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# When Does Channel-Output Feedback Increase the Capacity Region of the Two-User Linear Deterministic Interference Channel?

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**Abstract.** The two-user linear deterministic interference channel (LD-IC) with noisy channel-output feedback is fully described by six parameters that correspond to the number of bit-pipes between each transmitter and its corresponding intended receiver, i.e.,  $\vec{n}_{11}$  and  $\vec{n}_{22}$ ; between each transmitter and its corresponding non-intended receiver i.e.,  $n_{12}$  and  $n_{21}$ ; and between each receiver and its corresponding transmitter, i.e.,  $\check{n}_{11}$  and  $\check{n}_{22}$ . An LD-IC without feedback corresponds to the case in which  $\check{n}_{11} = \check{n}_{22} = 0$  and the capacity region is denoted by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ . In the case in which feedback is available at both transmitters,  $\check{n}_{11} > 0$  and  $\check{n}_{22} > 0$ , the capacity is denoted by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \check{n}_{11}, \check{n}_{22})$ . This paper presents the exact conditions on  $\check{n}_{11}$  (resp.  $\check{n}_{22}$ ) for observing an improvement in the capacity region  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \check{n}_{11}, 0)$  (resp.  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \check{n}_{22})$ ) with respect to  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ , for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$ . The exact conditions on  $\check{n}_{11}$  (resp.  $\check{n}_{22}$ ) to observe an improvement on either a single rate, both single rates, or the sum-rate capacity, for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$  are also presented in this paper.

**Key words:** Linear Deterministic Interference Channel, Noisy Channel-Output Feedback, Capacity Region.

## 1 Introduction

Channel-output feedback is an interference management technique that brings an unbounded increment on the degrees of freedom (DoF) of the two-user Gaussian interference channel (GIC) [1]. Essentially, in the very strong interference regime,

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the DoF gain provided by feedback can be arbitrarily large when the interference to noise ratios (INRs) grow to infinity. One of the reasons why feedback provides such a surprising benefit relies on the fact that it creates an alternative path to the existing point-to-point paths. For instance, in the two-user IC, feedback creates a path from transmitter 1 (resp. transmitter 2) to receiver 1 (resp. receiver 2) in which symbols that are received at receiver 2 (resp. receiver 1) are fed back to transmitter 2 (resp. transmitter 1) which decodes the messages and retransmits them to receiver 1 (resp. receiver 2). A capacity region enlargement due to the use of feedback is strongly dependent on several factors such as the interference regimes of both transmitter-receiver pairs. In the case in which one transmitter-receiver pair is in the very strong interference regime (the interfering signal is stronger than the intended signal) and the other is in the very weak interference regime (the interfering signal is weaker than the intended signal), the use of feedback in the former link does not enlarge the capacity region. Conversely, the feedback in the latter transmitter-receiver pair enlarges the capacity region depending on the signal to noise ratio (SNR) of the feedback link. Another metric to determine the benefits of feedback is the generalized DoF (GDoF), see [2] for a discussion on DoF and GDoF, as well as other metrics. The GDoF gain due to feedback in the IC depends on the topology of the network and the number of transmitter-receiver pairs in the network. For instance, in the symmetric  $K$ -user cyclic  $Z$ -interference channel, the DoF gain does not increase with  $K$  [3]. In particular, in the very strong interference regime, the DoF gain is shown to be monotonically decreasing with  $K$ . Alternatively, in the fully connected symmetric  $K$ -user IC with perfect feedback, the GDoF are shown to be identical to those in the two-user case and totally independent of the exact number of transmitter-receiver pairs [4]. Nonetheless, the network topology, the number of transmitter-receiver pairs and the interference regime are not the only parameters determining the effect of feedback. Indeed, the presence of noise in the feedback links turns out to be another relevant factor. In the case in which feedback links are impaired by additive Gaussian noise, the benefits of feedback become less interesting as reported in [5]. In [5], the capacity region of the GIC is approximated within a constant number of bits for the symmetric case. However, for certain values of the feedback SNRs, no improvement is observed in the capacity region. The exact values of the feedback SNRs beyond which the capacity region is enlarged depend on all the other channel parameters: two forward SNRs and two forward INRs. Hence, from the available descriptions of the capacity regions with and without feedback, it is a complex task to identify whether or not the existence of a feedback link with given SNR can enlarge the capacity region.

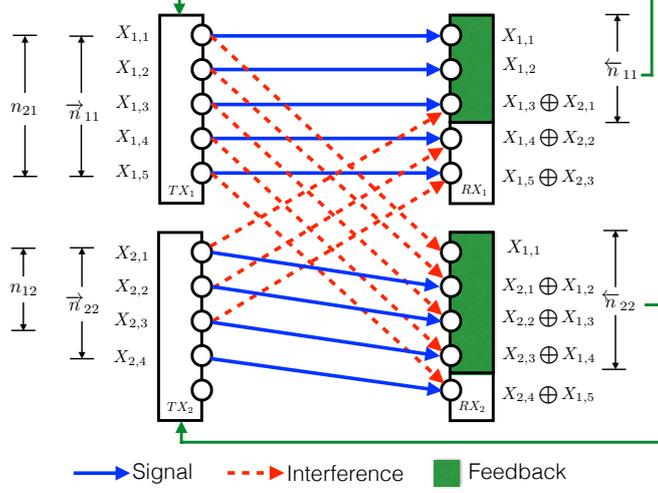
An alternative for dealing with the challenges described above is to study the GIC via its linear deterministic IC (LD-IC) approximation [6], for which the capacity region is known [7]. The two-user LD-IC with output-feedback is fully described by six parameters:  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) \in \mathbb{N}^6$ . There exist a mapping between the parameters describing the two-user LD-IC and the parameters describing the GIC. More specifically, there are two forward

SNRs ( $\overrightarrow{\text{SNR}}_i \geq 1$ ); two forward INRs ( $\overrightarrow{\text{INR}}_i \geq 1$ ); and two backward SNRs ( $\overleftarrow{\text{SNR}}_i \geq 1$ ), with  $i \in \{1, 2\}$ . In the LD-IC, the parameters of the GIC are mapped into the number of bit-pipes between each transmitter and its corresponding intended receiver, i.e.,  $\vec{n}_{ii} = \lfloor \log_2(\overrightarrow{\text{SNR}}_i) \rfloor$ ; between each transmitter and its corresponding non-intended receiver i.e.,  $n_{ij} = \lfloor \log_2(\overrightarrow{\text{INR}}_{ij}) \rfloor$ ; and between each receiver and its corresponding transmitter, i.e.,  $\overleftarrow{n}_{ii} = \lfloor \log_2(\overleftarrow{\text{SNR}}_i) \rfloor$ . An LD-IC without feedback corresponds to the case in which  $\overleftarrow{n}_{11} = \overleftarrow{n}_{22} = 0$  and the capacity region is denoted by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ . In the case in which feedback is available at both transmitters,  $\overleftarrow{n}_{11} > 0$  and  $\overleftarrow{n}_{22} > 0$ , the capacity is denoted by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$ .

This paper presents the exact conditions on  $\overleftarrow{n}_{11}$  (resp.  $\overleftarrow{n}_{22}$ ) for observing an improvement in the capacity region  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  (resp.  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$ ) with respect to  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ , for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$ . More specifically, it is shown that there exists a threshold for the number of bit-pipes in the feedback link of transmitter-receiver pair 1 (resp. 2), beyond which the capacity region of the two-user LD-IC with noisy channel-output feedback can be enlarged, i.e.,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  (resp.  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$ ), with strict inclusion. The exact conditions on  $\overleftarrow{n}_{11}$  (resp.  $\overleftarrow{n}_{22}$ ) to observe an improvement on either a single rate, both single rates, or the sum-rate capacity, for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$  are also presented in this paper. Surprisingly, these thresholds on the feedback parameters can be expressed in closed-form using relatively simple expressions that depend on the exact interference regimes of both transmitter-receiver pairs. Based on these results, several relevant engineering questions can be easily answered. Consider that feedback can be implemented in only one of the transmitter-receiver pairs. Several relevant questions arise in this setting. For instance, in which of the two transmitter-receiver pairs must the feedback link be implemented if the objective is to increase: (a) the individual rate of the transmitter-receiver pair in which feedback is implemented; (b) the individual rate of the other transmitter-receiver pair; or (c) the sum-rate of both transmitter-receiver pairs. In each of these scenarios, the feedback SNR, either  $\overleftarrow{n}_{11}$  or  $\overleftarrow{n}_{22}$ , must be higher than a given threshold for the capacity region enlargement to be observed. Interestingly, for each of these scenarios there exists a complete different answer. As a by-product, the exact values of  $\overleftarrow{n}_{11}$  or  $\overleftarrow{n}_{22}$  for which feedback is absolutely useless in terms of enlarging the capacity region are also identified.

## 2 Linear Deterministic Interference Channel with Noisy-Channel Output Feedback

Consider the two-user LD-IC, with parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$  described in Fig. 1. The parameters  $\vec{n}_{ii}, n_{ij}$  and  $\overleftarrow{n}_{ii}$  with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , are non-negative integers.  $\vec{n}_{ii}$  represents the number of bit-pipes between transmitter  $i$  and receiver  $i$ ;  $n_{ij}$  represents the number of bit-pipes



**Fig. 1.** Two-user linear deterministic interference channel with noisy channel-output feedback.

between transmitter  $j$  and receiver  $i$ ; and  $\overleftarrow{n}_{ii}$  represents the number of bit-pipes between receiver  $i$  to transmitter  $i$  (feedback). At transmitter  $i$ , with  $i \in \{1, 2\}$ , the channel-input  $\mathbf{X}_i^{(n)}$  at channel use  $n$ , with  $n \in \{1, \dots, N\}$ , is a  $q$ -dimensional binary vector  $\mathbf{X}_i^{(n)} = (X_{i,1}^{(n)}, \dots, X_{i,q}^{(n)})^\top$ , with  $q = \max(\overrightarrow{n}_{11}, \overrightarrow{n}_{22}, n_{12}, n_{21})$  and  $N$  the blocklength. At receiver  $i$ , the channel-output  $\overrightarrow{\mathbf{Y}}_i^{(n)}$  at channel use  $n$  is also a  $q$ -dimensional binary vector  $\overrightarrow{\mathbf{Y}}_i^{(n)} = (\overrightarrow{Y}_{i,1}^{(n)}, \dots, \overrightarrow{Y}_{i,q}^{(n)})^\top$ . The output-input relation during channel use  $n$  is given as follows

$$\overrightarrow{\mathbf{Y}}_i^{(n)} = \mathbf{S}^{q-\overrightarrow{n}_{ii}} \mathbf{X}_i^{(n)} + \mathbf{S}^{q-n_{ij}} \mathbf{X}_j^{(n)}, \quad (1)$$

and the feedback signal available at transmitter  $i$  at the end of channel use  $n$  is:

$$\overleftarrow{\mathbf{Y}}_i^{(n)} = \mathbf{S}^{(q-\overleftarrow{n}_{ii})^+} \overrightarrow{\mathbf{Y}}_i^{(n-d)}, \quad (2)$$

where  $d$  is a finite feedback delay, additions and multiplications are defined over the binary field,  $\mathbf{S}$  is a  $q \times q$  lower shift matrix, and  $(\cdot)^+$  is the positive part operator.

Transmitter  $i$  sends  $M_i$  information bits  $b_{i,1}, \dots, b_{i,M_i}$  by sending the codeword  $(\mathbf{X}_i^{(1)}, \dots, \mathbf{X}_i^{(N)})$ . The encoder of transmitter  $i$  can be modeled as a set of deterministic mappings  $f_i^{(1)}, \dots, f_i^{(N)}$ , with  $f_i^{(1)} : \{0, 1\}^{M_i} \rightarrow \{0, 1\}^q$  and  $\forall n \in \{2, \dots, N\}$ ,  $f_i^{(n)} : \{0, 1\}^{M_i} \times \{0, 1\}^{q(n-1)} \rightarrow \{0, 1\}^q$ , such that

$$\mathbf{X}_i^{(1)} = f_i^{(1)}(b_{i,1}, \dots, b_{i,M_i}) \text{ and} \quad (3)$$

$$\mathbf{X}_i^{(n)} = f_i^{(n)}(b_{i,1}, \dots, b_{i,M_i}, \overleftarrow{\mathbf{Y}}_i^{(1)}, \dots, \overleftarrow{\mathbf{Y}}_i^{(n-1)}). \quad (4)$$

At the end of the block, receiver  $i$  uses the sequence  $\mathbf{Y}_i^{(1)}, \dots, \mathbf{Y}_i^{(N)}$  to generate the estimates  $\hat{b}_{i,1}, \dots, \hat{b}_{i,M_i}$ . The average bit error probability at receiver  $i$ , denoted by  $p_i$ , is calculated as follows

$$p_i = \frac{1}{M_i} \sum_{\ell=1}^{M_i} \mathbb{1}_{\{\hat{b}_{i,\ell} \neq b_{i,\ell}\}}. \quad (5)$$

A rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is said to be achievable if it satisfies the following definition.

**Definition 1 (Achievable Rate Pairs).** *The rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is achievable if there exists at least one pair of codebooks  $\mathcal{X}_1$  and  $\mathcal{X}_2$  with codewords of length  $N$ , with the corresponding encoding functions  $f_1^{(1)}, \dots, f_1^{(N)}$  and  $f_2^{(1)}, \dots, f_2^{(N)}$  such that the average bit error probability can be made arbitrarily small by letting the block length  $N$  grow to infinity.*

Lemma 1 determines the set of all the rate pairs  $(R_1, R_2)$  that are achievable in the LD-IC with noisy channel-output feedback and parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ . Denote by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  the capacity region of the LD-IC with noisy channel-output feedback and parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ .

**Lemma 1 (Theorem 1 in [7]).** The capacity region  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  of the two-user LD-IC with noisy channel-output feedback is the set of non-negative rate pairs  $(R_1, R_2)$  that satisfy  $\forall i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ :

$$R_i \leq \min(\max(\vec{n}_{ii}, n_{ji}), \max(\vec{n}_{ii}, n_{ij})), \quad (6)$$

$$R_i \leq \min(\max(\vec{n}_{ii}, n_{ji}), \max(\vec{n}_{ii}, \overleftarrow{n}_{jj} - (\vec{n}_{jj} - n_{ji})^+)), \quad (7)$$

$$R_1 + R_2 \leq \min\left(\max(\vec{n}_{11}, n_{12}) + (\vec{n}_{22} - n_{12})^+, \max(\vec{n}_{22}, n_{21}) + (\vec{n}_{11} - n_{21})^+\right), \quad (8)$$

$$R_1 + R_2 \leq \max\left((\vec{n}_{11} - n_{12})^+, n_{21}\right) + \max\left((\vec{n}_{22} - n_{21})^+, n_{12}\right) \quad (9)$$

$$+ \left( (n_{12} - (\max(\vec{n}_{11}, n_{12}) - \overleftarrow{n}_{11})^+)^+ - (n_{12} - \vec{n}_{11})^+ \right)$$

$$- \min(\vec{n}_{11}, n_{21}) + \min\left((\vec{n}_{11} - n_{12})^+, n_{21}\right)^+$$

$$+ \left( (n_{21} - (\max(\vec{n}_{22}, n_{21}) - \overleftarrow{n}_{22})^+)^+ - (n_{21} - \vec{n}_{22})^+ \right)$$

$$- \min(\vec{n}_{22}, n_{12}) + \min\left((\vec{n}_{22} - n_{21})^+, n_{12}\right)^+,$$

$$2R_i + R_j \leq \max(\vec{n}_{jj}, n_{ji}) + \max(\vec{n}_{ii}, n_{ij}) + (\vec{n}_{ii} - n_{ji})^+ \quad (10)$$

$$- \min\left((\vec{n}_{jj} - n_{ji})^+, n_{ij}\right) + \left( (n_{ji} - (\max(\vec{n}_{jj}, n_{ji}) - \overleftarrow{n}_{jj})^+)^+ \right)$$

$$- (n_{ji} - \vec{n}_{jj})^+ - \min(\vec{n}_{jj}, n_{ij}) + \min\left((\vec{n}_{jj} - n_{ji})^+, n_{ij}\right)^+.$$

### 3 Preliminaries

#### 3.1 Definitions

Let  $\alpha_i \in \mathbb{Q}$  be the interference regime of transmitter-receiver pair  $i$ , with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ ,

$$\alpha_i = \frac{n_{ij}}{\vec{n}_{ii}}. \quad (11)$$

For each transmitter-receiver pair  $i$ , there exist five possible interference regimes (IRs): very weak (VWIR), i.e.,  $\alpha_i \leq \frac{1}{2}$ , weak IR (WIR), i.e.,  $\frac{1}{2} < \alpha_i \leq \frac{2}{3}$ , moderate IR (MIR), i.e.,  $\frac{2}{3} < \alpha_i \leq 1$ , strong IR (SIR), i.e.,  $1 < \alpha_i \leq 2$  and very strong IR (VSIR), i.e.,  $\alpha_i > 2$  [8]. The scenarios in which the desired signal is stronger than or equal to the interference ( $\alpha_i \leq 1$ ), namely very weak IR, weak IR and moderate IR, are referred to as low-interference regimes (LIRs). Conversely, the scenarios in which the desired signal is weaker than the interference ( $\alpha_i > 1$ ), namely strong IR and very strong IR, are referred to as high-interference regimes (HIRs). In the two-user LD-IC, it is possible to observe up to twenty-five possible interference regimes, given  $\alpha_1$  and  $\alpha_2$ . However, only fifteen cases are of real interest. This is because the transmitter-receiver pairs can be indifferently labeled and thus, for instance, studying the case in which  $\alpha_1 \leq \frac{1}{2}$  and  $\alpha_2 > 2$  is the same as studying the case in which  $\alpha_1 > 2$  and  $\alpha_2 \leq \frac{1}{2}$ .

The main results of this paper are presented using a list of events (Boolean variables) that are fully determined by the parameters  $\vec{n}_{11}, \vec{n}_{22}, n_{12}$ , and  $n_{21}$ . For instance, given the parameters  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21})$ , the events (12)-(26) describe some combinations of interference regimes that are particularly interesting.

$$E_1 : \quad \alpha_1 \leq 1 \wedge \alpha_2 \leq 1, \quad (i \text{ and } j \text{ in LIR}) \quad (12)$$

$$E_{2,i} : \quad \alpha_i \leq \frac{1}{2} \wedge 1 < \alpha_j \leq 2, \quad (i \text{ in VWIR and } j \text{ in SIR}) \quad (13)$$

$$E_{3,i} : \quad \alpha_i \leq \frac{1}{2} \wedge \alpha_j > 2, \quad (i \text{ in VWIR and } j \text{ in VSIR}) \quad (14)$$

$$E_{4,i} : \quad \frac{1}{2} < \alpha_i \leq \frac{2}{3} \wedge \alpha_j > 1, \quad (i \text{ in WIR and } j \text{ in HIR}) \quad (15)$$

$$E_{5,i} : \quad \frac{2}{3} < \alpha_i \leq 1 \wedge 1 < \alpha_j \leq 2, \quad (i \text{ in MIR and } j \text{ in SIR}) \quad (16)$$

$$E_{6,i} : \quad \frac{2}{3} < \alpha_i \leq 1 \wedge \alpha_j > 2, \quad (i \text{ in MIR and } j \text{ in VSIR}) \quad (17)$$

$$E_{7,i} : \quad \frac{1}{2} < \alpha_i \leq 1 \wedge \alpha_j > 1, \quad (i \text{ in WIR or MIR and } j \text{ in HIR}) \quad (18)$$

$$E_{8,i} : \quad \alpha_i > 1 \wedge \alpha_j \leq 1, \quad (i \text{ in HIR and } j \text{ in LIR}) \quad (19)$$

$$E_{9,i} : \quad \frac{1}{2} < \alpha_i \leq 1 \wedge 1 < \alpha_j \leq 2, \quad (i \text{ in WIR or MIR and } j \text{ in SIR}) \quad (20)$$

$$E_{10} : \quad \alpha_1 > 2 \wedge \alpha_2 \leq \frac{2}{3}, \quad (i \text{ in VSIR and } j \text{ in VWIR or WIR}) \quad (21)$$

$$E_{11,i} : \quad \alpha_i \leq \frac{1}{2} \wedge \alpha_j \leq 1, \quad (i \text{ in VWIR and } j \text{ in LIR}) \quad (22)$$

$$E_{12,i} : \frac{1}{2} < \alpha_i \leq \frac{2}{3} \wedge \alpha_j \leq \frac{2}{3}, \quad (i \text{ in WIR and } j \text{ in VWIR or WIR}) \quad (23)$$

$$E_{13,i} : \frac{2}{3} < \alpha_i \leq 1 \wedge \alpha_j \leq \frac{1}{2}, \quad (i \text{ in MIR and } j \text{ in VWIR}) \quad (24)$$

$$E_{14,i} : \frac{1}{2} < \alpha_i \leq \frac{2}{3} \wedge \frac{2}{3} < \alpha_j \leq 1, \quad (i \text{ in WIR and } j \text{ in MIR}) \quad (25)$$

$$E_{15,i} : \frac{1}{2} < \alpha_i \leq 1 \wedge \frac{1}{2} < \alpha_j \leq \frac{2}{3}, \quad (i \text{ in WIR or MIR and } j \text{ in WIR}) \quad (26)$$

Some other auxiliary events are considered. The event in which the signal from transmitter  $i$  is stronger in its intended receiver than in its non-intended receiver is denoted by

$$E_{16,i} : \vec{n}_{ii} > n_{ji}. \quad (27)$$

The event in which the sum of the bit-pipes in the direct links is higher than the sum of the bit-pipes in the cross-interference links is denoted by

$$E_{17} : \vec{n}_{11} + \vec{n}_{22} > n_{12} + n_{21}. \quad (28)$$

Finally, the event in which the bit-pipes in the direct link  $j$  are higher than the sum of bit-pipes in both cross-interference links is denoted by

$$E_{18,i} : \vec{n}_{jj} > n_{ij} + n_{ji}. \quad (29)$$

Combining the events (12)-(29), four main events can be determined:

$$S_{1,i} : (E_1 \wedge E_{16,i}) \vee (E_{2,i} \wedge E_{16,i}) \vee (E_{3,i} \wedge E_{16,i} \wedge E_{17}) \vee (E_{4,i} \wedge E_{16,i} \wedge E_{17}) \\ \vee (E_{5,i} \wedge E_{16,i} \wedge E_{17}) \vee (E_{6,i} \wedge \bar{E}_{16,j} \wedge E_{17}), \quad (30)$$

$$S_{2,i} : (E_{3,i} \wedge \bar{E}_{16,j} \wedge \bar{E}_{17}) \vee (E_{7,i} \wedge \bar{E}_{16,j} \wedge \bar{E}_{17}) \vee (\bar{E}_1 \wedge \bar{E}_{16,j}), \quad (31)$$

$$S_{3,i} : (E_1 \wedge \bar{E}_{16,i}) \vee (E_{2,i} \wedge \bar{E}_{16,i}) \vee (E_{3,i} \wedge E_{16,j} \wedge \bar{E}_{16,i}) \vee (E_{4,i} \wedge E_{16,j} \wedge \bar{E}_{16,i}) \\ \vee (E_{5,i} \wedge E_{16,j} \wedge \bar{E}_{16,i}) \vee (E_{6,i} \wedge E_{16,j}) \vee (\bar{E}_1 \wedge E_{16,j}) \vee (E_{8,i}), \quad (32)$$

$$S_{4,i} : (E_{11,i} \wedge E_{16,i} \wedge E_{16,j}) \vee (E_{12,i} \wedge E_{16,i} \wedge E_{16,j}) \vee (E_{13,i} \wedge E_{16,i} \wedge E_{16,j}) \\ \vee (E_{14,i} \wedge E_{16,i} \wedge E_{17}) \vee (E_{15,i} \wedge E_{16,i} \wedge E_{18,i}). \quad (33)$$

For all  $i \in \{1, 2\}$ , the events  $S_{1,i}$ ,  $S_{2,i}$  and  $S_{3,i}$  are mutually exclusive.

**Lemma 2.** For all  $i \in \{1, 2\}$  and for all  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$ , only one of the events  $S_{1,i}$ ,  $S_{2,i}$  and  $S_{3,i}$  is true.

*Proof:* The proof follows from verifying that for  $i \in \{1, 2\}$ , the events (30)-(32) are mutually exclusive. For instance consider that the event  $E_1 \wedge E_{16,i}$  in (30) is true. Then  $S_{1,i}$  is true and  $E_{2,i}$ ,  $E_{3,i}$ ,  $E_{4,i}$ ,  $E_{5,i}$ ,  $E_{6,i}$ ,  $E_{7,i}$  and  $E_{8,i}$  are false. Thus, all events in (31) and (32) are false. The same verification can be made for all the remaining events in (30). This proves that if  $S_{1,i}$  is true then  $S_{2,i}$  and  $S_{3,i}$  are simultaneously false. The same verification can be done for  $S_{2,i}$  and  $S_{3,i}$ . Finally following the same reasoning it can be verified that if any pair of the events  $\{S_{1,i}, S_{2,i}, S_{3,i}\}$  is false, the remaining event is necessarily true. This completes the proof. ■

### 3.2 Rate Improvement Metrics

The rate improvement metrics are given in terms of the following metrics [7]: (a) individual rate improvements  $\Delta_1$  and  $\Delta_2$ ; and (b) sum-rate improvement  $\Sigma$ , with  $\Delta_i \in \mathbb{R}_+$  and  $\Sigma \in \mathbb{R}_+$  for  $i \in \{1, 2\}$ .

Let  $\mathcal{C}_1 = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  and  $\mathcal{C}_2 = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  be the capacity region with noisy channel-output feedback and without feedback, respectively. In order to formally define  $\Delta_1$ ,  $\Delta_2$  and  $\Sigma$ , consider an LD-IC with parameters  $\vec{n}_{11}$ ,  $\vec{n}_{22}$ ,  $n_{12}$ ,  $n_{21}$ ,  $\overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ . The maximum improvement  $\Delta_i(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  of the individual rate  $R_i$  due to the effect of channel-output feedback with respect to the case without feedback is

$$\Delta_i(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) = \max_{R_j > 0} \sup_{\substack{(R_i, R_j) \in \mathcal{C}_1 \\ (R_i^\dagger, R_j) \in \mathcal{C}_2}} R_i - R_i^\dagger, \quad (34)$$

and the maximum sum rate improvement  $\Sigma(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22})$  with respect to the case without feedback is

$$\Sigma(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, \overleftarrow{n}_{22}) = \sup_{\substack{(R_1, R_2) \in \mathcal{C}_1 \\ (R_1^\dagger, R_2^\dagger) \in \mathcal{C}_2}} R_1 + R_2 - (R_1^\dagger + R_2^\dagger). \quad (35)$$

## 4 Main Results

### 4.1 Enlargement of the Capacity Region

In this subsection, the capacity region of an LD-IC with parameters  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21})$ , when feedback is available only at transmitter-receiver pair  $i$ , i.e.,  $\overleftarrow{n}_{ii} > 0$  and  $\overleftarrow{n}_{jj} = 0$ , is denoted by  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii})$  instead of  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  or  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$ , when  $i = 1$  or  $i = 2$ , respectively. Following this notation, Theorem 1 identifies the exact values of  $\overleftarrow{n}_{ii}$  for which the strict inclusion  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii})$  holds with  $i \in \{1, 2\}$ .

**Theorem 1.** Let  $i \in \{1, 2\}$ . Assume that  $S_{3,i} = \text{True}$ . Then, for all  $\overleftarrow{n}_{ii} \in \mathbb{N}$ ,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii})$ . Assume that either  $S_{1,i} = \text{True}$  or  $S_{2,i} = \text{True}$ . Then, for all  $\overleftarrow{n}_{ii} \leq \overleftarrow{n}_{ii}^*$ ,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii})$  and for all  $\overleftarrow{n}_{ii} > \overleftarrow{n}_{ii}^*$ ,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii})$ , with

$$\overleftarrow{n}_{ii}^* = \begin{cases} \max(n_{ji}, (\overleftarrow{n}_{ii} - n_{ij})^+) & \text{if } S_{1,i} = \text{True} \\ \overleftarrow{n}_{jj} + (\overleftarrow{n}_{ii} - n_{ij})^+ & \text{if } S_{2,i} = \text{True}. \end{cases} \quad (36)$$

Theorem 1 shows that under conditions  $S_{3,i}$  in (32), implementing feedback in transmitter-receiver pair  $i$  does not bring any capacity region enlargement. Alternatively, under conditions  $S_{1,i}$  in (30) and  $S_{2,i}$  in (31), the capacity region can be enlarged whenever  $\overleftarrow{n}_{ii} > \overleftarrow{n}_{ii}^*$ .

#### 4.2 Increase of the Individual Rate $R_i$ by Using Feedback in Link $i$

Theorem 2 (resp. Theorem 3) identifies the exact values of  $\overleftarrow{n}_{11}$  (resp.  $\overleftarrow{n}_{22}$ ) for which the individual rate  $R_1$  (resp.  $R_2$ ) can be improved, for parameters  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21})$  and  $\overleftarrow{n}_{22} = 0$  (resp.  $\overleftarrow{n}_{11} = 0$ ) in the LD-IC.

**Theorem 2.** Assume that either  $S_{2,1} = \text{True}$  or  $S_{3,1} = \text{True}$ . Then, for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  there always exists a  $(R'_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_1 = R'_1$ . Assume that  $S_{1,1} = \text{True}$ . When  $\overleftarrow{n}_{11} \leq \overleftarrow{n}_{11}^\dagger$ , for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  there always exists a  $(R'_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_1 = R'_1$ . When  $\overleftarrow{n}_{11} > \overleftarrow{n}_{11}^\dagger$ , for all  $(R'_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  there always exists a  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  such that  $R_1 > R'_1$ , with

$$\overleftarrow{n}_{11}^\dagger = \max(n_{21}, (\vec{n}_{11} - n_{12})^+). \quad (37)$$

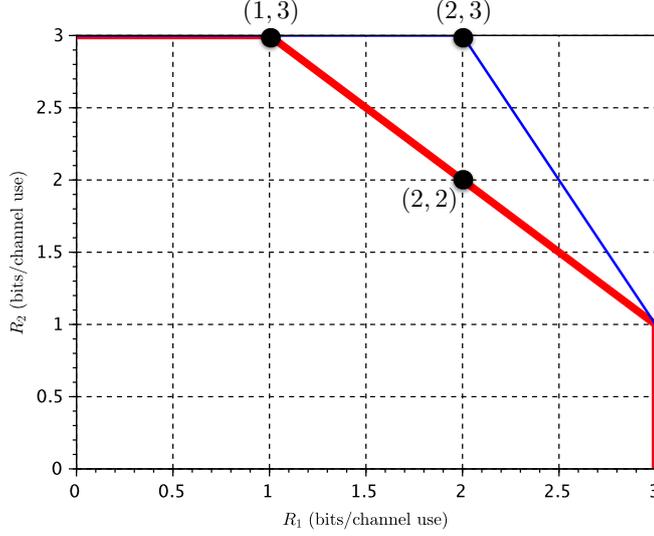
**Theorem 3.** Assume that either  $S_{2,2} = \text{True}$  or  $S_{3,2} = \text{True}$ . Then, for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$  there always exists a  $(R_1, R'_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_2 = R'_2$ . Assume that  $S_{1,2} = \text{True}$ . When  $\overleftarrow{n}_{22} \leq \overleftarrow{n}_{22}^\dagger$  for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$  there always exists a  $(R_1, R'_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_2 = R'_2$ . When  $\overleftarrow{n}_{22} > \overleftarrow{n}_{22}^\dagger$ , for all  $(R_1, R'_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  there always exists a  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$  such that  $R_2 > R'_2$ , with

$$\overleftarrow{n}_{22}^\dagger = \max(n_{12}, (\vec{n}_{22} - n_{21})^+). \quad (38)$$

Theorem 2 highlights that under conditions  $S_{2,1}$  in (31) and  $S_{3,1}$  in (32), the individual rate  $R_1$  can not be improved using feedback in transmitter-receiver pair 1, i.e.,  $\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) = 0$ . Alternatively, under condition  $S_{1,1}$  in (30), the individual rate  $R_1$  can be improved whenever  $\overleftarrow{n}_{11} > \overleftarrow{n}_{11}^\dagger$ , and thus,  $\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) > 0$ . It is worth noting that under condition  $S_{2,1}$  in (31), the capacity region can be enlarged but the individual rate  $R_1$  can not be improved. More specifically,  $(\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) = 0$  and  $\Delta_2(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) > 0)$ . On the other hand, when the individual rate  $R_1$  can be improved under condition  $S_{1,1}$  in (30) and  $\overleftarrow{n}_{11} > \overleftarrow{n}_{11}^\dagger$ , the individual rate  $R_2$  is also improved. That is,  $(\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) > 0$  and  $\Delta_2(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0) > 0)$  (see for instance Figure 2). The same analysis holds for Theorem 3 inverting the roles of the transmitter-receiver pairs.

#### 4.3 Increase of the Individual Rate $R_j$ by Using Feedback in Link $i$

Theorem 4 (resp. Theorem 5) identifies the exact values of  $\overleftarrow{n}_{22}$  (resp.  $\overleftarrow{n}_{11}$ ) for which the individual rate  $R_1$  (resp.  $R_2$ ) can be improved, for parameters  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21})$  and  $\overleftarrow{n}_{11} = 0$  (resp.  $\overleftarrow{n}_{22} = 0$ ) in the LD-IC.



**Fig. 2.** Capacity regions of  $\mathcal{C}(3, 3, 1, 1, 0, 0)$  (thick red line) and  $\mathcal{C}(3, 3, 1, 1, 3, 0)$  (thin blue line).

**Theorem 4.** Assume that  $S_{3,2} = \text{True}$ . Then, for all  $\overleftarrow{n}_{22} \in \mathbb{N}$ ,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$ . Assume that either  $S_{1,2} = \text{True}$  or  $S_{2,2} = \text{True}$ . When  $\overleftarrow{n}_{22} \leq \overleftarrow{n}_{22}^\ddagger$  for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$  there always exists a  $(R'_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_1 = R'_1$ . When  $\overleftarrow{n}_{22} > \overleftarrow{n}_{22}^\ddagger$ , for all  $(R'_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  there always exists a  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$  such that  $R_1 > R'_1$ , with

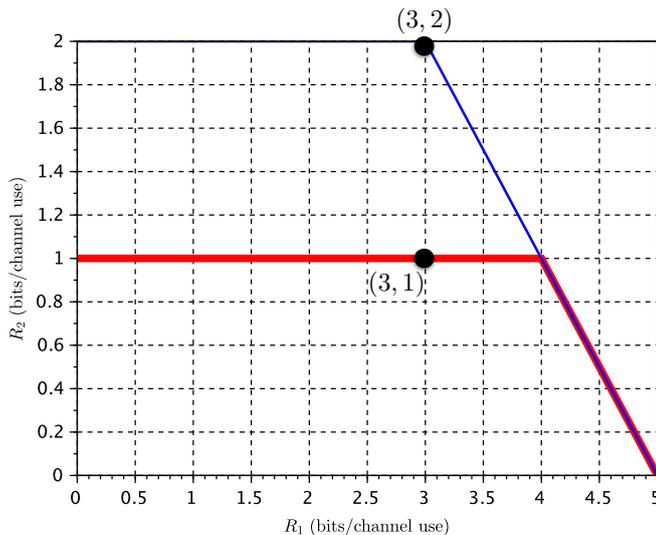
$$\overleftarrow{n}_{22}^\ddagger = \begin{cases} \max(n_{12}, (\vec{n}_{22} - n_{21})^+) & \text{if } S_{1,2} = \text{True} \\ \vec{n}_{11} + (\vec{n}_{22} - n_{21})^+ & \text{if } S_{2,2} = \text{True}. \end{cases} \quad (39)$$

**Theorem 5.** Assume that  $S_{3,1} = \text{True}$ . Then, for all  $\overleftarrow{n}_{11} \in \mathbb{N}$ ,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) = \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$ . Assume that either  $S_{1,1} = \text{True}$  or  $S_{2,1} = \text{True}$ . When  $\overleftarrow{n}_{11} \leq \overleftarrow{n}_{11}^\ddagger$  for all  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  there always exists a  $(R_1, R'_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  such that  $R_2 = R'_2$ . When  $\overleftarrow{n}_{11} > \overleftarrow{n}_{11}^\ddagger$ , for all  $(R_1, R'_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$  there always exists a  $(R_1, R_2) \in \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  such that  $R_2 > R'_2$ , with

$$\overleftarrow{n}_{11}^\ddagger = \begin{cases} \max(n_{21}, (\vec{n}_{11} - n_{12})^+) & \text{if } S_{1,1} = \text{True} \\ \vec{n}_{22} + (\vec{n}_{11} - n_{12})^+ & \text{if } S_{2,1} = \text{True}. \end{cases} \quad (40)$$

The relevance of Theorem 4 is that it shows that under conditions  $S_{3,2}$  in (32), implementing feedback in transmitter-receiver pair 2 does not bring any capacity region enlargement. That is,  $\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) = 0$  and  $\Delta_2(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) = 0$ , and thus, the individual rate

$R_1$  can not be improved. Alternatively, under conditions  $S_{1,2}$  in (30) and  $S_{2,2}$  in (31), the individual rate  $R_1$  can be improved whenever  $\overleftarrow{n}_{22} > \overleftarrow{n}_{22}^\ddagger$ , that is,  $\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) > 0$ . It is worth noting that under condition  $S_{2,2}$  in (31), the individual rate  $R_2$  can not be improved, that is,  $\Delta_2(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) = 0$ . When the individual rate  $R_1$  is improved under condition  $S_{1,2}$  in (30) and  $\overleftarrow{n}_{22} > \overleftarrow{n}_{22}^\ddagger$ , the individual rate  $R_2$  is also improved. That is,  $\Delta_1(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) > 0$  and  $\Delta_2(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22}) > 0$ . The same analysis holds for Theorem 5 inverting the roles of the transmitter-receiver pairs (see for instance Figure 3).



**Fig. 3.** Capacity regions of  $C(5, 1, 3, 4, 0, 0)$  (thick red line) and  $C(5, 1, 3, 4, 4, 0)$  (thin blue line).

#### 4.4 Increase of the Sum-Capacity

Theorem 6 identifies the exact values of  $\overleftarrow{n}_{ii}$  for which the sum-capacity can be improved, for parameters  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21})$  and  $\overleftarrow{n}_{jj} = 0$  in the LD-IC, with  $i \in \{1, 2\}$ .

**Theorem 6.** Assume that  $S_{4,i} = \text{False}$ . Then, for all  $\overleftarrow{n}_{ii} \in \mathbb{N}$   $C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii}) = C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ . Assume that  $S_{4,i} = \text{True}$ . When  $\overleftarrow{n}_{ii} \leq \overleftarrow{n}_{ii}^\ddagger$ , it follows that  $C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii}) = C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ . When  $\overleftarrow{n}_{ii} > \overleftarrow{n}_{ii}^\ddagger$ , it follows that  $C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii}) > C_{\text{sum}}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0)$ , with

$$\overleftarrow{n}_{ii}^\ddagger = \max(n_{ji}, (\vec{n}_{ii} - n_{ij})^+). \quad (41)$$

Theorem 6 identifies the conditions under which implementing feedback in transmitter-receiver pair  $i$  improves the sum-capacity whenever  $\overleftarrow{n}_{ii} > \overleftarrow{n}_{ii}^+$ , that is,  $\Sigma(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{ii}) > 0$ . It is worth noting that condition  $S_{4,i}$  implies that both receivers are in an LIR. However, the sum-capacity cannot always be improved in the different combinations of LIRs.

## 5 Conclusions

This paper presented the exact conditions on the feedback parameters  $\overleftarrow{n}_{11}$  and  $\overleftarrow{n}_{22}$ , beyond which the capacity region of the two-user LD-IC can be enlarged for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$ , i.e.,  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, \overleftarrow{n}_{11}, 0)$  (resp.  $\mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, 0) \subset \mathcal{C}(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}, 0, \overleftarrow{n}_{22})$ ), with strict inclusion. The exact conditions on  $\overleftarrow{n}_{11}$  (resp.  $\overleftarrow{n}_{22}$ ) to observe an improvement on either a single rate, both single rates, or the sum-rate capacity, for any 4-tuple  $(\vec{n}_{11}, \vec{n}_{22}, n_{12}, n_{21}) \in \mathbb{N}^4$  were also presented. Interestingly, there are conditions in the two-user LD-IC in which the use of feedback does not enlarge its capacity region.

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