Outputs analysis and dual immersion method for chaotic systems

Kokou A.A. Langueh, Octaviana Datcu, Jean-Pierre Barbot, Gang Zheng, Krishna Busawon

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K.A.A. Langueh, O. Datcu, J-P Barbot, G. Zheng and K. Busawon *

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Abstract

When secure data transmission is implemented through chaotic systems, the choice of the output is a preliminary problem. In this paper, the quality of the transmitted information is analysed with respect to the observability concept, for each potential output. Moreover, in order to overcome observability loss, a dual immersion technique is proposed.The use of high order sliding mode observer on a well known Lorenz system, allows to highlight the well founded of the proposed analysis and method.

Keywords: Chaotic systems, Observability, Singularity, Immersion, Secure data transmission.

1 Introduction

Since the works of Pecora and Caroll [19], it is well known that two chaotic systems can be synchronized. Following this fact, many authors have proposed secure data transmission schemes (see for example [5, 1, 17]) based on the synchronization of chaotic systems. Nevertheless, the output choice

*K.A.A Langueh is with QUARTZ Laboratory EA 7393, ENSEA, 6 Avenue du Ponceau, 95014 Cergy-Pontoise, O. Datcu is with Politehnica University of Bucharest, Faculty of Electronics, Telecommunications, and Information Technology and QUARTZ Laboratory, IPGP, ENSEA, 6 Avenue du Ponceau, 95014 Cergy-Pontoise, J-P. Barbot is with QUARTZ Laboratory EA 7393, ENSEA, 6 Avenue du Ponceau, 95014 Cergy-Pontoise, EPI Non-A, INRIA. G. Zheng is with EPI Non-A INRIA, Lille, Nord-Europe and K. Busawon is with Northumbria University, Newcastle, UK.

with respect to the synchronization, at the best of our knowledge, is less analyzed (see for example [13, 9]). In [18] Marels and Nijmeijer draw the link between unidirectional synchronization and observation. This allows to use the control system theory to analyze the synchronization of chaotic systems. Nevertheless, as chaotic systems are always nonlinear systems, linear control system theory can't be directly applied to such systems. This is due to the fact, for example, basic properties as stability, controllability and observability are generally local for nonlinear systems. It is the reason why in [11] local observability concepts and criteria are introduced. These lead to determine an observability singularity set [9]. This set is only constituted of 'forbidden' states (i.e. state which are unobservable). In this paper, starting from the results introduced in [11] and other recent works [13, 9] an analysis of the "best" output for secure data transmission is realized on the basic Lorenz circuit. It is important to mention that the observability analysis with respect to the output choice is only a preliminary step in the design of data secure transmission scheme. After that, many other problems occur, such as the choice of the ciphering method, the input choice,the method for retrieving information (see for example [6, 12]),... Moreover, when an output is chosen, it may exist an observability singularity set, which leads to lose information in some part of the state space. Hereafter it is proposed a solution, based on the immersion technique, to overcome this problem under very weak conditions. It is important to mention that, since the work of [22, 21], the immersion technique was extensively used in the observer design context. Nevertheless, this is usually used to recover the linearity property by diffeomorphism and output injection [3, 4, 24, 23], but generally with only a local diffeomorphism. In the majority of the mentioned papers, immersion was accomplished by adding a dynamic by means of output integration [3]. The stability of such extra dynamics can be problematic and an elegant solution is proposed in [23]. Nevertheless the problem of observability singularity was not tackled. A dual immersion technique using only extra differentiations is proposed in this paper. This method is close to the one proposed in [2], in another context. Moreover, for stability arguments it is chosen, in this paper, to impose exponentially stable dynamics instead of constant dynamics. The proposed approach is feasible thanks to the finite time differentiators as for example the one proposed in [7] (but for this method, the delay appears due to the data acquisition frame) or High Order Sliding Mode (HOSM) [15, 8].

The paper is organized as follows: in the next section some observability concepts, symbolic observability index and observability singularity defini-

tions are recalled and the problem statement is explained in the framework of Lorenz system. In section III, the dual immersion method is presented. After that, some recalls on HOSM differentiator are given in section IV. In section V, simulation examples, with respect to Lorenz system, are given. The paper ends with some conclusions and perspectives.

2 Some recalls and problem statement

Consider the following autonomous system:

$$\begin{aligned} \dot{x} &= f(x) \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, and $y \in \mathbb{R}$ is the output. f and h are supposed to be C^∞ vector fields. Rougthly speaking, in [11] in order to determine if or not the system (1) is observable (weakly locally observable) Hermann and Krener have introduced the following condition:

Proposition 1 *The system (1) is weakly locally observable if*

$$Rank\{d\mathcal{O}(\infty)\} = n \tag{2}$$

where

$$d\mathcal{O}(\infty)_{|x_0} = \begin{pmatrix} dh \\ \vdots \\ dL_f^{n-1}h \\ \vdots \\ dL_f^{n+k}h \\ \vdots \end{pmatrix}_{|x_0} \tag{3}$$

In (3) $L_f^i h = \frac{\partial L_f^{i-1} h}{\partial x} f$ is the usual Lie derivative and $dL_f^i h = \left(\frac{\partial L_f^i h}{\partial x_1}, \frac{\partial L_f^i h}{\partial x_2}, ..., \frac{\partial L_f^i h}{\partial x_n}\right)$ is the associated 1-form.

Remark 1 *If the system (1) is linear, then the weakly locally observability implies the classical global observability. Moreover, due to the Cayley Hamilton theorem, it is sufficient to compute the n first rows of $d\mathcal{O}(\infty)$ (denoted $d\mathcal{O}(n)$).*

Starting, from this linear argument and the fact that it is natural to design an observer of the same dimension that the original system in [20] the concept of regularly weakly locally observability was introduced. Roughly speaking, only the $n-1$ output derivatives are considered for recovering all the state. This leads to the following condition:

Proposition 2 *The system (1) is regularly weakly locally observable if*

$$Rank\{d\mathcal{O}(n)\} = n \tag{4}$$

with

$$d\mathcal{O}(n)_{|_{x_0}} = \begin{pmatrix} dh \\ \vdots \\ dL_f^{n-1}h \end{pmatrix}_{|_{x_0}} \tag{5}$$

Since $d\mathcal{O}(n)_{|_{x_0}}$ is only of full rank n around x_0, this implies that there might exist some $\bar{x} \in \mathbb{R}^n$ such that rank $d\mathcal{O}(n)_{|_{\bar{x}}} < n$. Due to this fact, let us define the following observability singularity set:

$$\mathcal{S}_n = \{x \in \mathbb{R}^n : \text{rank } \{d\mathcal{O}(n)_{|_x}\} < n\} \tag{6}$$

In this case, for the purpose of removing the singularities in $\mathcal{O}(n)_{|_x}$ defined in (6), it is necessary to increase the dimension of (5) by involving more derivatives of the output. This will be the purpose of the next section, but before, an example of the output choice is given hereafter.

Example 1 *Let us consider the Lorenz system*

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \tag{7}$$

Where $x = [x_1, x_2, x_3]^T \in \mathbb{R}^3$ is the state. Moreover, the parameter are chosen as follow $\sigma = 10$, $r = 28$ and $b = \frac{8}{3}$.

Case 1: *The output $y = x_1$*
For this output $d\mathcal{O}(3)_{|_x}$ is equal to

$$d\mathcal{O}(3)_{|_x} = \begin{pmatrix} 1 & 0 & 0 \\ -\sigma & \sigma & 0 \\ \sigma(r + \sigma - x_3) & -\sigma(\sigma + 1) & -\sigma x1 \end{pmatrix}_{|_x} \tag{8}$$

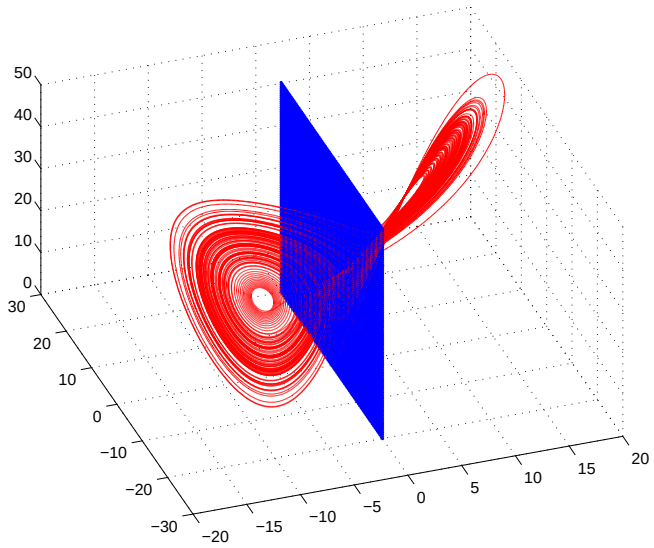

Figure 1: Lorenz attractor and the map $\mathcal{S}_3$

and then

$$\mathcal{S}_3 = \{x \in \mathbb{R}^3 : x_1 = 0\}$$

On the figure 1 it is shown that the set of observability singularity intersect the strange attractor. In fact the observability property is lost because only two derivatives of the output $y = x_1$ are considered. Nevertheless, if the third derivative is also considered $d\mathcal{O}(4)_{|x}$ becomes:

$$d\mathcal{O}(4)_{|x} = \begin{pmatrix} 1 & 0 & 0 \\ -\sigma & \sigma & 0 \\ \sigma(r+\sigma-x_3) & -\sigma(\sigma+1) & -\sigma x_1 \\ A & B & C \end{pmatrix}_{|x} \tag{9}$$

where $A = -\sigma[\sigma^2 + r\sigma(2\sigma+1) + 2x_1x_2 - (2\sigma+b+1)x_3]$, $B = \sigma[\sigma^2 + \sigma(r+1) + 1 - x_1^2 - \sigma x_3]$ and $C = \sigma[(2\sigma+b+1)x_1 - \sigma x_2]$. Then, the observability singularity set is:

$$\mathcal{S}_4 = \{x \in \mathbb{R}^3 : x_1 = x_2 = 0\}$$

this set does not intersect the strange attractor, see figure 2. Moreover, it is important to note that on $\mathcal{S}_4$, the system (7) is stable and consequently its behaviour is not chaotic.

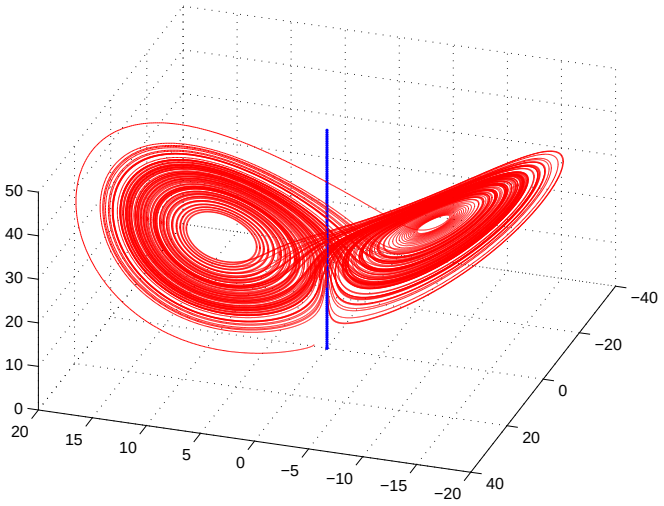

Figure 2: Laurenz attractor and the line $\mathcal{S}_4$

Case 2: *The output* $y = x_2$
In the same way as the case 1, it is obtained

$$\mathcal{S}_3 = \{x \in R^3 : 2x_2x_1^2 - brx_1 + r\sigma x_2 - \sigma x_2x_3 = 0\}$$

Case 3: *The output* $y = x_3$
Finally, for the third case, it is obtained

$$\mathcal{S}_3 = \{x \in R^3 : 2\sigma x_2^2 - 2rx_1^2 + 2 * x_1^2x_3 = 0\}$$

Comparing the set of observability singularities, for the sake of simplicity and space in the next only the firs case is considered.

The previous example, case 1, has shown that, even if the $\mathcal{O}(3)$ contains singularities, i.e. its rank is not equal to 3 for some $x \in \mathbb{R}^n$, it is still possible to obtain a higher dimensional map: $\mathcal{O}(4)$ which will not contain singularity on the strange attractor. Thus it may be interesting to design an observer of greater dimension than 3 which is equivalent to use a state space representation greater than 3. More precisely, (9) was obtained from the following injective function:

$$\mathcal{O}(4) = \left(y, \dot{y}, \ddot{y}, y^{(3)}\right)^T \tag{10}$$

Remark 2 *By extension $\mathcal{O}(n+k)$ is defined as follow:*

$$\mathcal{O}(n+k) = \left(y, \dot{y}, \ddot{y}, ..., y^{(n+k)}\right)^T \tag{11}$$

Setting $z = \mathcal{O}(4)$, the extended dynamics becomes

$$\dot{z} = \frac{\partial \mathcal{O}(4)}{\partial x} f(x)_{|x} = \begin{pmatrix} z_2 \\ z_3 \\ z_4 \\ L_f^3 h(x) \end{pmatrix}$$

for which a traditional high-gain observer [10] (or sliding mode observer [15]) can be easily designed to estimate z, noted as $\hat{z}$. Then, one can obtain the estimation of x, denoted $\hat{x}$ by:

$$\hat{x} = \underset{\hat{x} \in \mathbb{R}^n}{\arg\min} ||\hat{z} - \phi(\hat{x})||$$

which can be seen as an optimization problem. For the above optimization framework, the essential point is that the Jacobian of $\mathcal{O}(4)$ is a matrix of 4 by 3, thus non square, and this will cause difficulties when applying numerical methods to solve this optimization problem.

Motivated by this consideration, for a given $\mathcal{O}(n+k)$ defined in (11), hereafter, it is proposed to find a diffeomorphism ϕ by increasing the original state space. In the literature, this technique is called immersion and many authors use this immersion technique in order to obtain a specific normal form [23]. Most often, the immersion is realized by means of output integration. In this paper, it will be realized by a dual method which will be presented in the next section.

3 Dual immersion technique

3.1 Preliminary results

For a given $d\mathcal{O}(n)$ containing the singularities belonging to $\mathcal{S}_n$, this section will propose a constructive way to deduce an injective function $\mathcal{O}(n+k)$ defined in (11) by increasing the output derivative order. Then the following result is trivial.

Lemma 1 *Consider the observability singularity set $\mathcal{S}_n$ defined in (6), then the generated set $\mathcal{S}_{n+i}$ for $i \geq 1$ via differentiation is always included in $\mathcal{S}_{n+i-1}$, i.e.*

$$\mathcal{S}_{n+i} \subseteq \mathcal{S}_{n+i-1}$$

Proof. For $i \geq 0$, the following equation

$$\mathcal{O}(n+i) = \begin{bmatrix} \mathcal{O}(n+i-1) \\ L_f^{n+i_1} h \end{bmatrix}$$

is verified, hence

$$\text{rank}\{d\mathcal{O}(n+i)\} \geq \text{rank}\{d\mathcal{O}(n+i-1)\}$$

which implies that

$$\mathcal{S}_{n+i} \subseteq \mathcal{S}_{n+i-1}$$

■

Before presenting the dual immersion technique, the following assuption is reclame

Assumption 1 *It is assumed that system (1) is globally observable for $x \in \mathbb{R}^n$, or at least observable in the domain of interest D, i.e. there exists a least an integer $k \in \mathcal{Z}^+$ such that*

$$\text{rank}\{d\mathcal{O}(n+k)\} = n \tag{12}$$

for all $x \in \mathbb{R}^n$, or at least for all $x \in D$, where $\mathcal{O}(n+k)$ is defined in (11).

With the above assumption, we can state the following result.

Lemma 2 *For system (1), if Assumption 1 is satisfied, then there exists at least an integer $k \in \mathcal{Z}^+$ such that the following inclusion is satisfied:*

$$\emptyset = \mathcal{S}_{n+k} \subseteq \cdots \subseteq \mathcal{S}_{n+1} \subseteq \mathcal{S}_n$$

on R^n, or at least in D. Moreover, $z = \mathcal{O}(n+k)$ is such that rank$\{d\mathcal{O}(n+k)\} = n$, on R^n, or at least in D.

Proof. The above result is a direct consequence of the previous lemma. ■

The above results show that one can reduce the observability singularity set by just increasing the number of the derivative of the output. Consequently, one can find at least an integer $k \in \mathcal{Z}^+$ such that $z = \mathcal{O}(n+k)$

has a Jacobian of rank n on R^n, or at least in D. However, its Jacobian is not square (a matrix of $(n+k)\times n$). In what follows, we will propose a constructive way to calculate a new diffeomorphism $\phi(x,\xi)$ from the deduced $\mathcal{O}(n+k)$ in Lemma 2 via the technique of dual immersion, whose Jacobian $d\phi(x)$ will be square and invertible, at least on D.

3.2 Dual immersion approach

Consider again system (1), and suppose that we have already found a least integer $k \in \mathcal{Z}^+$ such that $z = \mathcal{O}(n+k)$ is a global injective function. Then let us present the following procedure to compute a global (or at least in D) diffeomorphism $\phi(x,\xi)$ (where $\xi \in \mathbb{R}^k$ will be defined hereafter).
Dual immersion algorithm

1. Initialization: Set $z_1 = \phi_1(x) = y$;
2. Compute $z_j = \phi_j(x) = y^{(j-1)}$ until a singularity of observability appears in the row $dL_f^{(j-1)}h$;
3. Define $\phi_j = y^{(j-1)} + \xi_1$ where ξ_1 is an additional state, with the following dynamics:
$$\dot{\xi}_1 = -\epsilon_1 \xi_1 \tag{13}$$
initialized at $\xi_1 = 0$.
4. - If $L_f^{(j+1)}h$ is not considered in (11), define $\phi_{i,j+1} = L_f^{(j)}h - \epsilon_1\xi_1$
   - If $L_f^{(j+1)}h$ is considered in (11), define $\phi_{i,j+1} = L_f^{(j)}h - \epsilon_1\xi_1 + \xi_2$, with
$$\dot{\xi}_2 = -\epsilon_2\xi_2 + \dot{\epsilon}_1\xi_1 - \epsilon_1^2\xi_1$$
5. repeat the operation until the last line of (11) is reached. The resulting matrix $d\{\phi\}$ becomes a square matrix and if there exist ϵ_j such that the matrix is regular for all x then the algorithm 3.2 converges.

This algorithm gives

Theorem 1 *For system (1), if Assumption 1 is satisfied and Algorithm 3.2 converges then $\phi(x,\xi)$ with $\xi \in \mathbb{R}^k$ is a global, or at least on D, diffeomorphism. Moreover, the Jacobian of the generated $\phi(x,\xi)$ via the dual immersion satisfies:*

$$\text{rank} d\phi(x,\xi) = n + k$$

for all $x \in \mathbb{R}^n$*, or at least in* D*.*

Proof. The proof follows from the algorithm, and assumption 1 is a necessary condition for the convergence of algorithm 3.2. ■

Roughly speaking, the dual immersion algorithm works as follow: considering that for the order $r = n + 1$, $\mathcal{O}(r)$ generates an "acceptable" (see [9] for chaotic systems) observability singularity set $\mathcal{S}_r$ and the singularity problem arrives in the last line of $\mathcal{O}(n)$, it is possible to define a new change of coordinate $z = \phi(x, \xi)$ as follows: $z_j = y^{(j-1)}$ until $j = n-1$ (i.e. until such a problem of observability singularity appears in the row $dL_f^{(n-1)}h$). and for $j = n$, z_j are assigned equal to $y^{(j-1)} + \xi_1$ where ξ_1 is an extra state, which verifies the following dynamics:

$$\dot{\xi}_1 = -\epsilon_1 \xi_1 \tag{14}$$

initialized at $\xi_1 = 0$. Finaly for $j = n+1$, z_j are assigned equal to $y^{(j)} + \dot{\xi}_1 = \dot{z}_n$.

Remark 3 *From the stability and the initialization of the dynamics (14) it is not necessary to simulate this dynamic in the observer design because it is equivalent of adding zero at* $y^{(j-1)}$*. In [2] the proposed solution consists to adding a constant in order to be able to design a High gain observer in the original coordinates.*

4 Recalls on high-order sliding-mode

In this paper, the proposed method is based on the so-called real-time exact robust HOSM differentiator [15, 16], which is recalled in the following.

Consider a signal $y(t) \in C^k$ (at least k times differentiable), let us suppose $(y, \cdots, y^{(k)}) = (z_1, \cdots, z_{k+1})$. The HOSM robust differentiator proposed in

[15] takes the following form:

$$\begin{aligned}
\dot{\hat{z}}_1 &= -\lambda_0 M^{\frac{1}{k}} |\hat{z}_1 - y|^{\frac{k}{k+1}} sign(\hat{z}_1 - y) + \hat{z}_2 \\
\dot{\hat{z}}_2 &= -\lambda_1 M^{\frac{1}{k-1}} |\hat{z}_2 - v_1|^{\frac{k-1}{k}} sign(\hat{z}_2 - v_1) + \hat{z}_3 \\
&\vdots \\
\dot{\hat{z}}_k &= -\lambda_{k-1} M^{\frac{1}{2}} |\hat{z}_k - v_{k-1}|^{\frac{1}{2}} sign(\hat{z}_k - v_{k-1}) + \hat{z}_{k+1} \\
\dot{\hat{z}}_{k+1} &= -\lambda_k M sign(\hat{z}_{k+1} - v_k)
\end{aligned}$$

where M is chosen to be bigger than the k-th derivative of $y(t)$, λ_i are positive design parameters, and the adjustment of those parameters is described in detail in [15]. Define the observation errors as: $e_i = z_i - \hat{z}_i$, thus the observation errors dynamics is given by:

$$\begin{aligned}
e_1 &= \hat{z}_1 - y \\
e_2 &= \dot{e}_1 = \lambda_0 M^{\frac{1}{k}} |e_1|^{\frac{k}{k+1}} sign(e_1) \\
&\vdots \\
e_k &= \dot{e}_{k-1} = \lambda_{k-1} M^{\frac{1}{2}} |e_{k-1}|^{\frac{1}{2}} sign(e_{k-1}) \\
e_{k+1} &= \dot{e}_k = \lambda_k M sign(e_k)
\end{aligned}$$

It has been proven in [14] that there exists t_0 such that $\forall t > t_0$ we have

$$e_i = z_i - \hat{z}_i = 0 \quad \text{for} \quad 1 \leq i \leq k+1$$

In the next section, a Lorenz example case 1 will be simulated and commented.

5 Lorenz example and immersion.

Consider again the system (7) with $y = x_1$, with initial conditions $x_1(0) \neq 0$, $x_2(0) \neq 0$ and $x_3(0) \neq 0$. It was shown, in the second section that $\mathcal{S}_4 = \{x \in R^3 : x_1 = x_2 = 0\}$ and that this set of observability singularity is not on the strange attractor, thus as D (the considered domain) is the strange attractor, then it is possible to use the dual immersion method.
Then $\mathcal{O}(n+k)$

$$\phi(x) = [y, \dot{y}, \ddot{y}, \dddot{y}]^T$$

has a regular Jacobian on D. So, the immersion is implemented as follows:

$$z = \phi(x,\xi) = \begin{pmatrix} y \\ \dot{y} \\ \ddot{y} + \xi \\ \dddot{y} + \dot{\xi} \end{pmatrix}$$

with $\dot{\xi} = -\varepsilon\xi$ and $\xi(0) = 0$ as initial condition. Then $d\phi(x,\xi)$ is a square matrix:

$$d\phi(x,\xi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\sigma & \sigma & 0 & 0 \\ \sigma(r+\sigma-x_3) & -\sigma(\sigma+1) & -\sigma x_1 & 1 \\ A & B & C & -\varepsilon \end{pmatrix}$$

with $A = -\sigma[\sigma^2 + r\sigma(2\sigma+1) + 2x_1x_2 - (2\sigma+b+1)x_3]$, $B = \sigma[\sigma^2 + \sigma(r+1) + 1 - x_1^2 - \sigma x_3]$ and $C = \sigma[(2\sigma+b+1)x_1 - \sigma x_2]$.
The ε parameter is chosen such that $d\phi(x,\xi) \neq 0$ (i.e. $\varepsilon x_1 \neq (2\sigma+b+1)x_1 - \sigma x_2$) on D. Thus, as no derivative of ε is considered in this exemple, this parameter is chosen as follows:
ε is equal to 10^4 if $det\{d\bar{\phi}(x,\varepsilon)|_{\varepsilon=10^4}\} \neq 0$ else $\varepsilon = 10$, this ensuring that $det\{d\bar{\phi}(x,\varepsilon)$ is equal to zero only if $x_1 = x_2 = 0$.

As the HOSM observer gives $\hat{z} = z$ in finite time, the estimated of x is also obtain in finite time as follows:

$$x = \phi^{-1}(z)|_\varepsilon =$$

$$\begin{bmatrix} z_1 \\ \frac{\sigma z1 + z2}{\sigma} \\ \begin{bmatrix} x_3 \\ \xi \end{bmatrix} = \begin{bmatrix} -\sigma y & 1 \\ \sigma[(2\sigma+b+1)y - \sigma x_2] & -\varepsilon \end{bmatrix}^{-1} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \end{bmatrix}$$

with $L_1 = \ddot{y} + (\sigma+1)\dot{y} - \sigma(r-1)y$ and $L_2 = \dddot{y} - [\sigma^2 + \sigma(r-1) + 1]\dot{y} + [\sigma^2(r-\sigma-1) + \sigma r]y + (\sigma y + \dot{y})y^2$

The simulation was done with initial conditions chosen randomly in $[0,1]$ for the system and the observer, moreover on the HOSM observer the paramer M is equal to 10^6. In Fig. 3, the exact values of is obtained after $1s$.

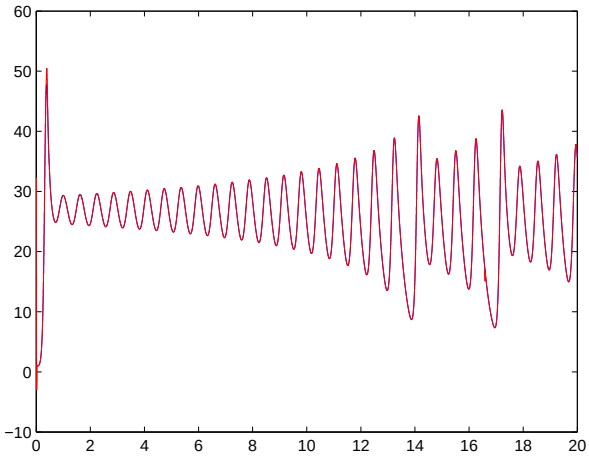

Figure 3: The stat x_3 and its estimate $\hat{x}_3$

6 Conclusion

In this paper an analysis of the observability, with respect to different outputs, was done. After that, a dual immersion method was proposed, in order to overcome some problems due to observability singularity set. This method is based on an immersion with a stable fictitious dynamics initialized at the equilibrium point. The inversion of the obtained diffeomorphism is not, generally, as simple as that in the Lorenz system with x_1 as output. In a future work, a numerical solution of the inverse diffeomorphism will be investigated by taking into account the particular structure of the obtained diffeomorphism.

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