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# A classical Force of Gravitation as an alternative to MOND?

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**Abstract:** In this paper we present an alternative and classical force of gravitation without dark matter or dark energy hypotheses. This model has common points with MOND but may be considered as an alternative to this theory. Indeed the laws of classical physics are respected. This model is a consequence of a study of forces which lead to conic trajectories and we compare our results with the Bertrand's theorem. At end an important part of this work is to propose an experiment on the Earth, in order to valid or invalid our results.

*Keywords:* Central force; conic; dark matter; force of gravitation; galaxies; modified gravitation; PACS Number:04.70.Kd

## 1. Introduction

It is well known that an important problem of the contemporary physics is to describe the celestial mechanics for large distances. For example, it appears that the classical laws of gravitation failed to describe the “flat” curves of rotation of the spirals galaxies without add, inside these galaxies, an important amount of a mysterious matter, called “dark matter” (1-3). Another important problem of the celestial mechanics is to explain the expansion, at an increasing rate, of the universe [4]. Indeed, classical theories of gravitation are always attractive. To solve this contradiction, physicians have to suggest, that another mysterious energy, called this time “dark energy”, exists inside the Universe [5, 6].

Problem is that there is no proof of the reality of the “dark matter” and the “dark energy” despite several important experiments on the Earth [7]. Moreover the most accurate study to try to detect them in the space seems also have failed [8].

Consequently, an important contemporary way of research is to modify the classical laws of Gravitation. Several alternative theories are in competition as for example, the  $f(R)$  theories of gravity [9], the scalar-tensor gravity theories [10, 11] or the Modified Newtonian dynamics (MOND) theory [12]. But it appears today that no one of these theories does unanimity. In particular in the case of MOND the laws of classical physics are not respected.

This point is the reason for what we are looking for such a law. In two previously papers, we presented a possible way to obtain an alternative and non relativistic force of gravitation [13, 14] with respect for classical physics. This work can be considered as an alternative to MOND and possesses common points with this theory. In particular we introduced an acceleration which is a constant of the Universe. Therefore as distinguished from MOND our original hypothesis is that trajectories of celestial bodies are always conics. This assumption led us to study with a mathematical point of view the family of forces which leads to conic trajectories and we present here a part of our results.

Indeed it appears that at the 19<sup>th</sup> century a theorem has been written about this problem, the Bertrand's theorem [15]. This work proved that there are only two central forces for which all orbits radially bounded are closed, the force of Newton and the Hooke's. These forces leads to conic trajectories with the difference that the Newton's is directed towards one of the foci and the Hooke's towards the

geometrical centre of the conic. The proof of this theorem has continued to draw attention and several different demonstrations has been recently published [16, 17].

Our own attempt to obtain a force of gravitation different from the Newton's in the case of large distances led us to study this theorem and to compare our results with its predictions. It is the reason for what in this paper we will present in a first part our mathematical results about the conics by comparison with this theorem. In a second part we will present the force of gravitation we obtained. At end we will purpose an experiment in order to valid or invalid our model.

## 2. Mathematical study of the forces which leads to conic trajectories

### 2.1 Theory

To obtain these forces we use the equation of the acceleration in polar coordinate. Therefore as distinguished from the Binet's equation we consider not only the radial acceleration but also the tangential acceleration. This original method (at our knowledge) allows obtaining all the forces which lead to conic trajectories and not only the central forces. We present here this method which has been published elsewhere [13, 14].

As usual in celestial mechanics we will use the polar system of coordinate  $(F; \vec{e}_R; \vec{e}_\theta)$  where  $F$  (foci of the conic) is the origin of the repair,  $r$  is the radial distance to the origin with the relation

$$F\vec{M} = r.\vec{e}_R$$

And the angle  $\theta$  is measured from the periapsis of the orbit. In this repair the acceleration is given by the classical relations

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_R + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

But the orbital shape is more concisely described by the reciprocal  $u = \frac{1}{r}$  as a function of  $\theta$  .

And by using the relations

$$\begin{aligned} \dot{r} &= \frac{d}{dt} r = \frac{d}{dt} \frac{1}{u} = -\frac{\dot{u}}{u^2} \\ \dot{u} &= \frac{d}{dt} u = \frac{d\theta}{dt} \frac{du}{d\theta} = \dot{\theta}u' \\ \dot{u}' &= \frac{d}{dt} u' = \frac{d\theta}{dt} \frac{d}{d^2\theta} u = \dot{\theta}u'' \end{aligned}$$

We obtain a generalization of the Binet's equation.

$$\vec{a} = \frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2 + 2u'^2u\dot{\theta}^2 - u'u^2\ddot{\theta}}{u^4}\vec{e}_R + \frac{-2u'u^2\dot{\theta}^2 + u^3\ddot{\theta}}{u^4}\vec{e}_\theta$$

Our method to obtain conic trajectories is to solve by substitution the system of equation

$$\begin{cases} a_R = \frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2}{u^4} - \frac{u'}{u}a_\theta = Y(u) - \frac{u'}{u}Z(u) \\ a_\theta = Z(u) \end{cases}$$

Where

$$Z(u) = \frac{-2u'u^2\dot{\theta}^2 + u^3\ddot{\theta}}{u^4}$$

And

$$Y(u) = \frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2}{u^4}$$

We introduce a function  $f(u)$  definite by

$$Y(u) = -A.f(u)$$

Where  $A$  is constant. To obtain  $r(\theta)$  as a conic, we have to solve a differential equation as

$$u''+u = B$$

Where  $B$  is a second constant. Consequently we have now to introduce a relation between  $Y(u)$  and  $\dot{\theta}$ . This relation is

$$\dot{\theta} = Cu\sqrt{f(u)}$$

Where  $C$  is a constant of the motion. Consequently

$$u''\dot{\theta}^2 - u\dot{\theta}^2 = -Au^2f(u)$$

And

$$u''+u = B = \frac{A}{C^2}$$

This differential equation leads now to the classical solution

$$r(\theta) = \frac{p}{1 + e\cos\theta}$$

The parameter  $p$  of the conic is

$$p = a(1 - e^2) = \frac{C^2}{A}$$

Where  $e$  is the eccentricity and  $a$  the semi major axis. Thus we obtain

$$C = \sqrt{A}\sqrt{a(1 - e^2)}$$

We have now to determine the tangential component of the acceleration and by using

$$\ddot{\theta} = \frac{C.\dot{r}}{r^2\sqrt{f}} \left[ \frac{1}{2}f'r - f \right]$$

Where

$$f' = \frac{d}{dr}f(r)$$

We obtain

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{2}\frac{Cf'\dot{r}}{\sqrt{f}} + C\frac{\dot{r}}{r}\sqrt{f}$$

Consequently our family of force (per unit mass) is with respect for the Newton's law of dynamics

$$\vec{F} = -Af.\vec{e}_R + \dot{r}^2 \left[ \frac{1}{2}\frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + C\dot{r}\sqrt{f} \left[ \frac{1}{2}\frac{f'}{f} + \frac{1}{r} \right] \vec{e}_\theta$$

Or, more simply

$$\vec{F} = -Af.\vec{e}_R + \dot{r}^2 \left[ \frac{1}{2}\frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + r\frac{dr}{d\theta}\dot{\theta}^2 \left[ \frac{1}{2}\frac{f'}{f} + \frac{1}{r} \right] \vec{e}_\theta \quad (2)$$

Where  $A$  and  $C$  are two constants. Their physical dimensions depend on the choice of  $f(r)$ . It is also interesting to determine the magnitude of the total speed

$$V = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{A}\sqrt{\frac{(2a-r)r}{a}}\sqrt{f(r)} \quad (3)$$

## 2.2 Nature of the force

The force given by equation (2) can be decomposed in two forces: the first part given by

$$\vec{F}_A = -Af \cdot \vec{e}_R$$

Is naturally always directed to  $F$ , foci of the conic. The second part of this force given by

$$\vec{F}_B = \dot{r}^2 \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + r \frac{dr}{d\theta} \dot{\theta} \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_\theta$$

Is tangent to the trajectory. Indeed, if we consider that the direction vector of tangent line to a curve is given, in polar coordinate, by

$$\vec{V}_D = \frac{dr(\theta)}{d\theta} \vec{e}_R + r(\theta) \vec{e}_\theta$$

And if we do the vector product  $\vec{F}_B \wedge \vec{V}_D$  we obtain

$$\vec{F}_B \wedge \vec{V}_D = \left( \frac{dr}{d\theta} \right)^2 \dot{\theta}^2 \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] r - \frac{dr}{d\theta} r \dot{\theta}^2 \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \frac{dr}{d\theta} = 0$$

Thus we prove that  $\vec{F}_B$  and  $\vec{V}_D$  are parallel. Consequently the force  $\vec{F}$  can be decomposed in two forces: a force  $\vec{F}_A$  which is directed to the foci and a force  $\vec{F}_B$  which is tangential to the trajectory.

## 2.3. The force is directed to $F$ : the force of Newton

If the force is directed to the foci  $F$  of the conic then the tangential part  $\vec{F}_B$  of the force has to be a null vector. Consequently we write

$$\vec{F}_B = \dot{r}^2 \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + r \frac{dr}{d\theta} \dot{\theta} \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_\theta = \vec{0}$$

The only solution is given by the relation

$$\frac{1}{2} \frac{f'}{f} + \frac{1}{r} = 0$$

The solving of this differential equation is

$$f(r) = \frac{C_1}{r^2}$$

Where  $C_1$  is constant. But this one does duplication with our previously constant  $A$  and consequently we can choose

$$C_1 = 1$$

We obtain

$$f(r) = \frac{1}{r^2}$$

And the force becomes simply

$$\vec{F} = -\frac{A}{r^2} \vec{e}_R$$

In the case of a point particle which orbits around a center of mass the constant is simply given by

$$A = GM$$

Where  $G$  is the universal constant of gravity and  $M$  the mass of the center of mass.

Consequently we prove here that it is only one possible force which is always directed to  $F$ . This point is in agreement with the Bertrand's theorem but can be considered complementary. Note that our way to obtain this result seems original.

#### 2.4. The force is directed to $O$ : the force of Hooke

To determine the family of forces which are directed to  $O$  we introduce a point, called  $I$  which is located between  $O$  and  $F$ . This distance  $FI$  is called  $\Delta$ . If the force is directed to  $I$  then the vector product

$$\vec{IM} * \vec{F} = \vec{0}$$

By noticing that, in our repair  $F\vec{e}_R\vec{e}_\theta$  the vector  $\vec{IM}$  is given by

$$\vec{IM} \begin{vmatrix} \Delta \cos \theta + r \\ -\Delta \sin \theta \end{vmatrix}$$

We obtain

$$\Delta \sin \theta . F_R + (\Delta \cos \theta + r) . F_\theta = 0$$

With the relations specific to the conics

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \text{ and } \dot{r} = \frac{eC\sin\theta}{a(1-e^2)} r\sqrt{f} \quad (4)$$

We obtain

$$\Delta = \frac{ae(2f + rf')}{f'(r-a) + 2f}$$

To respect our condition, and by noticing that the distance  $FO = ea$  we write that

$$\Delta = ea$$

Consequently we have to solve the differential equation

$$\frac{(2f + rf')}{f'(r-a) + 2f} = 1$$

The equation becomes

$$f' = 0$$

Consequently the function  $f(r)$  has to be constant. We called this one  $k$  and by using

$$f(r) = k$$

We obtain the expression of our force

$$\vec{F} = -kA \left[ \frac{a}{r}(1-e^2) + \frac{r}{a} - 1 \right] \vec{e}_R + ke\sin\theta \vec{e}_\theta$$

Its magnitude is given by

$$\|\vec{F}\| = A \frac{k}{a} \sqrt{a^2(1-e^2) + r^2 + a^2 - 2ar}$$

By noticing that the distance  $OM$ , where  $M$  is the point-particle and  $O$  the center of the conic, is given by

$$OM = \sqrt{a^2(1-e^2) + r^2 + a^2 - 2ar}$$

We obtain

$$\|\vec{F}\| = A \frac{k}{a} OM$$

This force is consequently a force of Hooke and its constant factor is

$$K = \frac{k}{a} A$$

We have proved here that it is only one central force which is located at  $O$  and which leads to a conic trajectory. As previously this mathematical demonstration is in agreement with the Bertrand's theorem but the demonstration seems original. Moreover note that the force of Hooke can lead to parabolic or hyperbolic trajectories, and not only circular or elliptic trajectories.

We present these results on the Figure 1.  $\vec{F}_1$  is the force of Newton,  $\vec{F}_2$  the Hooke's.

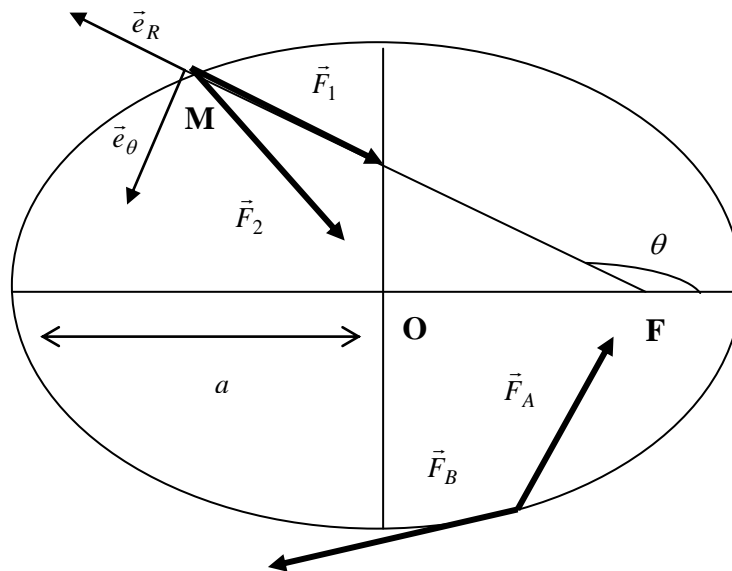


Figure 1. Representation of the forces

### 2.5. Central forces which lead to conic trajectories

To obtain a central force we are looking for the family of functions  $f(r)$  which leads to  $\Delta$  as a constant. Consequently we write the equation

$$\frac{d}{dr} \Delta = 0$$

This relation leads to

$$\frac{ea^2(2ff'' - 3f'^2)}{(f'(r+a) - 2f)^2} = 0$$

And the solving is

$$f(r) = \frac{1}{(C_1 r + C_2)^2}$$

Where  $C_1$  and  $C_2$  are constant. The position of  $I$  is given by

$$\Delta = \frac{aC_2 e}{C_2 + aC_1}$$

And the force becomes (Figure 2).

$$\vec{F} = A \frac{ar(C_2 - rC_1) - C_2(a^2(1 - e^2) + r^2)}{ar(C_1r + C_2)^3} \vec{e}_R + A \frac{eC_2}{(C_1r + C_2)^3} \text{Sin } \theta \cdot \vec{e}_\theta$$

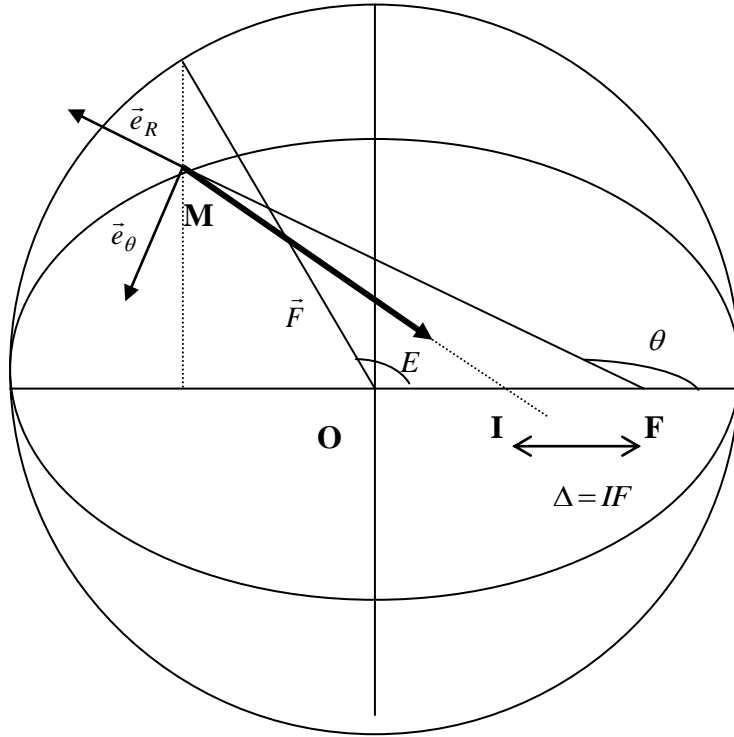


Figure 2. Representation of the central force

We can distinguish two limiting cases:

Firstly, if  $C_2 = 0$ : In this case the force becomes  $\vec{F} = -\frac{A}{C_1^2 r^2} \vec{e}_R$  and  $\Delta = 0$ : the center of the force is the foci of the conic. By choosing  $C_1 = 1$  we obtain the force of Newton.

Secondly, if  $C_1 = 0$ : In this case  $\Delta = ea$  and the center of the force is located at  $O$ , center of the conic. The force is given by

$$\vec{F} = A \frac{ar - a^2(1 - e^2) - r^2}{arC_2^2} \vec{e}_R + A \frac{e}{C_2^2} \text{Sin } \theta \cdot \vec{e}_\theta$$

This force is the Hooke's.

We can also determine a constant angular momentum at  $I$ . In order to study it, we begin to determine the angular momentum at  $F$ , foci of the conic. We use the relation

$$\vec{L}_F = r \cdot \vec{e}_R * (\dot{r} \cdot \vec{e}_R + r \dot{\theta} \cdot \vec{e}_\theta) = r^2 \dot{\theta} \cdot \vec{e}_Z$$

And by using equations (4) we obtain

$$\vec{L}_F = \frac{Cr}{C_1r + C_2} \cdot \vec{e}_Z$$

The angular momentum at  $I$  is given by



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$$\vec{L}_I = \vec{L}_F + I\vec{O} * (\dot{r}.\vec{e}_R + r\dot{\theta}.\vec{e}_\theta)$$

By noticing that

$$I\vec{O} = \Delta(\text{Cos } \theta.\vec{e}_R - \text{Sin } \theta.\vec{e}_\theta)$$

$\vec{L}_I$  becomes

$$\vec{L}_I = \vec{L}_F + \Delta[r\dot{\theta}.\text{Cos } \theta + \dot{r}\text{Sin } \theta]\vec{e}_Z$$

And

$$\vec{L}_I = \frac{Cr}{C_1r + C_2} + \frac{aC_2e}{C_2 + aC_1} [r\dot{\theta}.\text{Cos } \theta + \dot{r}\text{Sin } \theta]\vec{e}_Z$$

By using the relations specific to the conics (4) we obtain

$$\vec{L}_I = \frac{aC}{C_2 + aC_1} \vec{e}_Z \quad (m^2 s^{-1})$$

Consequently, the angular momentum is constant for all the forces and the trajectory is plane. Moreover we verify that our force is well central.

We can also giving the equation of time of the motion. As usual, we write this equation by using the eccentric anomaly  $E$ . Indeed, we have the relation

$$\dot{\theta} = \dot{E} \frac{\sqrt{1-e^2}}{1-e\text{Cos}E}$$

But we saw that

$$\dot{\theta} = \frac{C}{r} \sqrt{f(r)}$$

Consequently we obtain

$$\dot{E} \frac{\sqrt{1-e^2}}{1-e\text{Cos}E} = \frac{C}{r} \frac{1}{C_1r + C_2}$$

With the relation

$$r = a(1 - e\text{Cos}E)$$

We obtain

$$\dot{E}[C_1a(1 - e\text{Cos}E) + C_2] = \frac{C}{a\sqrt{1-e^2}} = \sqrt{\frac{A}{a}}$$

And by integration

$$C_1a(E - e\text{Sin}E) + C_2E = \sqrt{\frac{A}{a}}t$$

Consequently

$$E(C_1a + C_2) - eC_1a\text{Sin}E = \sqrt{\frac{A}{a}}t$$

By a factorization

$$(C_1a + C_2) \left[ E - e \frac{C_1a}{C_1a + C_2} \text{Sin}E \right] = \sqrt{\frac{A}{a}}t$$

And

$$E - e \frac{C_1a}{C_1a + C_2} \text{Sin}E = \frac{1}{(C_1a + C_2)} \sqrt{\frac{A}{a}}t = wt$$

Thus the mean motion is given by

$$w = \frac{1}{(C_1 a + C_2)} \sqrt{\frac{A}{a}}$$

If the force is the Newton's we obtain the equation of time of Kepler

$$E - e \sin E = \sqrt{\frac{A}{a^3}} t$$

And if the force is the Hooke's we obtain simply

$$E = \sqrt{\frac{A}{a C_2^2}} t$$

Consequently we see that the equation of time and the angular momentum can be described with simple expressions. Note that more generally these mathematical results seem original. Therefore it appears with respect for the Bertrand's theorem that the trajectories can't be closed. In this paper we won't study this point which will be the subject of another paper.

### **3. Alternative classical force of gravitation**

#### ***3.1. Our assumptions***

We will use now our mathematical results to try to build a possible force of gravitation. This one is building with respect for several assumptions. We list them here.

First of all we suppose that the classical physics can describe with a good approximation the motion of the stars and the actual dynamics of the universe. Indeed, if it is well known that the classical physics leads only to an approximation of the reality, it appears that this physics can in certain cases describe with a great precision the motion of celestial bodies, in particular when the speeds and the density of mass are low. It appears that these conditions are met at the periphery of the galaxies, and inside the clusters of galaxies.

Secondly, as we wrote it previously our fundamental idea is that the first law of Kepler is respected whatever the gravitational system (solar systems, galaxies, clusters of galaxies,..): if a point-particle interacts with a center of mass its trajectory is always a conic (circle, ellipse, parabola or hyperbola).

At end we suppose that the force of gravitation is central. Indeed, this kind of force, well known in classical physics is generally used to describe action-at-a-distance. This point makes different from our previously work. Indeed it appears that with this condition the choice of possible force is drastically reduced. In fact only one mathematical form is possible.

By taking into consideration these three assumptions and our mathematical results we will explore the possibility that the force of Hooke is the force of gravitation in the case of large distances. Indeed it appears that the only force which is directed toward the foci of the conic is the Newton's and we suppose that the force we are looking for is central. Moreover, our work has shown that we can link these two forces with a gradual evolution.

To explore this possibility we study two limiting cases of motion: firstly, the circular motion, in order to determine the curve of rotation of this force. Secondly, the parabolic motion, in order to determine the dynamics of the universe we expect.

### 3.2. Curve of rotation of the galaxies: determination of $K$

If our idea is correct, we should obtain a flat curve of rotation by a correct choice of the constant factor  $K$ . In order to determine it we study as usual the circular motion. Indeed, this motion describes with a good approximation the motion of the stars around the center of galaxies. In this particular motion, the eccentricity of the conic is given by  $e=0$  and  $a=r$ . Consequently, the force given by relation (2) becomes simply

$$\vec{F} = -kA\vec{e}_R$$

The curve of rotation is obtained by written the equality of the acceleration and the force

$$-\frac{V^2}{r} = -kA$$

Where  $V$  is the speed of the point-particle. We obtain

$$V = \sqrt{kAr}$$

And consequently

$$V = \sqrt{kAa}$$

In order to obtain a flat curve of rotation, and with respect for the physical dimension of  $k$  ( $m^{-2}$ ) we see that this factor has to be dependant of the semi-major axis, as

$$k = \frac{1}{r_0 a} \quad (5)$$

Where  $r_0$  is constant ( $m$ ). Moreover, we can determine this constant, because we have to respect the Tully Fisher law [18]. Indeed, this empirical relation suggests a correspondence between the visible mass of galaxies and the velocity as

$$V_\infty^4 \propto (M)$$

Where  $M$  is the total mass of the galaxy and  $V_\infty$  the constant speed of the stars. We saw that  $A$  has to be proportional to the mass thus

$$V_\infty = \sqrt{\frac{A}{r_0}} \propto A^{1/4}$$

And consequently

$$r_0 \propto A^{1/2}$$

This condition, with respect for physical dimensions, leads to write the relation

$$r_0 = \sqrt{a_0 A}$$

Where  $a_0$  is an acceleration. This acceleration has to be constant in the entire universe, and can be linked with the Milgrom's acceleration used in MOND [12]. With this relation  $V$  is constant for all the stars at the periphery of one galaxy and is in agreement with the Tully-Fisher law. We see that by using a constant acceleration, which has been introduced in another theory, we can describe the curve of rotation of galaxies without dark matter hypothesis. Note that unlike several other alternative theories our model is in agreement with classical physics.

By using relation (5) the force can be written

$$\vec{F} = -\frac{A}{ar_0} \left[ \frac{a}{r} (1 - e^2) + \frac{r}{a} - 1 \right] \vec{e}_R + \frac{A}{ar_0} e \sin \theta \cdot \vec{e}_\theta$$

We can now use this expression to study the parabolic motion.

### 3.3. Consequence on the parabolic motion: dynamics of the Universe

We see here the predictions of our model about the dynamics of the universe. To do it we study the limiting case of parabolic motion. The eccentricity is given by

$$e = 1$$

And, in the simplest case, the angular  $\theta$  is determined by

$$\theta = 0$$

Consequently our force becomes simply

$$\vec{F} = -\frac{A}{ar_0} \left[ \frac{r}{a} - 1 \right] \vec{e}_R$$

We can see that the sign of this force can be positive or negative, and is depending on the ratio  $\frac{r}{a}$ .

Indeed, if  $r < a$  the force is repulsive. However, it also appears that this force can be attractive if  $r > a$ . With this result we try to build a simple model about the dynamics of the Universe. We consider, for example, the interaction of two clusters of galaxies between themselves. We approximate their motion with a parabolic motion, and the force of gravitation is the force we present here.

If the clusters move away from one another it is possible that this force is repulsive. Consequently, their relative speed will increase, at least for a time. It appears that if our idea is correct this kind of interactions is actually majority inside the Universe. However when the distance between the clusters progresses this force becomes a day attractive. Consequently, we can easily describe the actual expansion of the Universe at an increasing rate, and simultaneously build a model of the expansion of the Universe compatible with the classical idea of “Big Bang / Big Crunch”. This simple model doesn’t need naturally a “dark energy” hypothesis, because the Universe is considerate as a kind of harmonic oscillator.

### 3.4. A general force of gravitation?

We saw that our model allows explaining the “flat” curve of rotation of the galaxies without dark matter. Moreover, it allows explaining the expansion of the Universe without dark energy. Consequently it seems that our hypothesis could describe these two different problems unlike majority of other alternative theories. However, it is certain that the force of Hooke can’t be the dominant force in the case of small distance, as solar systems for example. Consequently, the nature of force has to change gradually when the acceleration decreases.

To describe this evolution, we use our previously work about the central forces. We saw that the forces given by the relation

$$\vec{F} = A \frac{ar(C_2 - rC_1) - C_2(a^2(1 - e^2) + r^2)}{ar(C_1r + C_2)^3} \vec{e}_R + A \frac{eC_2}{(C_1r + C_2)^3} \sin \theta \cdot \vec{e}_\theta$$

Are central and directed toward a point  $I$  definite by

$$FI = \frac{aC_2e}{C_2 + aC_1}$$

We can determine the constants for our limiting cases: if  $r \ll r_0$  the force is the Newton's and consequently  $C_2 = 0$ . If  $r \gg r_0$  the force is the Hooke's consequently  $C_2 = \sqrt{r_0 a}$  and  $C_1 = 0$ . We can list our results in the table 1.

		$C_1$	$C_2$
Small distances	$a \ll r_0$	1	0
Large distances	$a \gg r_0$	0	$\sqrt{r_0 a}$
General case		$g\left(\frac{a}{r_0}\right)$	$\sqrt{r_0 a} \left[ 1 - g\left(\frac{a}{r_0}\right) \right]$

Tab 1. Coefficients of the central force

In this table the function  $g\left(\frac{a}{r_0}\right)$  is defined by the following relation

$$0 \leq g\left(\frac{a}{r_0}\right) \leq 1$$

Indeed, by using this relation we obtain a graduation between our two limiting cases. Moreover the distance  $FI$  becomes

$$FI = \frac{ae\sqrt{r_0 a}(1-g)}{\sqrt{r_0 a}(1-g) + ag}$$

We see that

$$\text{Limit}FI_{g \rightarrow 1} = 0$$

And

$$\text{Limit}FI_{g \rightarrow 0} = ae$$

Consequently the center of force  $I$  is well located between  $O$  and  $F$  (Figure 2.).

#### 4. A possible experiment inside the solar system: a test on the free fall motion

Our model indicates that in our solar system, a part of the force of gravitation should be due to the force of Hooke. Naturally, this part should be a tiny amount of the force of gravitation, because the Newton's law is well verified. Moreover, this part should increase with the distance to the sun, and be more important at the periphery of the system than at its center. At end, this part won't modify the trajectories of celestial bodies, because these trajectories are always conic.

We can use this reasoning to propose an experiment inside our solar system or perhaps at the surface of the Earth. Indeed if trajectories are not modified, it isn't the case of the equation of time. Consequently it appears that a classical experiment, the study of the free fall motion, could help to valid or invalid our model. We will now try to prove it by using our equations about the conics.

##### 4.1. Equation of the free fall motion

We saw in part 2. that the radial acceleration is given by

$$a_R = -Af + \dot{r}^2 \left[ \frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right]$$

And, by using the relations specifics to the conics

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \quad \text{and} \quad \dot{r} = \frac{dr}{dt} = \frac{d\theta}{dt} \frac{dr}{d\theta} = \dot{\theta} \frac{dr}{d\theta}$$

We obtain, after simplification

$$a_R = -\frac{1}{2} \frac{A}{ar} (2f(r)(a^2(1-e^2) + r^2 - ar) + f'(r)(ra^2(1-e^2) + r^3 - 2r^2a))$$

And, in the case of free fall motion ( $e=1$ ) the radial acceleration becomes

$$\ddot{r} = -\frac{1}{2} \frac{A}{a} (2f(r)(r-a) + f'(r)(r^2 - 2ra)) \quad (5)$$

If we consider that the Newton's force is the correct force of gravitation, valid in all the cases, then the radial acceleration has to be given by

$$\ddot{r} = -\frac{A}{r^2}$$

Consequently we can write a differential equation

$$-\frac{A}{r^2} = -\frac{1}{2} \frac{A}{a} (2f(r)(r-a) + f'(r)(r^2 - 2ra))$$

And after simplification

$$-\frac{1}{r^2} = -\frac{1}{2a} (2f(r)(r-a) + f'(r)(r^2 - 2ra))$$

The solving of this equation is

$$f(r) = \frac{1}{r^2} \left[ \frac{-2a + C_1 r}{-2a + r} \right]$$

Where  $C_1$  is constant. With this relation and by using (3) the total speed becomes

$$V = \sqrt{\frac{A(2a - C_1 r)}{ar}}$$

However this speed has to be equal to zero if

$$r = 2a$$

(Case of the free fall motion without initial speed). Thus we have to write that  $C_1 = 1$  and

$$f(r) = \frac{1}{r^2}$$

With this expression of  $f(r)$  the force is the Newton's. Consequently, we can't obtain a force different from this force which leads to the same acceleration, in the case of the free fall motion. This point indicates that if a force of gravitation different from the Newton's is valuable for large distances we should detect an anomaly around this law. As we wrote it previously, this anomaly should be a tiny proportion of it but it is possible to think that with a contemporary precision, we could detect it. Note that this test is only a modern version of the experiments of Galilee. Moreover note that it should detect all variations around the Newton's law and not only our model. At end it appears that this kind of anomaly has been suspected in the past [19].

#### 4.2. Our prediction

If our model is correct we should obtain in the case of free fall motion a radial acceleration given by

$$\ddot{r} = A \frac{ar(C_2 - rC_1) - C_2r^2}{ar(C_1r + C_2)^3}$$

In the solar system  $C_2 \approx 0$  by using the relation

$$C_1 = g \text{ and } C_2 = \sqrt{r_0 a}(1 - g)$$

We do a series for  $g \rightarrow 1$  and we obtain

$$\ddot{r} = -\frac{A}{r^2} + (1 - g)A\sqrt{\frac{r_0}{a}} \left[ \frac{2a - r}{r^3} \right]$$

The first term is naturally the Newton's. The additive term we should obtain is

$$a_D = (1 - g)A\sqrt{\frac{r_0}{a}} \left[ \frac{2a - r}{r^3} \right]$$

Note that this term is equal to zero at the beginning of the free fall motion. Indeed at this point we have the relation  $r = 2a$ . Note also that if  $r > 2a$  this term becomes negative. At end if our model is correct we should detect a small modification around the Newton's law given by the previously relation.

## 5. Discussion

### 5.1. Physical interpretation

In this paper, we tried to show that the force of Hooke can describe several astronomical observations. However we can try to discuss this assumption with a physical point of view.

Firstly, we can say that the force of Hooke exist already in the gravitation. It is the force which interacts with a point-particle inside a sphere where the density of mass is uniform. This is the consequence of the theorem of Gauss and this result is well known since centuries. For example, in a sphere the acceleration is given by

$$\ddot{r} = -\frac{GM_{\text{int}}}{r^2} \cdot \bar{e}_R$$

Where  $M_{\text{int}}$  is the total mass contained inside the sphere of radius  $r$ . If the point particle is dropped out from a distance  $a$  of the center and if the mass density is uniform we obtain

$$\ddot{r} = -\frac{C}{a^3} r \cdot \bar{e}_R$$

Where  $C$  is constant. We can compare this expression with the acceleration we obtained in a comparable motion with our force

$$\ddot{r} = -\frac{A}{r_0 a^2} r \cdot \bar{e}_R$$

We see that the expressions are proximate. With this analogy we can suggest a physical interpretation of our force: progressively, when the distance to the center of force increases, the point particle which orbits around the center of mass "considers" that this mass is progressively «diluted» inside a closed volume. This volume is depending on the semi major axis of its trajectory and on the constant  $r_0$ . At the end of this evolution the density of mass inside this closed volume is uniform and the force becomes entirely the Hooke's.

Note that this kind of interpretation was already used in another action-at-a-distance, for example to build the model of the atom of Thomson at the 19<sup>th</sup> century.

### **5.2. A way to build a relativistic theory?**

The theory we are trying to build is classical, and consequently can't describe exactly the gravitation. But if this theory is correct it has to be a limiting case of a relativistic theory. This point is a theorist test for this model. Indeed without experiment we can't prove that this model is correct. But we can notice that the solution is simple and seems in agreement with the astronomical observations. The test we suggested in this paper is naturally difficult to implement but it is perhaps possible, with contemporary precision, to do it. At end, building a relativistic theory should be naturally very difficult but, if the force is always central, as we suggested it in this paper, perhaps a simple solution exists.

### **5.3. Is this idea so surprising?**

We want discuss this last point. Effectively it can be surprising that a so simple force could be the force of gravitation in the case of large distances. Naturally the last word will be for the experiments, for example if dark matter is discovered or perhaps if our own experiment is done a day. But if we compare this idea with other perhaps it isn't worth. I think especially to the idea of "dark energy" which seems really problematic. All things considered this idea of "the force of Hooke" has perhaps the virtue of simplicity.

## **6. Conclusion**

In this paper we present our results about an alternative and classical force of gravitation. This force allows describing the "flat" curve of rotation of the spiral galaxies and the expansion at an increasing rate of the Universe. This model is building by using an acceleration which is constant inside the Universe, and which is comparable to the acceleration used in MOND. Moreover we present an experiment on the Earth to valid or invalid our model. At end we present a mathematical study of the forces which leads to conic trajectories.

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