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*A MODELING OF MICHELSON - MORLEY INTERFEROMETER
IN A PLATONIC FOUR-DIMENSIONAL SPACE*

*UNE MODÉLISATION DE L'INTERFÉROMÈTRE DE MICHELSON - MORLEY
DANS UN ESPACE QUADRIDIMENSIONNEL PLATONICIEN*

Alain Jégat

Abstract

To illustrate the article hal-01165196 (« A Platonic model for the special theory of relativity ») is proposed here a modeling of the famous Michelson - Morley experiment in the late 19th century whose result led to venture the hypothesis of the invariance of the speed of light c in any inertial reference frame .

Considering the movement of the earth, this experiment was to detect a variation in the speed of light over paths of identical length, but whose travel times were expected different. However experience has shown that travel times are always identical.

This modeling provides a geometrical explanation for this result in a Platonic four-dimensional space.

Résumé

Pour illustrer l'article hal-01081576 (« Un modèle platonicien pour la théorie de la relativité restreinte ») est proposée ici une modélisation de la célèbre expérience de Michelson-Morley réalisée à la fin du 19^{ème} siècle dont le résultat a conduit à avancer l'hypothèse de l'invariance de la célérité c de la lumière dans le vide, dans tout référentiel galiléen.

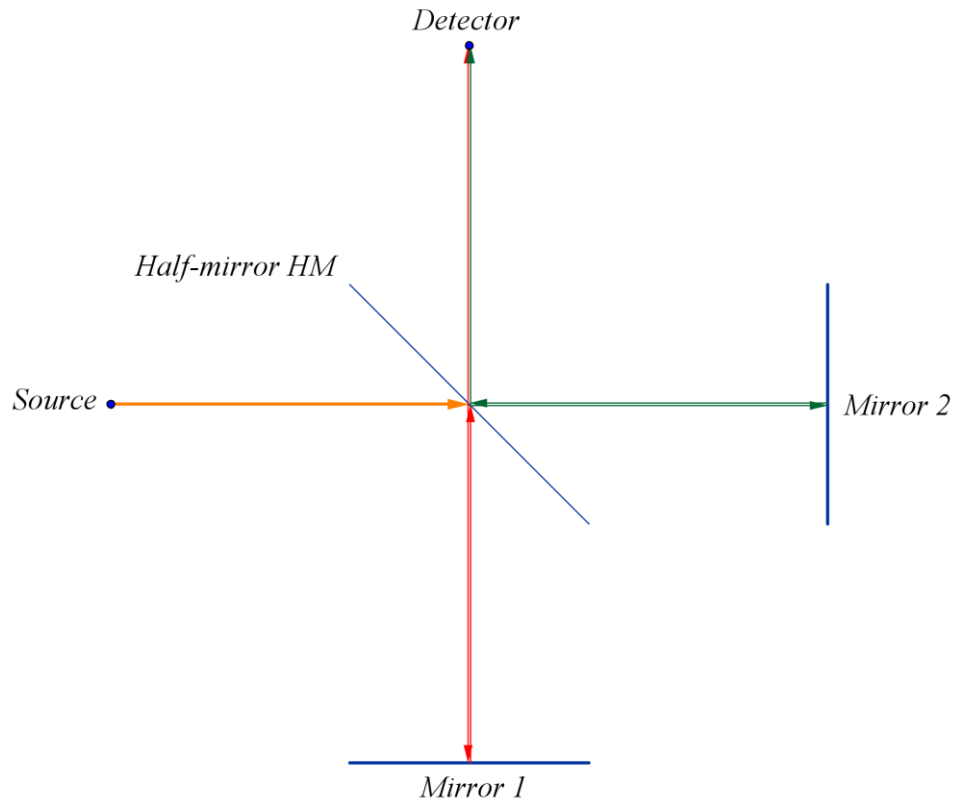
Compte-tenu du mouvement de la Terre, cette expérience devait déceler une variation de la vitesse de la lumière sur des trajets de longueurs identiques, mais dont les durées de parcours étaient attendues différentes. Or l'expérience a montré que les temps de parcours sont toujours identiques.

Cette modélisation propose une explication géométrique de ce résultat dans un espace quadridimensionnel platonicien.

1. A brief reminder of the experiment

A light wave L is splitted by a half mirror HM into two sub-beams which converge after passing through two different pathways (see diagram \textcircled{D} below).

Considering the movement of the earth, this experiment was to detect a variation in the speed of light over paths of identical length, but whose travel times were expected different. However experience has shown that travel times are always identical.



2. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

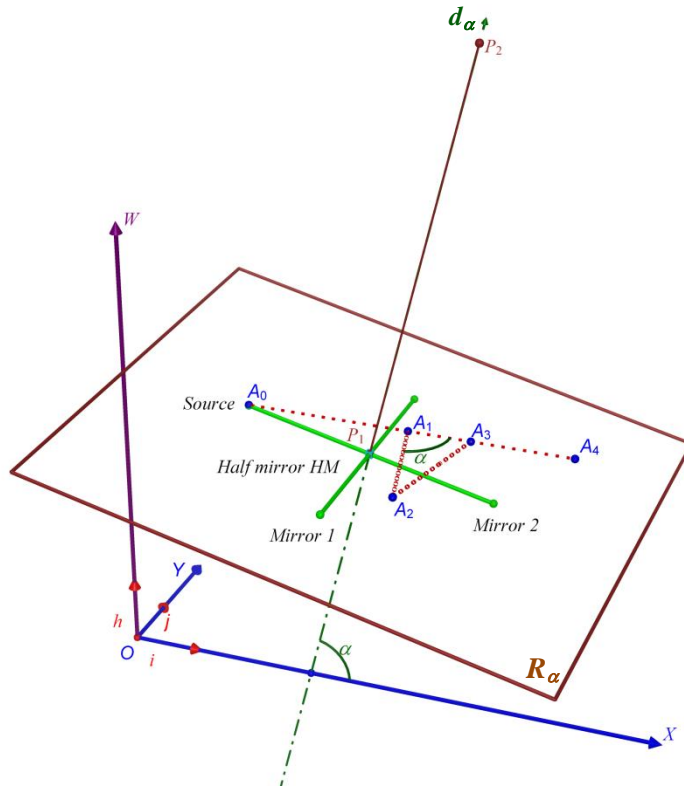
« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pré-publication hal-01165196, version 1).

$(O, \vec{i}, \vec{j}, \vec{k}, \vec{h})$ is a frame for the four-dimensional Euclidean space whose axes are denoted (OX) , (OY) , (OZ) , (Ow) ; the direction of the projection is that of the vector \vec{h} .

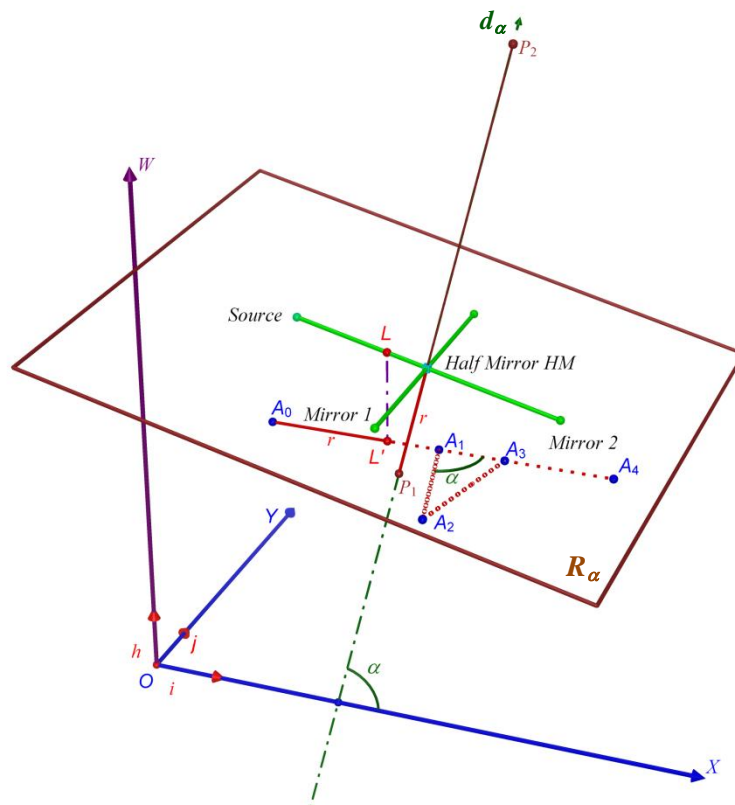
3. Modeling

We can model this situation as follows: the experimental device belongs to the reference frame R_α moving in the direction d_α (see diagrams below).

At the beginning of the experiment, the half mirror HM is at the point P_1 and the source is at the point A_0 .



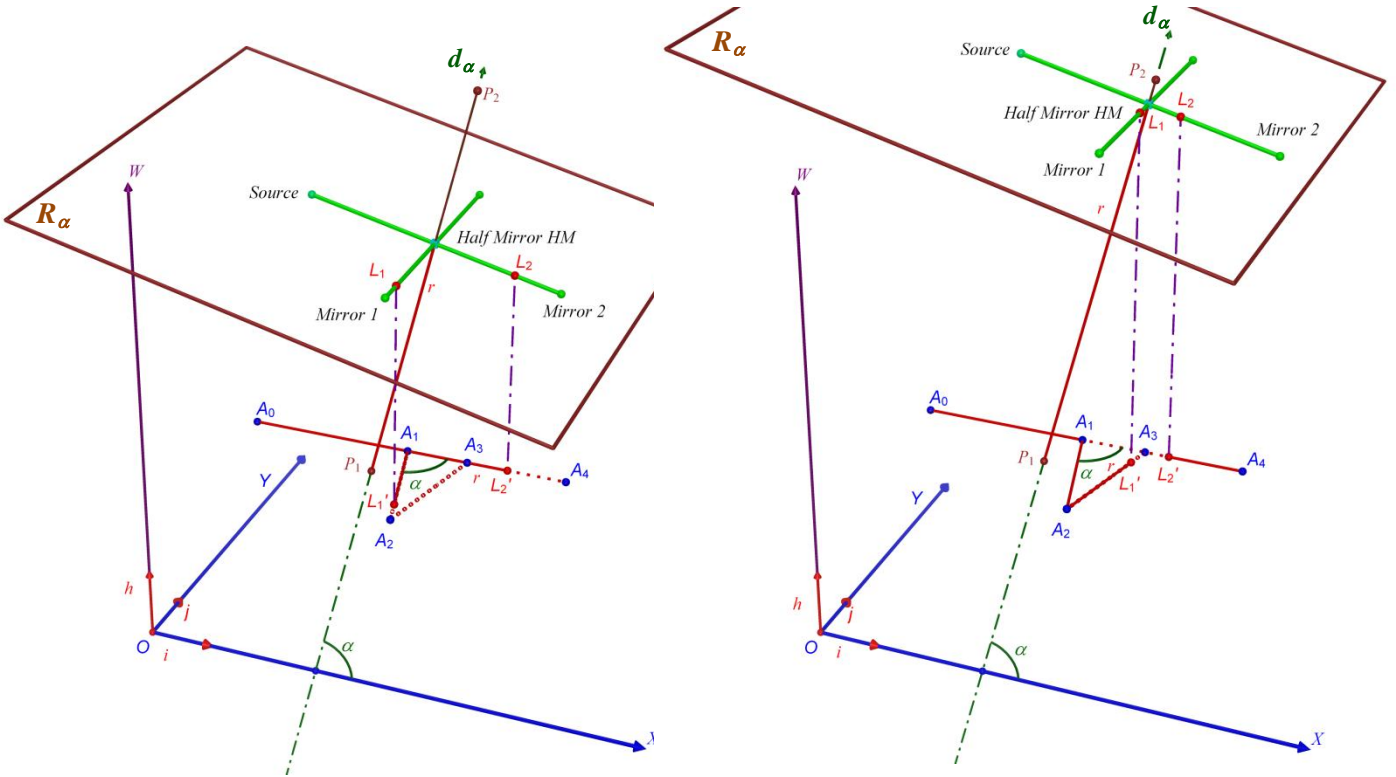
The light wave emitted by the source, whose path perceived by the reference frame R_α is the one indicated in the introduction (see diagram \textcircled{D}), actually travels the following paths (represented by the dotted lines), in a hyperplane orthogonal to the direction of projection \vec{h} :



- The initial beam travels the segment A_0A_1 (which corresponds projectively in the reference frame R_α to the path *Source-HM*). Note that during this travel (and the following), the distance r traveled by L' is equal to the distance r traveled by HM .

Designating by l the arm length *Source-HM* we have: $A_0A_1 = \frac{l \sin \alpha}{1 - \cos \alpha}$.

(See detailed calculations at the end of the article, particularly about the angle of reflection $\widehat{A_3A_1A_2}$.)



- Then the sub-beam L'_1 travels the path $A_1A_2-A_2A_3$ (which, taking account of the movement of the reference frame R_α , corresponds projectively for the sub-beam L_1 to the round trip *HM-Mirror1* then *Mirror1-HM*); with $A_1A_2 = A_2A_3 = \frac{l}{\sin \alpha}$.

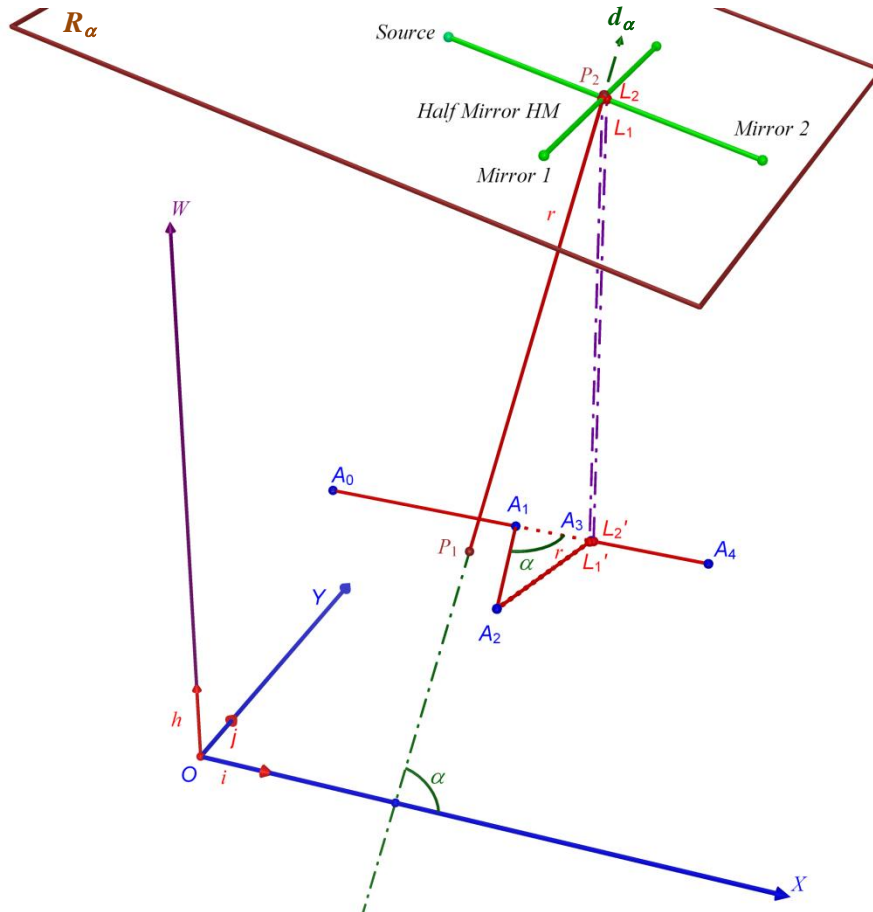
- While the sub-beam L'_2 travels the path $A_1A_4-A_4A_3$ (which, taking account of the movement of the reference frame R_α , corresponds projectively for the sub-beam L_2 to the round trip *HM-Mirror2* then *Mirror2-HM*);

with $A_1A_4 = \frac{l \sin \alpha}{1 - \cos \alpha}$ and $A_4A_3 = l \tan\left(\frac{\alpha}{2}\right)$.

Both sub-beams L'_1 and L'_2 meet at A_3 (corresponding projectively into R_α to the meeting of the sub-beams L_1 and L_2 at the new position of the half mirror HM), after having traveled a distance Δr equal to P_1P_2 , traveled by the reference frame containing the device.

$$\text{With } \Delta r = A_0A_1 + A_1A_4 + A_4A_3 = A_0A_1 + A_1A_2 + A_2A_3 = P_1P_2 = l \left(\frac{2}{\sin \alpha} + \frac{\sin \alpha}{1 - \cos \alpha} \right).$$

(The last part of the path to the detector being traveled by the beam finally reconstituted presents it, no real interest.)



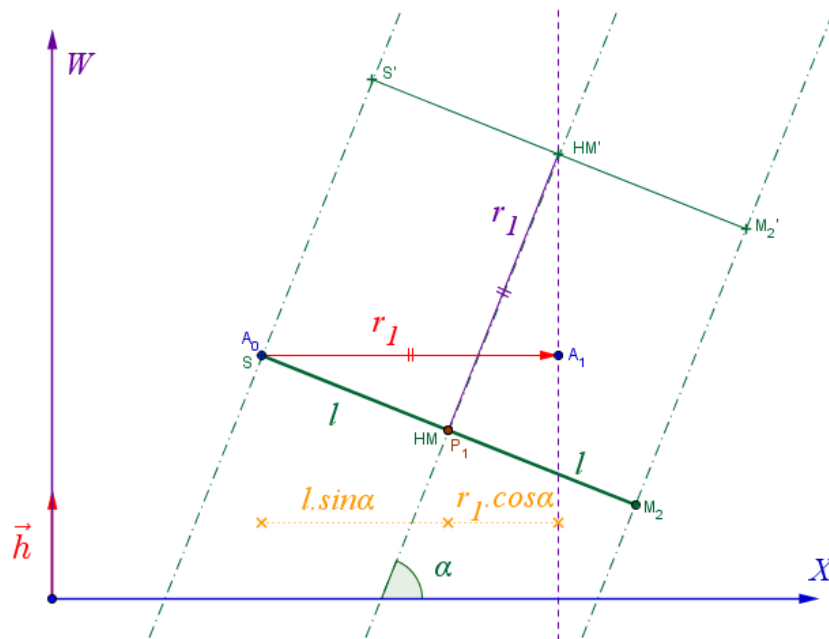
4. Conclusion

The equations thus established, true regardless of the angle α (non-multiple of π), reflect the fact that when an observer located in HM perceives the return of the sub-beam L_1 , he simultaneously perceives the return of the sub-beam L_2 , whatever the speed of the Galilean reference frame related to the earth.

Measurements done by Michelson and Morley fit that simultaneous perception, whose result has led to venture the hypothesis of the invariance of the speed of light c in any inertial frame ... Then, subsequently, the development of the Platonic model that offers here a purely geometrical point of view of this result.

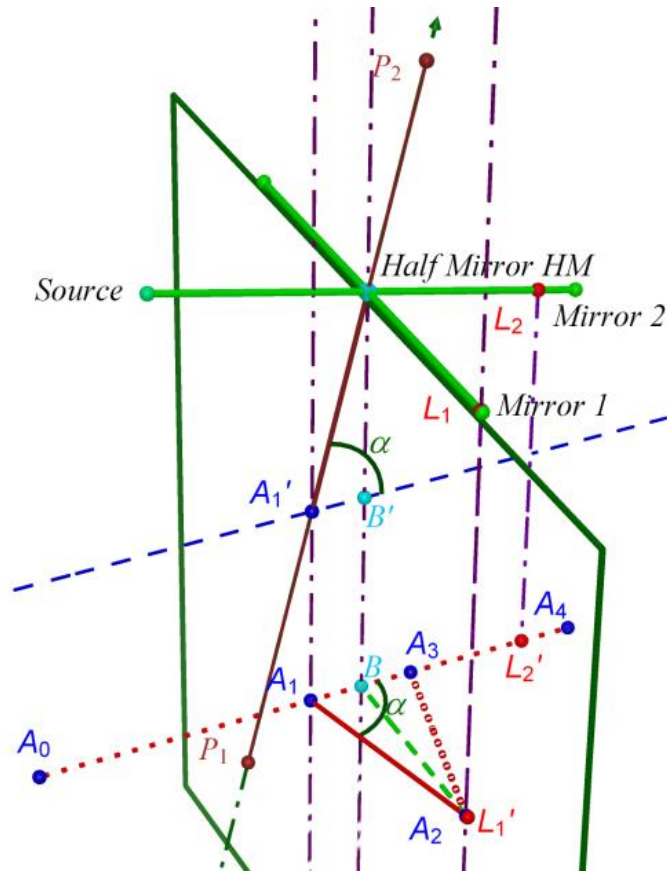
5. Additional Calculations

①



From : $r_1 = l \sin \alpha + r_1 \cos \alpha$, with $r_1 = A_0A_1$, we obtain : $A_0A_1 = \frac{l \sin \alpha}{1 - \cos \alpha}$.

②



This diagram shows the modeling of the experiment when the sub-beam L_1 is perceived at the mirror 1; which corresponds to the positioning of L'_1 at A_2 .

If A'_1 denotes the position of the half mirror HM while the beam is at A_1 we have $A'_1 HM = A_1 A_2 = \Delta r$.

Furthermore, considering the plane $L'_1 L_1 HM$, orthogonal to $(A_0 A_4)$ and the intersecting points B and B' shown in the diagram, we obtain $A_1 B = A'_1 B'$, then $\widehat{BA_1 A_2} = \widehat{B'A'_1 L_1 S} = \alpha$ (congruent triangles) *.

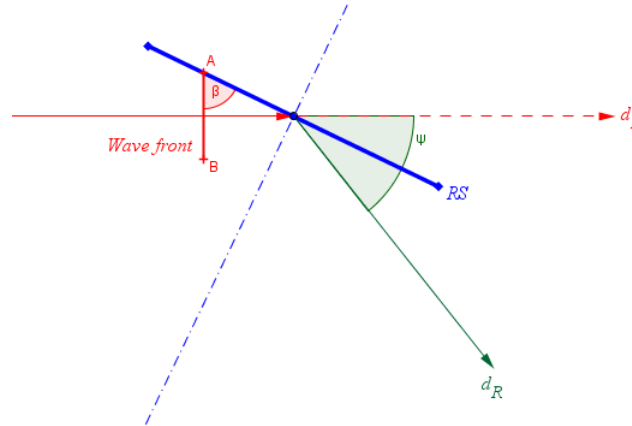
Then we obtain $\Delta r = A_1 A_2 = \frac{l}{\sin \alpha}$ and, for reasons of symmetry, $A_2 A_3 = \frac{l}{\sin \alpha}$.

To finish, $A_4 A_3 = A_1 A_4 - A_1 A_3 = \frac{l \sin \alpha}{1 - \cos \alpha} - 2 \frac{l}{\tan \alpha} = l \tan \left(\frac{\alpha}{2} \right)$.

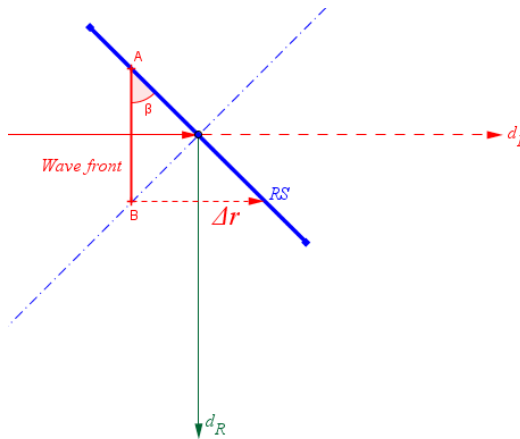
(* See note ③ for a physical justification of the result $\widehat{A_3 A_1 A_2} = \alpha$.)

⑤ We can argue the following physical reasoning for measuring the angle of reflection of the sub-beam L'_1 :

- in a conventional manner, a light beam encountering a reflecting surface RS is deflected and we have: $\psi = (\vec{d}_R, \vec{d}_I) = \pi - 2\beta$, where β is a measure of the angle formed by an infinitesimal wavefront and the reflective mirror.

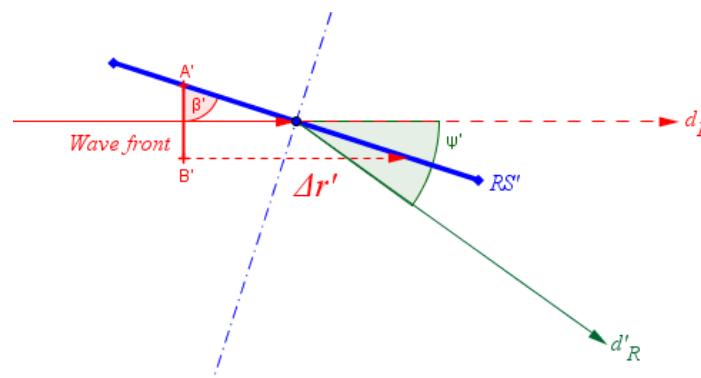


- In the experiment of Michelson-Morley, we have $\beta = \frac{\pi}{4}$ and $\psi = \frac{\pi}{2}$, angles measured into R_α .

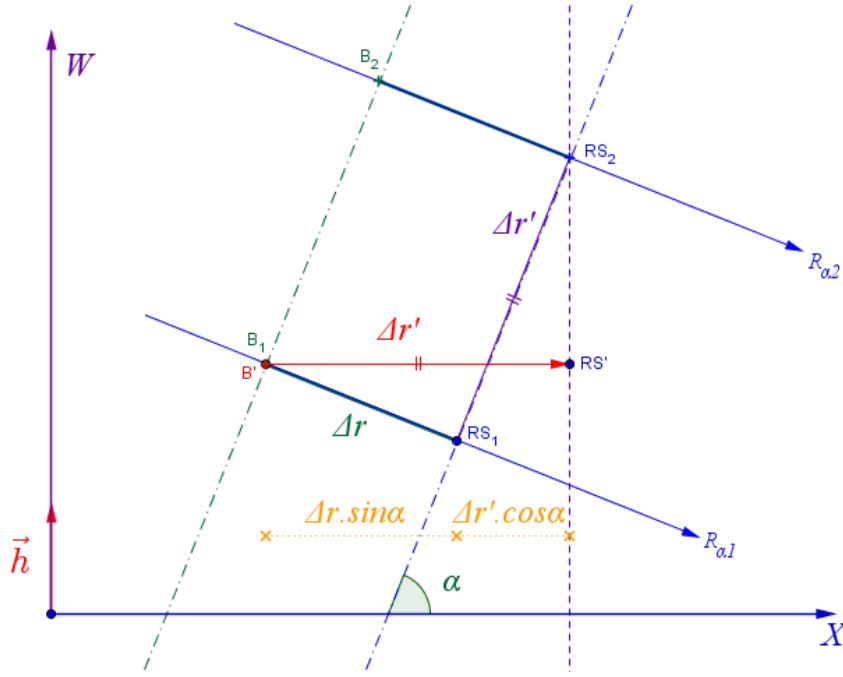


Thus, if AB denotes the infinitesimal width of the wavefront at the time of its first contact with RS and Δr the distance traveled by the beam to the second contact, one has in this case : $AB = \Delta r$.

- In fact, in the four-dimensional projective space, we have the following situation:
 - ✓ at the first contact, the perceived beam into R_α at A is in fact at A' and the one perceived at B is actually at B' ; with $A'B' = AB = \Delta r$.



- ✓ Taking account of the movement of the reference frame R_α to which is linked the device, we have then $\Delta r' = \frac{\Delta r \cdot \sin \alpha}{1 - \cos \alpha} = \frac{\Delta r}{\tan\left(\frac{\alpha}{2}\right)}$.



Thus, when the wavefront is in contact (projectively) for the second time with the reflective surface, it has traveled in 4D space the distance $\Delta r'$ and the corresponding angle β' is such that:

$$\tan \beta' = \frac{\Delta r'}{A'B'} = \frac{\Delta r'}{\Delta r} = \frac{1}{\tan\left(\frac{\alpha}{2}\right)};$$

$$\text{i.e. } \beta' = \frac{\pi}{2} - \frac{\alpha}{2}.$$

If we use the classical law of reflection recalled at the beginning of this remark, we therefore have the following result:

$$\psi' = \pi - 2\beta' = \alpha.$$

6. Bibliography

Jégat, Alain (2014) , " De l'Allégorie de la Caverne à la Relativité Restreinte", Les Éditions du Net, ISBN: 978-2-312-02454-7.

Staley, Richard (2009), "Albert Michelson, the Velocity of Light, and the Ether Drift", Einstein's generation. The origins of the relativity revolution, Chicago: University of Chicago Press, ISBN 0-226-77057-5.

Maxwell, James Clerk (1880), "On a Possible Mode of Detecting a Motion of the Solar System through the Luminiferous Ether", *Nature* 21: 314–5.

Miller, A.I. (1981). Albert Einstein's special theory of relativity. Emergence (1905) and early interpretation (1905–1911). Reading: Addison–Wesley. p. 24.
ISBN 0-201-04679-2.

Feynman, R.P. (1970), "The Michelson–Morley experiment (15-3)", *The Feynman Lectures on Physics 1*, Reading: Addison Wesley Longman, ISBN 0-201-02115-3.

Schumacher, Reinhard A. (1994). "Special Relativity and the Michelson-Morley Interferometer". *American Journal of Physics* 62: 609–612.

Einstein A. (1916), *Relativity: The Special and General Theory*, New York: H. Holt and Company.

Jeroen van Dongen (2009), "On the Role of the Michelson–Morley Experiment: Einstein in Chicago", *Archive for History of Exact Sciences* 63 (6): 655–663.

Swenson, Loyd S., Jr. (2013) [1972]. *The Ethereal Aether: A History of the Michelson-Morley-Miller Aether-drift Experiments, 1880–1930*. University of Texas Press. ISBN 978-0-292-75836-0.

Michelson, A. A.; et al. (1928). "Conference on the Michelson–Morley Experiment Held at Mount Wilson, February, 1927". *Astrophysical Journal* 68: 341–390.