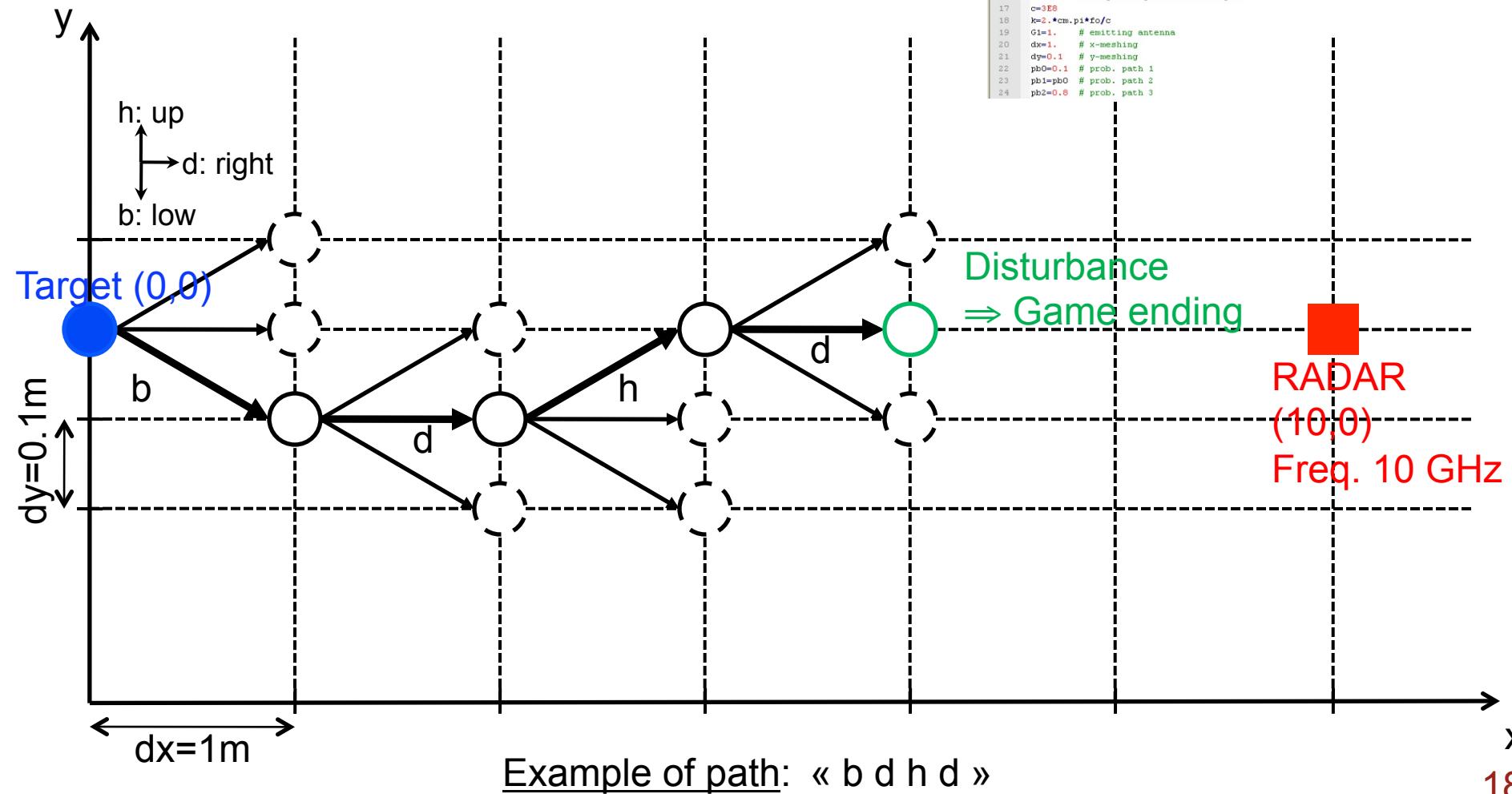
$$z'_{\alpha} = \begin{bmatrix} 2z_0 & z_{32} \\ z_{32} & 2z_0 \end{bmatrix}$$



# NUMERICAL DESCRIPTION (2/2)

## modelling displacement (meshing)

- « Open » playground (no hiding)
- « Python » computation



- Gain

- Inverse of the lowest distance reached

- « Choice / gain » projection

- Definition of criteria to range similar families
  - Choosing « evolution directions »
    - Straightforward (« bb...bb », « dd...dd ») ...)
    - Change of direction (« bdh...bh ») ...)

- « Go-getter »

- Straightforward path following:  $g1 \rightarrow \text{« b »}$ ,  $g2 \rightarrow \text{« h »}$  or  $g3 \rightarrow \text{« d »}$
  - Probability:  $P(\text{« g1 »})=0.1$ ,  $P(\text{« g2 »})=0.1$ ,  $P(\text{« g3 »})=0.8$

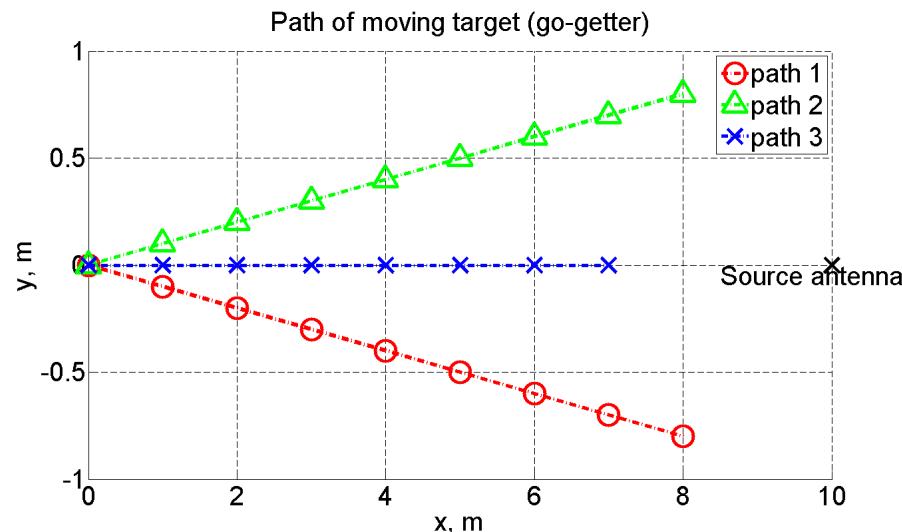
- « Undecided »

- Ups and downs paths:  $u1$ ,  $u2$ ,  $u3$
  - Probability:  $P(\text{« u1 »})=0.25$ ,  $P(\text{« u2 »})=0.25$ ,  $P(\text{« u3 »})=0.5$

- « Random »

- Randomly distributed choices, 3 simulations:  $r1$ ,  $r2$ ,  $r3$
  - Probability:  $P(\text{« r1 »})=1/3$ ,  $P(\text{« r2 »})=1/3$ ,  $P(\text{« r3 »})=1/3$

# NUMERICAL RESULTS: « GO-GETTER »



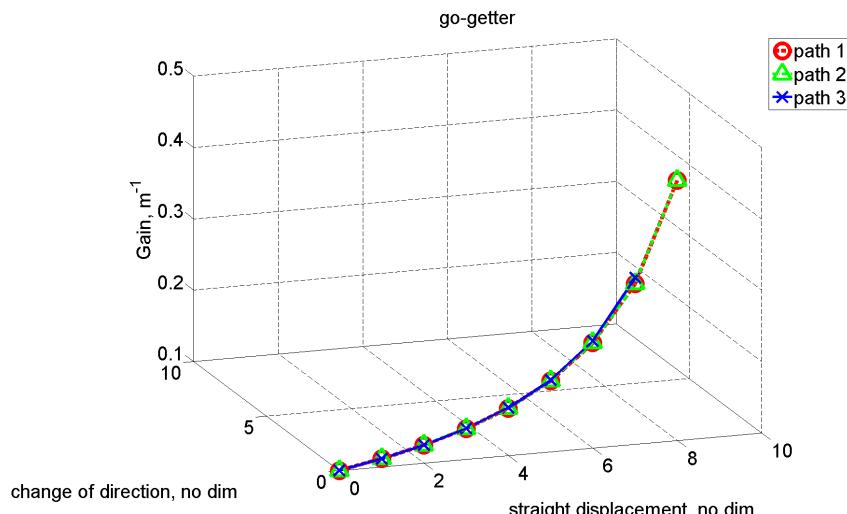
$\text{Gain} = 1/d_{\min}$   
 $\text{EGP} = \text{Partial Gain Expectation}$   
 (including probability)  
 $\text{EG} = \sum_i(\text{EGP}_i)$

i	1	2	3
$d_{\min}$	2.15	2.15	3.00
$\text{EGP}_i$	$4.64 \times 10^{-9}$	$4.64 \times 10^{-9}$	$6.99 \times 10^{-2}$

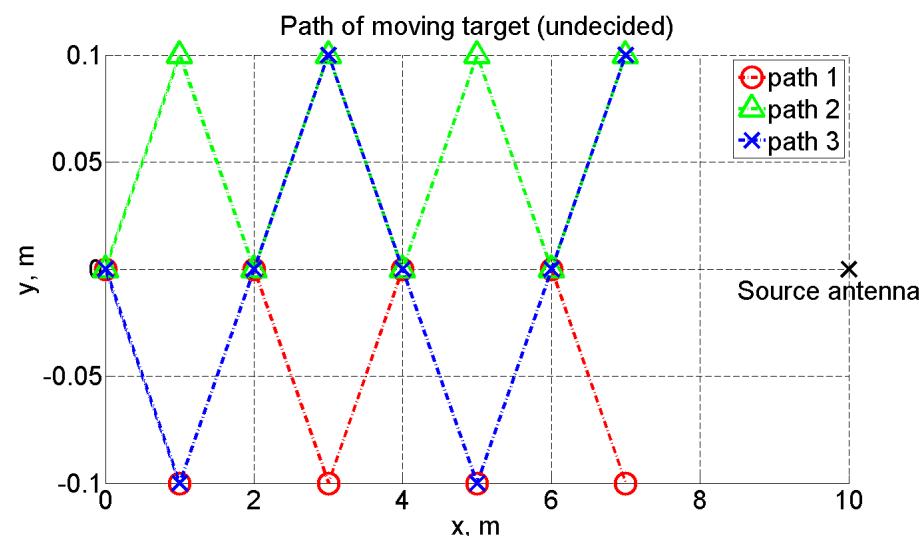
$$\text{EG} = 6.99 \times 10^{-2}$$

Potential gain higher for paths 2 and 3 (symmetrical)

Highest EGP for path 3 (straightforward)



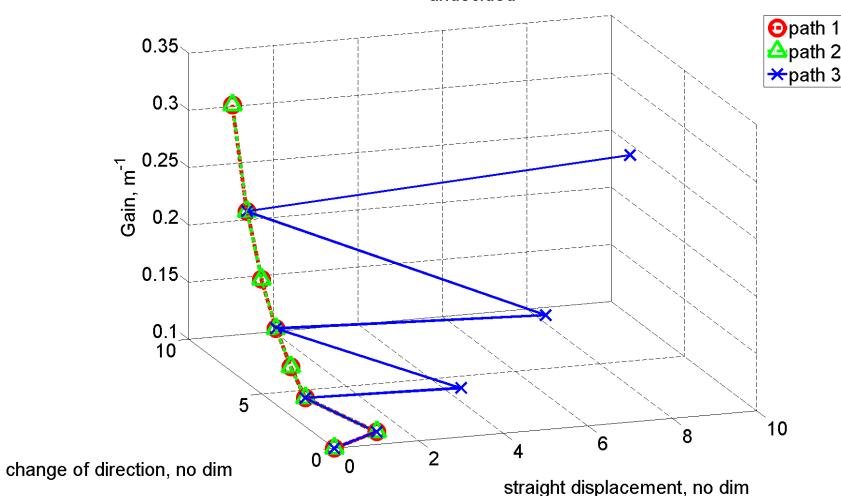
# NUMERICAL RESULTS: « UNDECIDED »



	1	2	3
$d_{\min}$	3.00	3.00	3.00
$\text{EGP}_i$	$2.03 \times 10^{-5}$	$2.03 \times 10^{-5}$	$2.60 \times 10^{-3}$

$$\text{EG} = 2.64 \times 10^{-3}$$

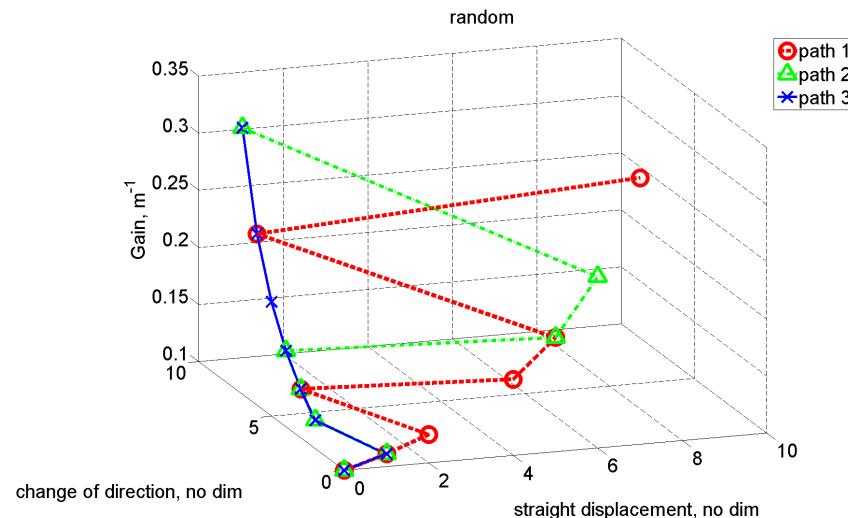
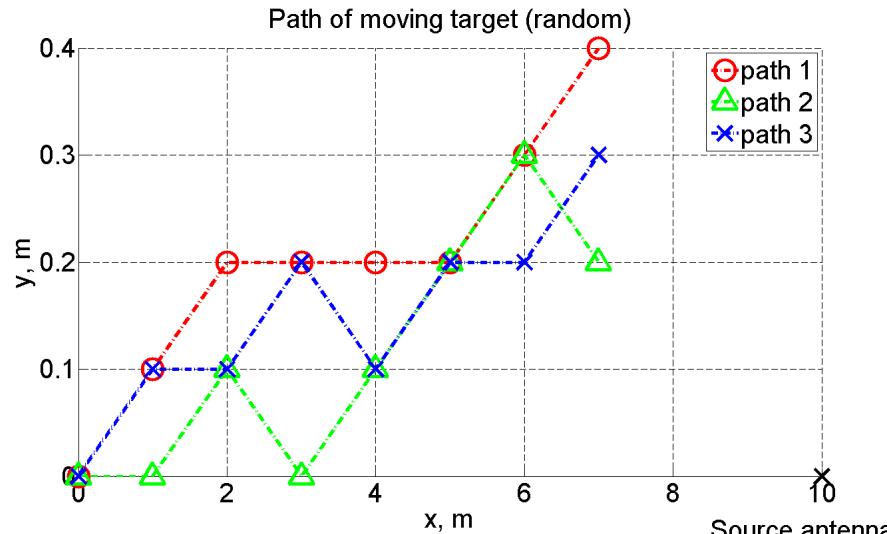
undecided



Similar results for paths 1, 2 & 3

Lower EGP & EG than in case « Go-getter »

# NUMERICAL RESULTS: « RANDOM »



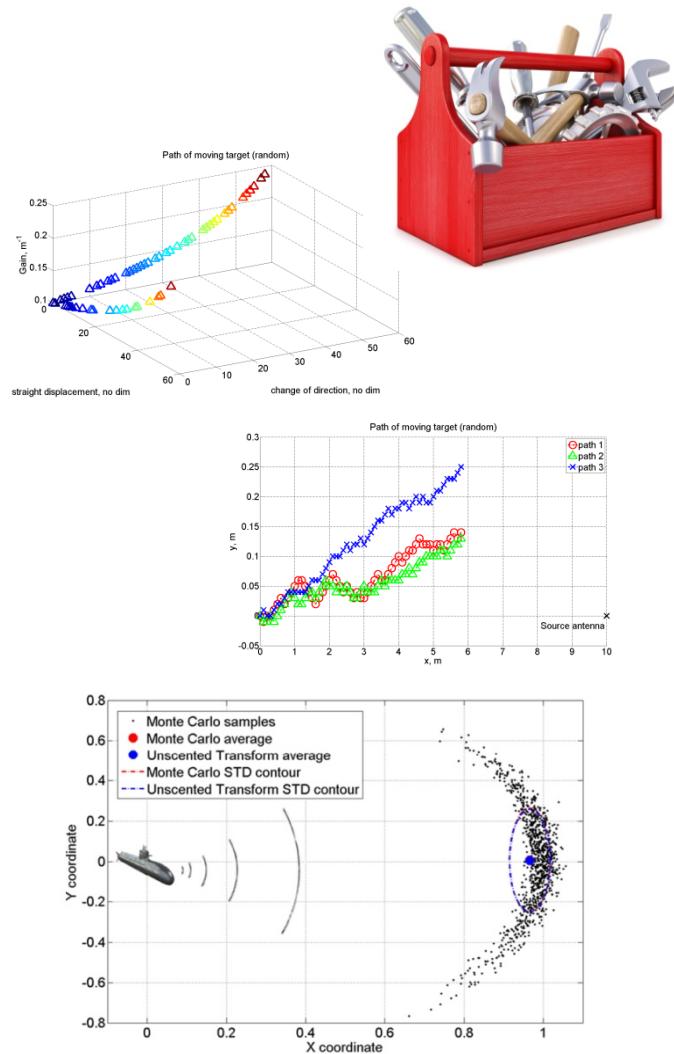
	1	2	3
$d_{\min}$	3.00	3.00	3.00
$\text{EGP}_i$	$1.52e-4$	$1.52e-4$	$1.52e-4$

Equivalent results for paths 1, 2 & 3  
(expected)

Lowest EGP & EG  
⇒ No decision about priority paths...

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# CONCLUSION & PROSPECTS



- **Toolbox for complex processes modelling**
  - Huge diversity of models (codes, analytical formalisms, ...)
  - Aims  $\neq$  Common & Universal modelling
- **Illustration from EM game modelling**
  - Tensorial Analysis of Networks formalism
  - Gamma matrix, tenfolds
  - Evolution tree, probability & gain
- **Forthcoming works**
  - Complete description of complex issue (electronic war...)
  - Enriching previous models  
→ Probability, uncertainty, reliability...

**M. Ferber, Méthodologie de prise en compte a priori de la CEM dans l'optimisation robuste de systèmes d'électronique de puissance, PhD Thesis, University of Lyon, December 2013**

**B. Ravelo et al., modelling of complex RF environment effect on RF circuits EMC analysis, RADIO2014, Mauritius, 2014**