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AN INTERFACE APPROACH FOR EQUIVALENT CONTACT STIFFNESSES IN CONTACT PROBLEMS

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Abstract

In this paper, a spring-like micromechanical contact model is proposed, aiming to catch in an accurate way the mechanical behavior of two rough surfaces in contact under a closure pressure. The contact region between two elastic bodies is treated as a thin damaged interphase characterized by a periodical distribution of non-interacting penny-shaped cracks. By combining a homogenization approach with an asymptotic technique, the analytical expressions of the tangential and normal contact stiffnesses are obtained. Besides, analytical formulations describing the evolution of contact and no-contact radii with respect to the closure pressure are proposed, resulting to be consistent with theoretical asymptotic predictions and with the Hertz contact theory. The proposed model has been successfully validated through some comparisons with theoretical and experimental results available in literature.

Sommario

Nel presente lavoro si propone un modello di contatto micromeccanico a rigidzze distribuite allo scopo di descrivere in modo accurato il comportamento meccanico di due superfici rugose in contatto e soggette ad una pressione di chiusura. La zona di contatto tra due corpi elastici è assimilata ad un'interfase sottile e microfessurata, caratterizzata da una distribuzione periodica di fessure circolari non interagenti tra loro. Attraverso un approccio di omogeneizzazione combinato con una tecnica asintotica, si deducono le espressioni analitiche delle rigidzze di contatto tangenziali e normale. Si propone, inoltre, una descrizione semplificata delle aree di contatto e di non contatto in funzione della pressione di chiusura, consistente con i comportamenti limite teorici e con la teoria di contatto Hertziano. Il modello proposto è stato validato con successo attraverso una serie di confronti con risultati teorici e sperimentali disponibili in letteratura.

Keywords: interfacial contact stiffness; wavy surfaces; roughness; imperfect interfaces.

1. INTRODUCTION

Analytical and numerical modeling of contact problems related to rough surfaces can be surely considered as an open and challenging research topic, strictly associated to many industrial applications. From a computational point of view, it is possible to identify a class of modeling problems in which it is neither possible nor useful to account for a fine and detailed description of the contact regions, although contact may strongly affects, the overall mechanical response for the problem in object. In these cases, a possible

strategy is based on modeling contact features by means of equivalent stiffness and dashpot distributions at the contact nominal interface.

One of the earliest contact model for elastic rough surfaces was proposed by Greenwood and Williamson [1]. This model implies a Hertzian contact solution [2] for curved elastic nominally-flat surfaces by considering a population of non-interacting asperities following a given statistical distribution. Starting from the analytical solution of Westergaard [3], Johnson et al. [4] developed a contact model that concerns the elastic contact between a two-dimensional wavy surface and a flat plane. They found the expressions of the contact area in the asymptotic limit cases of early contact and of nearly full contact conditions. Krolkowski and Szczepec [5] provided an analytical formulation of the interface stiffness both in normal and in the tangential directions, that combines the Hertz-Mindlin theory [2], the Greenwood-Williamson contact model [1], and the Johnson model [4,6]. Their statistical approach models the contact between rough surfaces as the contact between two elastic wavy surfaces, ideally covered with asperities of spherical shape. Yoshioka and Scholz [7] proposed a contact model for elastic problems via a statistical approach that allows to account for possible oblique contact among asperities.

Many studies can be found in the specialized literature addressing experimental characterization of the mechanical behavior of rough surfaces in contact under closure-pressure conditions [8-11], providing estimates for normal and tangential contact stiffnesses. For instance, Sherif and Kossa [8] employed an experimental technique based on the evaluation of the local natural frequencies at the contact region. Gonzalez-Valadez et al. [9] proposed experimental results based on ultrasonic tests. As a matter of fact, experimental approaches highlight that: stresses are highly concentrated in the contact region and they are mainly not affected by the shape of the bodies in contact at a suitable distance from the contact area [4,6]; an hysteresis phenomenon occurs at the interface level (as a result of the plastic deformation localized at the asperity tips) in the case of cycling loads [9]; null values of the interfacial stiffnesses are achieved when the closure pressure tends to zero [9].

In this paper a spring-like contact model is proposed. Normal and tangential interfacial stiffnesses per unit area are consistently derived by coupling a homogenization approach for microcracked media under the non-interacting approximation (NIA) [12-15], and arguments of asymptotic analysis [16-19]. The model is detailed in Section 2, and its validation is provided, by comparing numerical results with both theoretical and experimental findings, in Section 3. Finally, some conclusions are traced in Section 4.

2. CONTACT MODEL

In what follows and as a notation rule (Einstein summation rule is assumed): $(\mathbf{a} \otimes \mathbf{b})_{ij} = a_i b_j$ is the dyadic product between vectors; $(\mathbf{A} \cdot \mathbf{b})_i = A_{ij} b_j$ is the tensor product between a two-rank tensor and a vector; $(\mathbf{A} * \mathbf{B})_{ik} = A_{ij} B_{jk}$ is the standard tensor product between two-rank tensors; $\mathbf{A} : \mathbf{B} = A_{ij} B_{ji}$ is the double contraction between two-rank tensors.

2.1 General framework

Let two continuous bodies, comprising of linearly elastic isotropic materials (E_i, ν_i with $i = 1, 2$, being Young modulus and Poisson ratio, respectively), be in contact via non-conforming rough surfaces. Moreover, referring to the local tangent plane π at the contact nominal interface, let K_N^C and K_T^C be the normal and tangential incremental contact stiffnesses per unit nominal contact area, defined as:

$$K_N^C = \frac{d\mathcal{F}_N}{dw} \quad ; \quad K_T^C = \frac{d\mathcal{F}_T}{ds} \quad (1)$$

where w and s are the relative displacements of far points in the normal- and tangential-to-the-contact-plane directions, respectively, and \mathcal{F}_N and \mathcal{F}_T are the normal and tangential forces transmitted through the unit contact area.

Let a local Cartesian frame related to the orthonormal basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be introduced, with \mathbf{e}_3 orthogonal to π . Moreover, let the contact surfaces be characterized by a periodical distribution of asperities, and let the randomness aspect of the roughness be regularized by assuming a simply regular periodic wavy-like

shape of the contacting surfaces in the reference configuration. The two wavy surfaces are assumed to be geometrically isotropic, with a bi-sinusoidal shape characterized by wavelength λ and amplitude Δ (with $\Delta \ll \lambda$). As a result, the tangential contact stiffness K_T^C is isotropic in the local tangent plane π . Moreover, any sliding phenomena between the nominal surfaces in contact is assumed to be prevented at the macroscale, and a frictionless behavior is locally considered at the asperity scale.

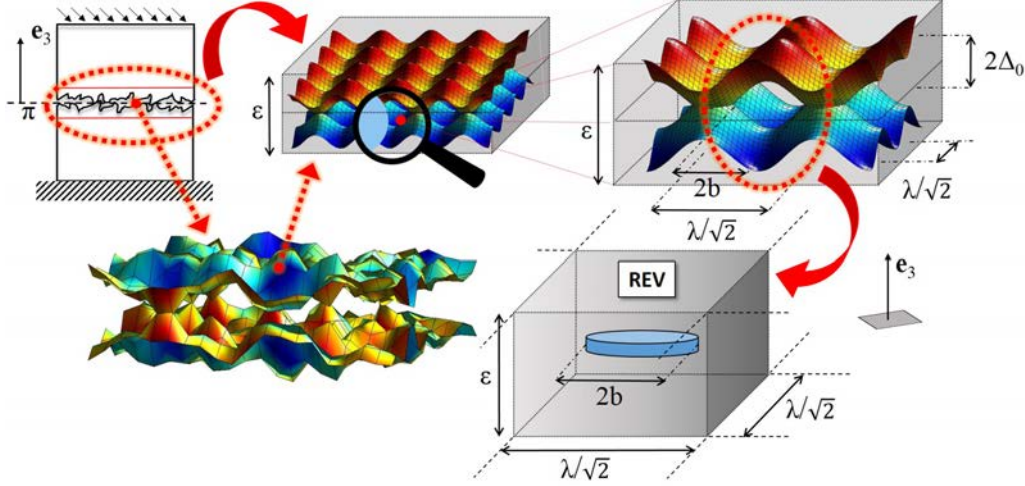


Figure 1: Exemplary sketch of the "regularized" contact/interphase zone with the identification of the REV (representative elementary volume) herein considered

As it is well established [3,4], when two non-conforming surfaces are in contact under pressure conditions, the no-contact areas can be assumed to behave as almost independent penny-shaped cracks. Accordingly, it is possible to identify a ε -thick representative elementary volume (REV) at the contact interface (Fig. 1), and to treat the whole ε -thick contact zone as an imperfect interphase characterized by a periodical distribution of non-interacting penny-shaped cracks. In the following reference is made to an isotropic interphase weakened by one family of penny-shaped microcracks only, characterized by a transversely isotropic crack distribution with symmetry axis \mathbf{e}_3 and with an average radius b . Moreover, the undamaged interphase is assumed to behave as a linearly elastic material, and crack faces undergo frictionless conditions.

The adopted imperfect-interface approach, proposed by authors in former papers [20-22], is employed to consistently derive effective mechanical properties at the contact zone, in terms of contact stiffness distributions, by coupling a NIA-based homogenization approach for microcracked media [12-15] and the matched asymptotic method [17-19,23]. Basic elements of the imperfect-interface procedure are detailed in the following.

2.2 Imperfect interface approach

2.2.1 Homogenization of the microcracked interphase

Within the framework of the NIA [15], each penny-shaped crack, embedded into a Cauchy-type stress-field $\boldsymbol{\sigma}$, does not experience mechanical interactions by surrounding cracks. Let $\mathbf{n} = \mathbf{e}_3$ be the unit vector normal to the crack middle surface Γ on π , and let \mathbf{u}^+ and \mathbf{u}^- be the displacements at the top and bottom crack boundaries, respectively. Denote also as $\mathbf{u}_{cod} = \langle \mathbf{u}^+ - \mathbf{u}^- \rangle = [\int_{\Gamma} (\mathbf{u}^+ - \mathbf{u}^-) d\Gamma] / |\Gamma|$ the average measure of the displacement jump through the crack, in the following referred to as crack opening displacement (COD) vector. In agreement with the homogenization technique employed in [12-14], and considering a plane-stress assumption, \mathbf{u}_{cod} can be expressed in terms of the stress vector

$\mathbf{T}_n = \boldsymbol{\sigma} \cdot \mathbf{n}$ as [12]:

$$\mathbf{u}_{cod} = \beta \frac{b}{E_0} [(\mathbf{n} \otimes \mathbf{n}) \cdot \mathbf{T}_n] + \gamma \frac{b}{E_0} [\mathbf{T}_n - (\mathbf{n} \otimes \mathbf{n}) \cdot \mathbf{T}_n] \quad (2)$$

where $\beta = 16(1 - \nu_0^2)/3\pi$, $\gamma = 16(1 - \nu_0^2)(1 - \frac{\nu_0}{2})^{-1}/3\pi$, and E_0 and ν_0 are the Young modulus and the Poisson ratio of the undamaged interphase, respectively. It is worth pointing out that E_0 and ν_0 can be obtained in terms of the elastic properties (E_i, ν_i with $i = 1, 2$) of the two materials in contact, as the result of a homogenization of the undamaged ε -thick REV. Thereby, the complementary elastic potential $f(\boldsymbol{\sigma})$ of the cracked material can be expressed as [14]:

$$f = f_0(\boldsymbol{\sigma}) + \Delta f = f_0 + \frac{8(1 - \nu_0^2)}{3(1 - \frac{\nu_0}{2})E_0} \rho \left\{ (\mathbf{n} \otimes \mathbf{n}) : (\boldsymbol{\sigma} * \boldsymbol{\sigma}) - \frac{\nu_0}{2} [(\mathbf{n} \otimes \mathbf{n}) : \boldsymbol{\sigma}]^2 \right\} \quad (3)$$

where

$$f_0 = \frac{1 + \nu_0}{2E_0} [(\mathbf{n} \otimes \mathbf{n}) : (\boldsymbol{\sigma} * \boldsymbol{\sigma})] - \frac{\nu_0}{2E_0} [(\mathbf{n} \otimes \mathbf{n}) : \boldsymbol{\sigma}]^2 \quad (4)$$

is the complementary elastic potential of the undamaged interphase, expressed in terms of the compliance tensor $\mathbb{S}_0 = \mathbb{S}_0(E_0, \nu_0)$, Δf is a perturbation contribution depending on microstructural crack features, and ρ is the scalar crack density, expressed in agreement with [24] by (Fig. 1):

$$\rho = \frac{b^3}{|REV|} = \frac{2b^3}{\lambda^2 \varepsilon} \quad (5)$$

Due to Eq. (3), the effective compliance tensor \mathbb{S} of the microcracked interphase can be derived as:

$$(\mathbb{S})_{ijkl} = (\mathbb{S}_0)_{ijkl} + (\Delta \mathbb{S})_{ijkl} = \frac{\partial^2 f}{\partial \sigma_{ij} \partial \sigma_{kl}} \quad (6)$$

where $\Delta \mathbb{S}$ is the contribution compliance tensor associated to Δf . Accordingly, the effective moduli of the cracked interphase in the normal (N) and tangential-to-interface (T) directions are obtained as:

$$E_N = E_0 \left[1 + \frac{16(1 - \nu_0^2)}{3} \rho \right]^{-1} ; \quad G_{NT} = G_0 \left[1 + \frac{8(1 - \nu_0)}{3(1 - \frac{\nu_0}{2})} \rho \right]^{-1} \quad (7)$$

2.2.2 Asymptotic expansion method and interface stiffnesses

Let the REV thickness ε (see Fig. 1) be considered as a small parameter. Accordingly, an asymptotic expansion with respect to ε can be conveniently performed starting from the effective mechanical properties in Eqs. (7) of the cracked ε -thick interphase. The asymptotic technique herein adopted and briefly outlined in the following, refers to the so-called *matched asymptotic expansion* method employed in [17-19].

In detail, the elastic equilibrium problem for the interphase region and for the bodies in contact can be formulated in terms of asymptotic expansions with respect to ε of the Cauchy-type stress field ($\boldsymbol{\sigma}$), of the strain field (\mathbf{e}), and of the displacement field (\mathbf{u}):

$$\begin{aligned} \boldsymbol{\sigma}^\varepsilon &= \boldsymbol{\sigma}^0 + \varepsilon \boldsymbol{\sigma}^1 + o(\varepsilon) \\ \mathbf{u}^\varepsilon &= \mathbf{u}^0 + \varepsilon \mathbf{u}^1 + o(\varepsilon) \\ \mathbf{e}(\mathbf{u}^\varepsilon) &= \varepsilon^{-1} \mathbf{e}^{-1} + \mathbf{e}^0 + O(\varepsilon) \end{aligned} \quad (8)$$

Moreover, reference is herein made to a *soft* interface assumption [19] within the interphase domain. Thereby, the material stiffness tensor in the ε -thick interphase region can be expanded as:

$$\mathbb{C}^\varepsilon = \varepsilon \mathbb{C} + o(\varepsilon) \quad (9)$$

where $\mathbb{C} = \mathbb{S}^{-1}$ is computed in agreement with Eqs. (7). By performing a rescaling process of the equilibrium problem for the domains under investigation (bodies in contact and interphase) and by adopting a matching procedure as described by Rizzoni et al. [19], the *soft* interface law, relating the stress vector \mathbf{T}_n at the nominal contact interface (with $\mathbf{n} = \mathbf{e}_3$) and the displacement jump $[\mathbf{u}]$ at the interface, can be expressed as:

$$\mathbf{T}_n = \text{diag} [K_T \quad K_T \quad K_N] \cdot [\mathbf{u}] \quad (10)$$

where the tangential (T) and normal (N) stiffnesses result in:

$$K_T = \frac{3 E_0 \lambda^2 (2 - \nu_0)}{64 b^3 (1 - \nu_0^2)} \quad ; \quad K_N = \frac{3 E_0 \lambda^2}{32 b^3 (1 - \nu_0^2)} \quad (11)$$

2.3 Effective contact stiffnesses

In agreement with Johnson et al. [4,6], the average radius b of each penny-shaped microcrack is assumed to be dependent on the closure nominal pressure \bar{p} (Fig. 1), such that $0 \leq 2b \leq \frac{\lambda}{\sqrt{2}}$.

In detail, $2b \leq \frac{\lambda}{\sqrt{2}}$ when $\bar{p} < p^*$ and $2b \rightarrow 0^+$ when $\bar{p} \rightarrow p^*$, where p^* is the average value of the nominal pressure which brings the surfaces into complete contact, in the following referred to as complete contact pressure. According to the Hertzian elastic contact theory it is defined as:

$$p^* = \sqrt{2\pi} E' \frac{\Delta_0}{\lambda} \quad (12)$$

with $(E')^{-1} = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}$ being the reduced Hertzian modulus and Δ_0 being the amplitude of the bi-sinusoidal shape modeling rough surfaces at the reference configuration. The contact regions are described by introducing the contact radius a , such that $a \rightarrow 0^+$ when $\bar{p} \rightarrow 0^+$ and $a \rightarrow \frac{\lambda}{\sqrt{2}}$ when $\bar{p} \rightarrow p^*$. With reference to the sketch in Fig. 2, contact areas are described as circular (i.e., contact point condition) and no-contact zones are almost square in shape when $\bar{p} \rightarrow 0^+$.

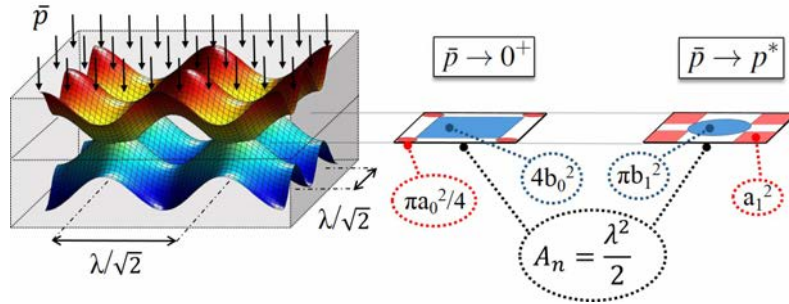


Figure 2: Outline of the asymptotic behaviors of contact and no-contact areas

On the contrary, in the near complete closure condition ($\bar{p} \rightarrow p^*$), the contact areas are assumed to be almost square in shape and the no-contact area as quasi-circular.

Let a_0 and b_0 (respectively, a_1 and b_1) be the values of the no-contact and contact radii, respectively, when $\bar{p} \rightarrow 0^+$ (resp., $\bar{p} \rightarrow p^*$). In agreement with asymptotic estimates provided by Johnson et al. [4], the following relationships are assumed to hold:

$$a_0 \left(\frac{\bar{p}}{p^*} \right) = \frac{\lambda}{\sqrt{2}} \sqrt{\left(\frac{3 \bar{p}}{8\pi p^*} \right)^{2/3}} \quad (13)$$

$$b_0 \left(\frac{\bar{p}}{p^*} \right) = \sqrt{\frac{\lambda^2}{8} \left[1 - \pi \left(\frac{3}{8\pi} \frac{\bar{p}}{p^*} \right)^{2/3} \right]} \quad (14)$$

$$a_1 \left(\frac{\bar{p}}{p^*} \right) = \frac{\lambda}{2\sqrt{2}} \sqrt{1 - \frac{3}{2\pi} \left(1 - \frac{\bar{p}}{p^*} \right)} \quad (15)$$

$$b_1 \left(\frac{\bar{p}}{p^*} \right) = \sqrt{\frac{3\lambda^2}{4\pi^2} \left(1 - \frac{\bar{p}}{p^*} \right)} \quad (16)$$

The contact and no-contact radius evolution with \bar{p} is simply described by the following area-based weighted averages:

$$a \left(\frac{\bar{p}}{p^*} \right) = \sqrt{\frac{\pi a_0^2 \left(1 - \frac{\bar{p}}{p^*} \right) + 4a_1^2 \frac{\bar{p}}{p^*}}{\pi \left(1 - \frac{\bar{p}}{p^*} \right) + 4 \frac{\bar{p}}{p^*}}} \quad (17)$$

$$b \left(\frac{\bar{p}}{p^*} \right) = \sqrt{\frac{4b_0^2 \left(1 - \frac{\bar{p}}{p^*} \right) + \pi b_1^2 \frac{\bar{p}}{p^*}}{4 \left(1 - \frac{\bar{p}}{p^*} \right) + \pi \frac{\bar{p}}{p^*}}} \quad (18)$$

It is worth pointing out that Eqs. (17) and (18) satisfy the consistency condition:

$$A_c + A_{nc} = A_n = \frac{\lambda^2}{2} \quad (19)$$

where $A_c = a^2 \left[\pi \left(1 - \frac{\bar{p}}{p^*} \right) + 4 \frac{\bar{p}}{p^*} \right]$ is the contact area and $A_{nc} = b^2 \left[4 \left(1 - \frac{\bar{p}}{p^*} \right) + \pi \frac{\bar{p}}{p^*} \right]$ is the no-contact one for a given value \bar{p} of the nominal closure pressure. Moreover, they recover the asymptotic relationships previously introduced (Eqs. (13-16)), and in particular the following limits for $\bar{p} \rightarrow 0^+$ hold:

$$a \rightarrow a_0 \rightarrow O\left(\left(\frac{\bar{p}}{p^*}\right)^{\frac{1}{3}}\right) \quad ; \quad b \rightarrow b_0 \rightarrow \frac{\lambda}{2\sqrt{2}} \quad (20)$$

Following the classical Hertzian theory for the elastic contact case, theoretical estimates of interfacial contact stiffnesses in normal (N) and tangential (T) directions can be expressed as [26]:

$$K_N^{th} = \frac{4E'a}{\lambda^2} \quad ; \quad K_T^{th} = K_N^{th} \frac{2(1-\nu_0)}{(2-\nu_0)} \quad (21)$$

It immediately results that, due to the limit conditions in Eqs. (20), stiffnesses expressed by Eqs. (11) do not recover the physical behavior associated to stiffness null values when $\bar{p} \rightarrow 0^+$ and described by the Hertzian theory (Eqs. (21)). In order to enforce such a physical constraint, the effective contact stiffnesses are arranged as:

$$K_i^C = K_i \left[1 - e^{-\gamma_i^0 \left(\frac{\bar{p}}{p^*} \right)^{1/3}} \right] \quad i = N, T \quad (22)$$

where K_N and K_T are obtained from the imperfect interface approach (Eqs. (11)). It is worth to remark that stiffnesses expressed by Eq. (22) straight recover the asymptotic behavior for $\bar{p} \rightarrow 0^+$ provided by the Hertzian estimate (Eqs. (21) and (20)) when model parameters γ_N^0 and γ_T^0 assume the theoretical values:

$$\gamma_{N,th}^0 = (9\pi)^{-1/3} \quad ; \quad \gamma_{T,th}^0 = 2\gamma_{N,th}^0 \frac{2(1-\nu_0)}{(2-\nu_0)^2} \quad (23)$$

In particular, for $\nu_0 = 0.3$ it results: $\gamma_{N,th}^0 = 0.33$, $\gamma_{T,th}^0 = 0.32$.

3. RESULTS AND DISCUSSIONS

The experimental results obtained by Gonzalez-Valadez et al. [9] are herein chosen to validate the proposed model. They provide experimental measures, by means of ultrasonic pulser-receivers, of interfacial contact stiffnesses for steel specimens in contact through rough nominally-flat surfaces under closure pressure conditions. The specimens were subjected to loading-unloading cycles of compressive pressure in a hydraulic frame operating in loading control mode. The load is applied in a quasi-static way, up to the nominal pressure value of 400 MPa. Steel specimens are characterized by the following mechanical properties: $E = 200$ GPa and $\nu = 0.3$. Accordingly, the Hertzian reduced modulus value is $E' = 109.89$ GPa, and $E_0 = E$ and $\nu_0 = \nu$. Moreover, in agreement with data proposed in [9], contact rough surfaces in the reference configuration can be modeled as regularized shapes (see Fig. 1) characterized by $\lambda = 130$ μm and $\Delta_0 = 1.58$ μm .

A comparison procedure among numerical results based on the proposed model and experimental data by Gonzalez-Valadez et al. [9] relevant to the 11-th loading cycle is carried out, deriving (via a least-squares fitting procedure) the optimal values for parameters γ_N^0 and γ_T^0 (Fig. 3).

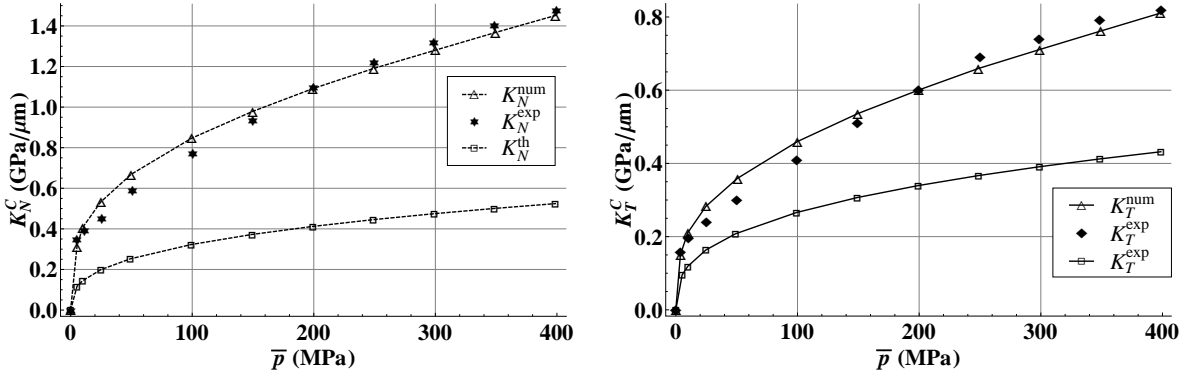


Figure 3: Comparison among numerical and experimental [9] results for normal (on the left) and tangential (on the right) contact stiffnesses vs. closure pressure \bar{p} . Hertz-based theoretical predictions are also provided

As a result, the best-fitting values are: $\gamma_{N,num}^0 = 0.96$, $\gamma_{T,num}^0 = 0.58$. It is worth pointing out that this numerical estimates for γ_N^0 and γ_T^0 are in the same order of magnitude of the corresponding Hertz-based theoretical ones: $\gamma_{N,th}^0 = 0.33$, $\gamma_{T,th}^0 = 0.32$.

Previously-proposed results are obtained by referring to the complete contact pressure p^* introduced in Eq. (12). The latter strictly holds in elastic regime. Nevertheless, due to localization mechanisms associated to tips plastic deformation and fatigue effects, the pressure that brings the surfaces into a complete contact condition can be retained as a function of the pressure loading history, as well as of the number of loading cycles. Accordingly, a suitable description of the actual closure pressure can be introduced as:

$$p_h^* = h p^* = h \sqrt{2\pi} E' \frac{\Delta_0}{\lambda} \quad (24)$$

where $h \leq 1$ is a history-based correction parameter.

As a really first approximation, in the case of loading cycles characterized by the same maximum value of the closure pressure, a measure of h can be put in the form $h = \frac{\Delta}{\Delta_0}$, with Δ a measure of the actual amplitude for the wavy surfaces.

With reference to the experimental results by Gonzalez-Valadez et al. [9] the following estimates can be consistently considered: $\Delta = 1.18$ (at the 11-th loading cycle) and $h = 0.75$. By adopting p_h^* computed

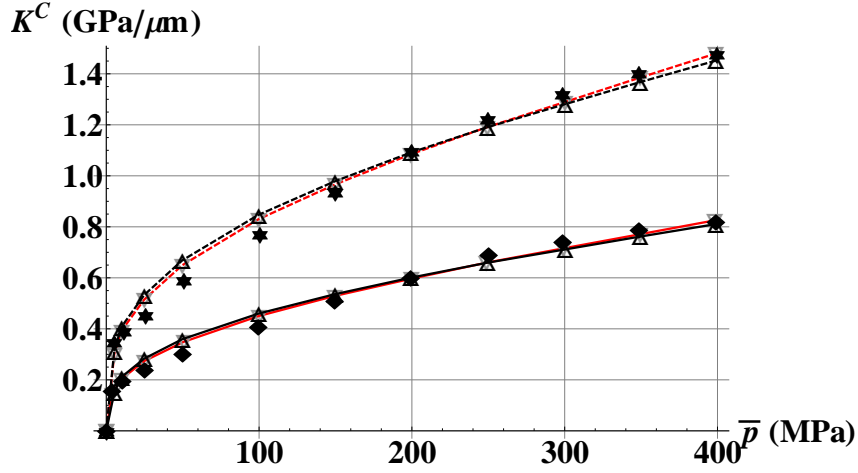


Figure 4: Numerical and experimental contact stiffnesses vs. normal pressure \bar{p} , computed considering both the Hertzian-based closure pressure p^* (the same notation rule in Fig. 3 applies) and the corrected one p_h^* (red curves, $--\nabla-- K_N^{num}$ and $—\nabla— K_T^{num}$)

with the previously-introduced value of h , the numerical predictions of the contact stiffnesses remain in good agreement with the benchmark experimental results, when best-fitting values of parameters γ_N^0 and γ_T^0 are: $\gamma_{N,num}^0 = 0.83$ and $\gamma_{T,num}^0 = 0.51$. Thereby, a history-based correction of the closure pressure leads to a consistent reduction of the γ^0 -type parameters towards their theoretical prediction (Fig. 4).

Many studies on contacting rough surfaces [1,2,4,5] revealed that the ratio of tangential to normal stiffness is solely dependent on the Poisson ratio of the contacting materials. Under the assumption that the two bodies in contact are made by the same material, it results in:

$$\frac{K_T^C}{K_N^C} = A \phi(\nu) \quad (25)$$

where $0.5 \leq A \leq 2$ is a constant and $\phi(\nu)$ is a function of the Poisson ratio ν . In the case of the classic Hertz-Mindlin contact theory [1,2,5], $A = 2$ and $\phi(\nu) = (1 - \nu)/(2 - \nu)$.

Many approaches are available in literature, providing relationships identical to that in Eq. (25), characterized by the same function $\phi(\nu)$ as in the Hertz-Mindlin theory but with different values of A . Sherif and Kossa [8] found $A = \pi/2$. Yoshioka and Scholz [7] obtained the approximated estimate $A = 0.71$. Moreover, following the contact model by Baltazar et al. [10], coefficient A can be expressed as $A = \frac{2\xi}{\Psi}$, where ξ and Ψ are correction factors accounting for the geometrical misalignments with respect to the shear and longitudinal directions, respectively. The resulting values of the tangential to normal stiffness ratio for the above cited models are plotted in Fig. 5, in comparison with the available experimental data, [9] and with present numerical results.

It is worth noting that the experimental curves show a significant dependence of the stiffness ratio on the closure pressure, that is successfully reproduced by the proposed model.

4. CONCLUSIONS

In the present paper a new micromechanical method for obtaining the interfacial stiffnesses of rough surfaces in contact under closure pressure condition is proposed. To this aim a homogenization technique for microcracked media [12-15] and an asymptotic expansion method [17,19] are consistently combined in the framework of an imperfect interface model. Normal and tangential-to-interface contact stiffnesses have been analytically established. Comparisons among present results with theoretical predictions and available experimental data highlighted soundness and effectiveness of such an approach. Moreover, proposed results proved the model to be able in catching the proper dependence of the contact stiffness ratio on the closure pressure, fully in agreement with the experimental evidence.

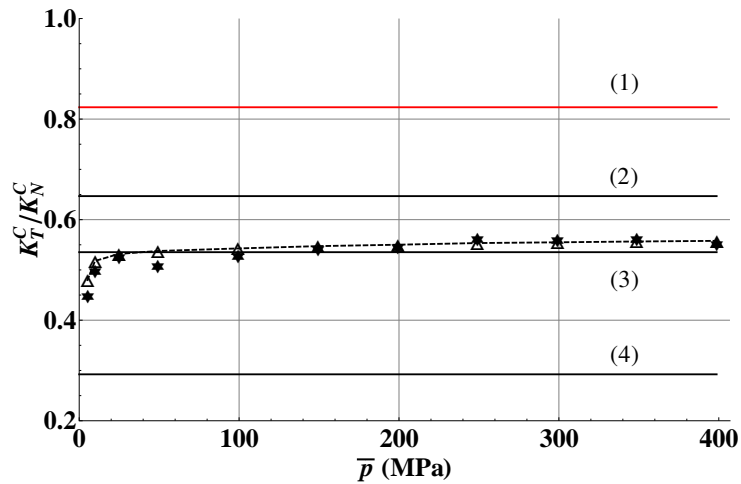


Figure 5: Tangential-to-the-normal stiffness ratio as function of the closure pressure. Comparison among present results ($-- \triangle --$, computed with p^*) and both experimental (by [9], \star) and theoretical prediction: (1) Hertz-Mindlin [2], (2) Sherif and Kossa [8], (3) Baltazar et al. [10], (4) Yoshioka and Scholz [7]. Results relevant to the model by Baltazar et al. [10] have been computed considering $\xi = 0.65$ and $\Psi = 1$

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