

# Notes on wave-induced current predictors Mirko Fiacchini

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## Notes on wave-induced current predictors

Mirko Fiacchini \*

#### Abstract

The objective here is to build a predictor of the wave effect on the tidal current. The tide is, in fact, influenced by the superficial wave, in particular when the measure is taken at relatively low depths. Thus, while a cyclic behaviour of the tide, with periods of the order of some hours, is relatively easy to be forecast, the effect of the wave might be predicted employing the available measures of the tidal current. This effect is often characterized by much smaller periods, some second in general, and is less regular the tidal main oscillation. The predictiors proposed are dynamical models, based on observers, that can provide the future estimation of the value of the wave-induced current.

### **1** Problem formulation

The predictor might be a dynamical system whose input is the present measure of the current at time t, denote it v(t), and whose output is the prediction of the value of the current time  $t + \delta$ , denote it as  $v_p(t + \delta)$ .

We make some assumptions on the current model. First, we can assume the main oscillation of the current due to tide evolution is slow enough with respect to the variations dues to the wave such that the first can be neglected in the current model. Such assumption is not restrictive, since, although the tide is not constant, its value can be forecast with precision and then can be assumed to be known at every instant and in the future. In other words, it would be sufficient to remove the tide value from the measured current, to decouple the two effects on the current.

The second assumption is that the oscillation of the current due to the wave is periodic with know period T. Concerning this assumption, it is reasonable to suppose that the main behavior of the wave could be periodic. On the other hand, a slow variation of the period around a nominal value should be expected. Moreover, also in case the period could be considered fixed, supposing the exact knowledge of its value could be not realistic. On the other hand one of the beneficial feature of the use of an observer-based prediction is that the effects of the possible uncertainties in the parameters in general, the period in particular, can be compensated by the inherent robustness of the observer-based predictor.

From the theory related to the signal analysis, every periodic signal of period T can be exactly represented as the sum of sinusoids whose period are multiple of T, that is as its Fourier series representation. That is, from periodicity of v(t), we have that

$$v(t) = \alpha_0 + \sum_{j=1}^{\infty} \alpha_j \cos\left(\frac{2\pi}{T}jt\right) + \beta_j \sin\left(\frac{2\pi}{T}jt\right),$$

<sup>\*</sup>M. Fiacchini is with GIPSA-lab, Grenoble Campus, 11 rue des Mathématiques, BP 46, 38402 Saint Martin d'Hères Cedex, France

where the Fourier coefficients are

$$\alpha_0 = \frac{1}{T} \int_0^T v(t) dt, \quad \alpha_j = \frac{2}{T} \int_0^T v(t) \cos\left(\frac{2\pi}{T} jt\right) dt, \quad \beta_j = \frac{2}{T} \int_0^T v(t) \sin\left(\frac{2\pi}{T} jt\right) dt, \quad (1)$$

for all  $j \in N$  with j > 1. Thus, any periodic signal is the sum of the infinite number of sinusoids that have frequencies which are multiple of *T* (plus a constant terms). Nevertheless, in general, the attention can be often limited to the first components, those with lower frequencies, in practice. Indeed the signal one is interested in, involves in general a finite bandwidth, being the higher frequencies more related to the noise than to the proper signal.

Then, without loss of generality, we assume hereafter that the periodic effect of the wave on the current has a nominal part given by the linear combination a finite number of sinusoids

$$v_n(t) = \alpha_0 + \sum_{j=1}^{N_s} \alpha_j \cos\left(\frac{2\pi}{T}jt\right) + \beta_j \sin\left(\frac{2\pi}{T}jt\right), \qquad (2)$$

where the parameter  $N_s$  is assumed big enough to catch the main current behaviour. The mismatch between the real evolution and the nominal one will be considered as an exogenous term d(t), representing both the noise and the modelling approximation, that is

$$v(t) = v_n(t) + d(t).$$
(3)

#### 2 Continuous-time predictor

Consider the nominal part of the effect of the waves on the current, i.e. (2) and notice that every signal which is the sum of sine and a cosine of the same frequency is the output of perfect linear oscillator. In fact, given the linear dynamical system

$$\dot{x}(t) = \begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix} x(t),$$

$$y(t) = Cx(t),$$
(4)

its solution has the form

$$y(t) = Ce^{At}x(0) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \cos(bt) & -\sin(bt) \\ \sin(bt) & \cos(bt) \end{bmatrix} x(0),$$

and then

$$y(t) = (C_1 x_1(0) + C_2 x_2(0)) \cos(bt) + (C_2 x_1(0) - C_1 x_2(0)) \sin(bt).$$
(5)

Hence, the Fourier coefficients are univocally determined by an appropriate choice of the initial condition x(0) (with for instance  $C = [1 \ 1]$ ), and vice-versa. This means that every signal of the form (2) can be generated by an autonomous system composed by  $N_s$  linear undamped oscillators, that is by

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -f & 0 & 0 & \cdots & 0 & 0 \\ 0 & f & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 2f & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -N_s f \\ 0 & 0 & 0 & 0 & 0 & \cdots & N_s f & 0 \end{bmatrix} x(t) = Ax(t),$$
(6)  
$$v_n(t) = Cx(t) = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} x(t),$$

with  $f = 2\pi/T$  and for an appropriate  $x(0) \in \mathbb{R}^{2N_s+1}$ . Then, constructing an observer, that is a dynamical systems that permits to reconstruct the state of a system from its output, would lead implicitly to identify the Fourier parameters as in (5) and thus the evolution of the signal  $v_n(t)$ . Notice, that the first state dynamics is an integrator and is related to the mean value of  $v_n$  and then to  $\alpha_0$ .

We define as observer for the system (6) the classical Luenberger observer, that is the system

$$\dot{z}(t) = Az(t) + L(Cz(t) - v(t)), 
y(t) = Cz(t),$$
(7)

with  $z \in \mathbb{R}^{2N_s+1}$  and  $L \in \mathbb{R}^{1 \times (2N_s+1)}$  such that the matrix A + LC is a Hurwitz stable matrix, i.e. its eigenvalues have negative real part. Among the infinite possible choices of L, we considered the one that solves an optimal LQR problem. The resulting error between the system and the observer states, defined e = z - x, is the solution of the following differential equation

$$\dot{e}(t) = \dot{z}(t) - \dot{x}(t) = (A + LC)e(t),$$

and then converges to zero, like the value  $y(t) - v_n(t)$ . Then the value of y(t) converges exponentially fast to  $v_n(t)$ . Moreover, since the state *z* converges to *x* then also the Fourier parameters of the output signals of both systems converges to the same value. This means that, if the state of the system was not affected by an external additional perturbation, once the two outputs are close (or the same) the stay close (or the same) and then the prediction of the value  $v_n(t + \delta)$  is given by

$$\bar{v}_c(t) = C e^{A\delta} z(t), \tag{8}$$

where z(t) is the observer state at time t.

Thus, summarizing, the system given by

$$\dot{z}(t) = Az(t) + L(Cz(t) - y(t)),$$
  

$$\bar{v}_c(t) = Ce^{A\delta}z(t),$$
(9)

provides a prediction of the value of  $v_n(t + \delta)$  and the prediction mismatch converges to zero exponentially fast if the original systems if not affected by additive perturbation.

Considering the uncertainty (or the variation) in the period T, this would be reflected as an uncertainty (or a variation) of the parameter f and then in the matrix A. Since the observer for linear systems can cope also with the parametric uncertainties, then it is evident that the assumption made above on the knowledge of T is not too restrictive.

#### **3** Discrete-time predictor

Following reasoning analogous to the case illustrated in the previous section, a discretetime predictor can be built. The approach is implicitly based on the discrete-time Fourier series, and the related theory. This claims that a discrete-time periodic signal  $v_d(k)$ , with  $k \in \mathbb{N}$  and period N, i.e. such that  $v_d(k) = v_d(k+N)$  for all  $k \in \mathbb{N}$ , can be written as

$$v_d(k) = \sum_{n=0}^{N-1} c_n e^{ik\Delta n},$$
 (10)

with  $\Delta = 2\pi/N$ , for all  $k \in \mathbb{N}$ . The parameters  $c_n$  are the Fourier coefficients and are given by

$$c_n = \frac{1}{N} \sum_{k=0}^{N-1} v_d(k) e^{ik\Delta n},$$
(11)

for all  $n \in \mathbb{N}_{N-1}$ . Notice that, for discrete-time signals, the maximal number of harmonics is finite and univocally determined by the period N, or, equivalently, by the time increment  $\Delta$ . Notice also that, although the Fourier coefficients (11) are complex in general, it follows from (10) that the signal  $v_d(k)$  have to be composed by a finite number of sinusoids, being it real valued.

As for the continuous-time case, consider the following discrete-time linear autonomous system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} \cos(\Delta) & -\sin(\Delta) \\ \sin(\Delta) & \cos(\Delta) \end{bmatrix} x(k) = R(\Delta)x(k), \\ y(k) &= Cx(k) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} x(k), \end{aligned}$$
(12)

one has that the solution at k = 1 is given by

$$y(1) = (C_1 x_1(0) + C_2 x_2(0)) \cos(\Delta) + (C_2 x_1(0) - C_1 x_2(0)) \sin(\Delta).$$

Moreover, since

$$R(\alpha)R(\beta) = \begin{bmatrix} \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) & -\cos(\alpha)\sin(\beta) - \sin(\alpha)\cos(\beta) \\ \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) & -\sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta) \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = R(\alpha + \beta),$$

then

$$y(k) = (c_1 x_1(0) + c_2 x_2(0)) \cos(k\Delta) + (c_2 x_1(0) - c_1 x_2(0)) \sin(k\Delta),$$
(13)

which means that, as for the continuous-time case, every discrete-time sinusoid of period  $N \in \mathbb{N}$  can be expressed as the output of (12), for an appropriate initial condition.

Assume that, as for the continuous-time case, one knows that the wave current oscillation is periodic of period T and the relevant harmonics are related to the frequencies  $f_j = f \cdot j$  with  $j \in \mathbb{N}_{N_s}$  and  $f = 2\pi/T$ . Then the angle variations between two instants are given by

$$\Delta_j = f_j au_s = rac{2\pi}{T} j rac{T}{N} = j rac{2\pi}{N} = j \Delta,$$

where  $\tau_s = T/N$  is the sampling period and thus the nominal signal  $v_{n,d}(k)$  is the output of the discrete-time linear autonomous system

with

$$A_{d} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cos(\Delta) & -\sin(\Delta) & 0 & 0 & \cdots & 0 & 0 \\ 0 & \sin(\Delta) & \cos(\Delta) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cos(2\Delta) & -\sin(2\Delta) & \cdots & 0 & 0 \\ 0 & 0 & 0 & \sin(2\Delta) & \cos(2\Delta) & \cdots & 0 & 0 \\ \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \cos(N_{s}\Delta) & -\sin(N_{s}\Delta) \\ 0 & 0 & 0 & 0 & 0 & \cdots & \sin(N_{s}\Delta) & \cos(N_{s}\Delta) \end{bmatrix},$$

$$C_{d} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix},$$
(15)

where the first state dynamics is an integrator and is related to the mean value of  $v_{n,d}$ , as for the continuous-time case. A predictor can be built for the system (14), based on the discrete-time Luenberger observer, as

$$z_d(k+1) = A_d z_d(k) + L_d(C_d z(k) - v_d(k)),$$
  

$$\bar{v}_d(k) = C_d A_d^{\delta_d} z(k),$$
(16)

for all  $k \in \mathbb{N}$  and with

$$\delta_d = \arg\min_{d\in\mathbb{N}} \left| d - \frac{\delta N}{T} \right|,$$

the prediction horizon in terms of discrete instants and after a rounding with respect to  $\delta$  and  $L_d$  designed such that  $A_d + L_dC_d$  is Schur.



Figure 1: Nominal  $v_n$ , measured v and reconstructed current.

### **4** Numerical application

To illustrate the proposed prediction method we first built a periodic signal  $v_n(t)$  as defined in (2) with the period T = 25s, the number of sinusoids  $N_s = 5$  and the Fourier parameters  $\alpha_0, \alpha_j$  and  $\beta_j$ , for  $i = 1, \dots, N_s$  generated randomly. In particular the values of the Fourier coefficients are uniformly distributed between  $\pm M$  with M = 0.5.

Then, measures of the current evolution in the period, affected by the uncertainty d, (uniformly distributed and bounded in  $\pm 0.5M$ ) are generated at every instant 0.25k, with k = 0, ..., N and N = 100. Figure 1 represents the nominal value  $v_n$  of the current, the measured one v, i.e. affected by disturbance d, and the reconstructed current, that is the function obtained by computing approximations of the Fourier coefficients

$$\alpha_{0} = \frac{1}{N} \sum_{k=0}^{N} v(k), \quad \alpha_{j} = \frac{2}{N} \sum_{k=0}^{N} v(k) \cos\left(\frac{2\pi}{N} jk\right), \quad \beta_{j} = \frac{2}{N} \sum_{k=0}^{N} v(k) \sin\left(\frac{2\pi}{N} jk\right).$$
(17)

These coefficients, as approximations of the Fourier ones, give a good estimation of the real value of the current, provided that the wave-induced current is a periodic signal of the form (2)-(3), or, equivalently, the sum of terms as in (5). See Figure 1, in which the current and the corrupted measures are shown together with the current reconstructed by using (17). This means that, if the shape of the wave does not change in time, then the numerical approximation of the Fourier coefficients provide a good estimation, and then a good prediction, of the current. Nevertheless, it is natural to suppose that the wave shape might evolve in time. This is equivalent to say that the terms multiplying the sinusoids in (5) (that are in practice the Fourier coefficients), are not constant, but evolves in time. For this reason, observers-based predictions, that implicitly reconstruct the actual state and then the time varying parameters, should be employed to catch the time varying shape of the wave.

To generated the time varying wave current, the matrices  $A \in \mathbb{R}^{(2N_s+1)\times(2N_s+1)}$  and *C* are built, as in (6) and the observer LQR gain is computed for Q = I and R = 10. The value of the current is generated as the output of the system

$$\dot{x}(t) = Ax(t) + d(t),$$
  

$$v(t) = Cx(t) + g(t),$$

whith initial conditions generated randomly with bound M. Recall that the initial values of the Fourier coefficients are univocally determined by the initial condition of the state, see (5). The signals d(t) and g(t) are obtained interpolating discrete uniformly distributed random signals, with sampling time 25s and maximal amplitude 0.05M. Notice that the signal d(t) and g(t), whose values are depicted in Figure 2, leads to the variation of the parameters in the terms (5), that is the cause of the wave shape evolution.



Figure 2: Signals d(t) and g(t).

The observer is fed by the system output and provides at time t an estimation of the actual value of the current and the prediction at  $t + \delta$ , with  $\delta = 15s$ .

Then, the discrete-time predictor is built, with  $N_s = 5$  and  $\Delta = 2\pi/N$ . The signal v(t) is sampled with sampling time  $\tau_s = T/N = 0.25s$  and the resulting discrete-time signal is used to feed the discrete-time predictor (16), with horizon  $\delta_d = \delta \cdot N/T = 60$  samples.



Figure 3: Measured v, estimated and 15s-ahead predicted current.

Therefore, the discrete-time observer is directly obtained by posing  $\delta_d = 0$ . The results of the continuous and discrete-time observer as well as the predictor are depicted in Figure 3. It can be noticed that the behaviors of the two observers and predictors are rather similar.

Notice, in Figure 3, how fast the estimations converge to the real value of the current, while the predictions need some measurements to produce a reliable value. Moreover, reasonably, the prediction is in general less precise than the estimation, as the latter employs measurements not available for the prediction. It can be notice that the prediction errors, depicted in Figure 4, converge to small values.



Figure 4: 15s-ahead prediction errors.

Finally, some value have been computed to quantify the precision of the prediction,

with respect to the forecasting horizon. In particular, we numerically computed the mean squared error for the continuous and discrete time predictor, that is

$$MSE_{c} = rac{1}{T_{f} - T_{0}} \int\limits_{T_{0}}^{T_{f}} e_{c}(t)^{2} dt, \qquad MSE_{d} = rac{ au_{s}}{T_{f} - T_{0}} \sum_{k=T_{0}/ au_{s}}^{T_{f}/ au_{s}} e_{d}(k)^{2},$$

with  $e_c(t) = v(t) - \bar{v}_c(t)$  and  $e_d(k) = v(\tau_s k) - \bar{v}_d(k)$ , and the mean value and variance for both predictors:

$$\mu_{c} = \frac{1}{T_{f} - T_{0}} \int_{T_{0}}^{T_{f}} e_{c}(t) dt, \qquad \qquad \mu_{d} = \frac{\tau_{s}}{T_{f} - T_{0}} \sum_{k=T_{0}/\tau_{s}}^{T_{f}/\tau_{s}} e_{d}(k),$$
  
$$\sigma_{c} = \frac{1}{T_{f} - T_{0}} \int_{T_{0}}^{T_{f}} (e_{c}(t) - \mu_{c})^{2} dt, \qquad \qquad \sigma_{d} = \frac{\tau_{s}}{T_{f} - T_{0}} \sum_{k=T_{0}/\tau_{s}}^{T_{f}/\tau_{s}} (e_{d}(k) - \mu_{d})^{2}.$$

The simulation time has been fixed by  $T_0 = \delta$  and  $T_f = 200 + \delta$  while the prediction horizon has been varied between 0 and 25, that is  $\delta = 0, 5, ..., 25$ . The values above are normalized with respect to

$$\mathbf{v} = \frac{1}{T_f - T_0} \int_{T_0}^{T_f} |\mathbf{v}(t)| dt,$$

that is a measure of the amplitude of the current. Moreover, for every value of  $\delta$ , 20 simulations have been performed and the mean values computed. This because every simulation depends on the initial condition and the realization of the signals d(t) and g(t), that are randomly generated as above. The obtained results are given in Table 4.

Forecasting horizon $\delta$	0	5	10	15	20	25
$MES_c/v$	0.0004	0.0188	0.0200	0.0183	0.0152	0.0168
$MES_d/v$	0.0001	0.0198	0.0201	0.0180	0.0152	0.0175
$\mu_c/v$	-0.0004	0.0004	-0.0030	0.0032	-0.0016	-0.0026
$\mu_d/\nu$	-0.0001	-0.0002	-0.0024	0.0020	-0.0000	-0.0024
$\sigma_c/v$	0.0004	0.0188	0.0198	0.0180	0.0151	0.0166
$\sigma_d/v$	0.0001	0.0197	0.0198	0.0177	0.0151	0.0174

Table 1: Simulation results for bounds on d(t) and g(t) equal to 0.05M = 0.025.

To infer on the effect of the wave changes, that is of the input d(t) and g(t), we compute the same parameters for different values of their amplitude. The bound on the randomly generated values of d(t) and g(t) employed for the experience above was 0.05M = 0.025. The results for bounds 0.2M and 0.5M have recollected in Tables 4 and 4, respectively.

Forecasting horizon $\delta$	0	5	10	15	20	25
$MES_c/v$	0.0058	0.2052	0.2740	0.2474	0.2887	0.2117
$MES_d/v$	0.0011	0.2215	0.2809	0.2508	0.2879	0.2170
$\mu_c/v$	0.0000	-0.0322	0.0049	0.0012	-0.0034	0.0062
$\mu_d/\nu$	0.0011	-0.0281	-0.0022	0.0027	-0.0054	0.0024
$\sigma_c/v$	0.0055	0.2031	0.2718	0.2455	0.2849	0.2097
$\sigma_d/v$	0.0011	0.2192	0.2790	0.2494	0.2845	0.2150

Table 2: Simulation results for bounds on d(t) and g(t) equal to 0.2M = 0.1.

Forecasting horizon $\delta$	0	5	10	15	20	25
$MES_c/v$	0.0286	1.2243	1.1295	1.0610	1.0657	1.0500
$MES_d/v$	0.0041	1.2728	1.1563	1.0661	1.0621	1.0731
$\mu_c/\nu$	0.0173	0.0107	-0.0222	0.0497	0.0231	-0.0173
$\mu_d/\nu$	0.0021	0.0081	-0.0182	0.0359	0.0186	-0.0037
$\sigma_c/\nu$	0.0253	1.2126	1.1208	1.0471	1.0530	1.0342
$\sigma_d/v$	0.0040	1.2646	1.1487	1.0553	1.0491	1.0617

Table 3: Simulation results for bounds on d(t) and g(t) equal to 0.5M = 0.25.

From the analysis of the three tables above, it can inferred that, as expected, the predicted precision depends on the amplitude of the signals d(t) and g(t). This is reasonable since those signals are to be intended as the causes of wave shape evolution. In fact, for their unpredictable random nature, their effects cannot be forecast and then the prediction error must increase with their amplitude.

On the other hand, it can be noticed that the prediction accuracy seems not to be substantially influenced by the forecasting horizon. This less intuitive effect is due, in our opinion, to the assumption posed in this notes. We supposed, in fact, that the current signal is characterized by a finite number of harmonics and that such number of harmonics, as well as their frequencies, are known and used in the predictor design. Thus, the only source of uncertainty are the generator signals d(t) and g(t), that are randomly generated, bounded and have null mean. It could be reasonable, then, that the prediction errors could be proportional to their amplitude and not to the forecasting horizon. We think that it would be interesting to test the predictor with real measures of the wave-induce current. On the other hand, it has to be pointed out and recall that the assumptions posed in this notes are not very restrictive. Recall, in fact, that every periodic signal can be approximated with the desired precision by considering a sufficiently high number of harmonics. Moreover, every signal, periodic of not, can be seen as a periodic one with time varying Fourier coefficients. Thus, our impression is that the method presented in this notes might fit well for predicting real current signals, conditioned to an adequate tuning of the models and the parameters determining them.

## 5 Conclusions and future works

These notes present a dynamical observer method to estimate the future wave current based on the current and past measures. The main benefit of this approach is the fact it can cope efficiently with a relatively slow evolution of the wave shape. This predictors are aimed at providing an accurate estimation of the future maximal active power generated by a controlled tidal turbine, by feeding with their prediction the optimization-based model of the controlled turbine developed by Mazen Alamir, see [Alamir(2015)]. Thus, the coupling of the predictor with the LP model of the tidal turbine could provide, with relatively low computational burden, an accurate picture of the future active power that can be delivered by any turbine. The forecast of the power production can be employed to take decisions in advance to prevent the effects of future potentially undesired behaviours or to improve the the overall system performance.

# References

[Alamir(2015)] Mazen Alamir. Scalable active power model of a controlled tidal turbine. Technical report, CNRS, GIPSA-lab, Control Systems Department, 2015.