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# A Queuing Model for Road Traffic Simulation

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**Abstract.** We present in this article a stochastic queuing model for the road traffic. The model is based on the  $M/G/c/c$  state dependent queuing model, and is inspired from the deterministic Godunov scheme for the road traffic simulation. We first propose a variant of  $M/G/c/c$  state dependent model that works with density-flow fundamental diagrams rather than density-speed relationships. We then extend this model in order to consider upstream traffic demand as well as downstream traffic supply. Finally, we show how to model a whole road by concatenating road sections as in the deterministic Godunov scheme.

**Keywords:** traffic flow modeling, queuing systems, state dependent.

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## INTRODUCTION

The dynamics of traffic flows in road networks is complex, and is submitted to stochastic disturbances. Congestion is a phenomenon that arises on local as well as large areas, whenever traffic demand exceeds traffic supply. As usual in road traffic models and in associated numerical schemes, a link of the network is modelled as a sequence of road sections, for which fundamental diagrams (laws on capacity) are given (estimated). As the sequence of sections is in series, if any section is not performing optimally, the whole link will not be operating efficiently. Deterministic traffic models taking into account these dynamics are very known. We base here on the LWR first order model [8, 10], for which numerical schemes have been performed since decades; see for example [7, 2]. In the Godunov scheme [7], as well as in the cell-transmission model (CTM) [2], traffic demand and supply functions are defined and used. The demand/supply framework provides a comprehensive foundation for first-order node models. Flow interactions in these models typically result from limited inflow capacities of the downstream links. Recently, this framework has been supplemented with richer features such as conflicts within the node [13]. In [9], the authors presented a dynamic network loading model that yields queue length distributions. It is a discretized, stochastic instance of the Kinematic wave model (KWM), whereas the stochastic CTMs constitute stochastic instances of discretized KWMs.

We propose here a stochastic traffic model based on the queuing model of [11, 12] and on the Godunov scheme [6, 7] of the LWR traffic model [8, 10]. We first rewrite the  $M/G/c/c$  state dependent queuing model [11, 12] on a road section, by considering density-flow fundamental diagrams rather than density-speed ones. By this, we consider the traffic demand and supply functions for the section, and derive a model for a road with a downstream supply. Finally, we present a model of two road sections in tandem. The model we present here can also be used to the analysis of travel times through road traffic networks (derive probability distributions, reliability indices, etc), as in [5, 4, 3], where an algebraic deterministic approach is used.

## A SHORT REVIEW IN $M/G/C/C$ SYSTEMS

We give in this section a short review on the  $M/G/c/c$  state dependent queuing model [11, 12]. A link of a road network is modelled with  $c$  servers set in parallel, where  $c$  is the maximum number of cars that can move on the road. The velocity of cars is assumed to be dependent on the number  $n$  of cars on the road, according to a non-increasing density-speed relationship. For example, in the linear case, one have  $V(n) = c - n + 1$ , where  $V$  denotes the car-speed (velocity). The arrival process of cars into the link is assumed to be Poisson with rate  $\lambda$ , while the service rate of the  $c$  servers depend on the number of cars on the road. A normalized service rate  $f(n)$  is considered, and is taken  $f(n) = V(n)/V(1) \leq 1$ . In the linear case, for example, we have  $f(n) = (c - n + 1)/c$ . We notice here that  $V(1)$  is the speed corresponding to one car in the road (ie. the free speed).

The stationary probability distribution  $P_n = P(N = n)$  of the number of customers  $N$  in the  $M/G/c/c$  state dependent model, is given as follows.

$$P_0 = \left( 1 + \sum_{n=1}^c \frac{(\lambda L/V(1))^n}{\prod_{i=1}^n if(i)} \right)^{-1}, \quad P_n = \frac{(\lambda L/V(1))^n}{\prod_{i=1}^n if(i)} P_0, \quad n = 1, \dots, c. \quad (1)$$

where  $L$  is the length of the road section.

From  $P_n$ , other important performance measures can be easily derived. The blocking probability  $P_c = (\lambda L/v_n)^c / \prod_{i=1}^c if(i) P_0$ . The throughput  $\theta = \lambda(1 - P_c)$ . The expected number of cars in the section  $\bar{N} = \sum_{n=1}^c nP_n$ . The expected service time  $W = \bar{N}/\theta$ .

## ROAD SECTION MODEL

We slightly modify the  $M/G/c/c$  model of MacGregory Smith, by defining the normalized service rate as the ratio of the flow by the maximum flow, rather than the speed by the free speed. This modification will permit us to consider the demand and supply functions of a road section, and then to use them in the case where tow (or many) sections are set in tandem.

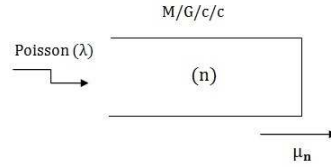


FIGURE 1. One road section.

In this section, we present the  $M/G/c/c$  state dependent queuing model on one road section, for which we consider a triangular fundamental traffic diagram.

$$Q(\rho) = \min(v_f \rho, w(\rho_j - \rho)),$$

where  $\rho$ ,  $Q(\rho)$ ,  $v_f$ ,  $w$ , and  $\rho_j$  denote respectively the car-density in the road section, the car-flow, the free speed, the backward wave speed, and the jam-density. The demand and the supply functions  $\Delta(\rho)$  and  $\Sigma(\rho)$  respectively are then given as follows.

$$\Delta(\rho) = \min(v_f \rho, q_{\max}), \quad \Sigma(\rho) = \min(q_{\max}, w(\rho_j - \rho)),$$

where  $q_{\max} = \rho_j / (1/v_f + 1/w)$ .

The service rate  $\mu_n$  of the road section, depends on the number  $n$  of cars in the road, and is given by the car-flow on the section.

$$\mu_n = \min(v_f \rho, w(\rho_j - \rho)) = \min\left(v_f \frac{n}{L}, w\left(\frac{c-n}{L}\right)\right).$$

The normalized service rate is then fixed to

$$q_n = \frac{q_n}{q_{\max}} = \frac{\min(v_f \rho, w(\rho_j - \rho))}{q_{\max}} = \frac{\min(v_f \frac{n}{L}, w(\frac{c-n}{L}))}{q_{\max}}.$$

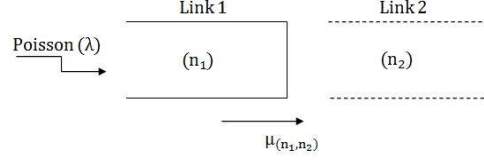
The stationary probability distribution of the number of cars on the road section is then given as follows.

$$p_n = \frac{(\lambda q_{\max})^n \left(\frac{L}{v_f}\right)^{n_c} \left(\frac{L}{w}\right)^{n-n_c}}{\prod_{i=1}^{n_c} i^2 \prod_{i=n_c+1}^{n-n_c} i(c-i)} p_0, \quad \text{with} \quad p_0 = \left( 1 + \sum_{n=1}^c \frac{(\lambda q_{\max})^n \left(\frac{L}{v_f}\right)^{n_c} \left(\frac{L}{w}\right)^{n-n_c}}{\prod_{i=1}^{n_c} i^2 \prod_{i=n_c+1}^{n-n_c} i(c-i)} \right)^{-1}, \quad (2)$$

where  $n_{cr} = \rho_{cr} L$  is the number of cars corresponding to the critical car-density.

## A ROAD SECTION WITH A DOWNSTREAM SUPPLY

We model here a road section with a state dependent  $M/G/c/c$  queuing model, as in the last section, but we consider here that the service is constrained by the flow supply of the downstream section. We assume that the flow supply of the downstream section is stochastic and that the stationary probability distribution of the cars in the downstream (fictive) section is given.



**FIGURE 2.** Two links in tandem.

We use the same notations as above, but with an additional index indicating the road section: 1 for section 1 and 2 for the downstream section (section 2). We assume triangular fundamental diagrams for the two sections.

The car-flow outgoing from section 1 and entering to section 2 is assumed to be given by the minimum between the traffic demand on section 1 and the traffic supply of section 2.

$$q_{12} = \min(\Delta_1(\rho_1), \Sigma_2(\rho_2)) = \min\left(v_{f1} \frac{n_1}{L_1}, q_1^{\max}, q_2^{\max}, w_2 \left(\frac{c_2 - n_2}{L_2}\right)\right).$$

Therefore, the normalized service rate  $f(n_1, n_2)$  of section 1 is given as follows.

$$f(n_1, n_2) = \frac{q_{12}}{q_1^{\max}} = \min\left(v_{f1} \frac{n_1}{L_1}, q_1^{\max}, q_2^{\max}, w_2 \left(\frac{c_2 - n_2}{L_2}\right)\right) / q_1^{\max}.$$

We have here, an  $M/G/c/c$  system on section 1 (with  $c_1$  servers), parameterized by the traffic supply downstream of the section (the number of cars in section 2). Again, following MacGregor Smith model, the stationary probability distribution of the number of cars on section 1, parameterized by the number of cars on section 2, is given by

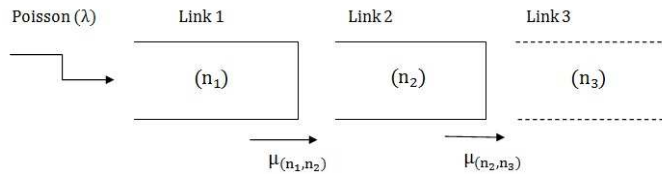
$$P_{n_1|n_2} = P(N_1 = n_1 | N_2 = n_2) = \frac{(\lambda q_1^{\max})^{n_1}}{\prod_{i=1}^{n_1} f(i, n_2) i} P_{0|n_2}, \quad \text{with } P_{0|n_2} = \left(1 + \sum_{n_1=1}^{c_1} \frac{(\lambda q_1^{\max})^{n_1}}{\prod_{i=1}^{n_1} f(i, n_2) i}\right)^{-1}. \quad (3)$$

Then, the stationary distribution of the cars on section 1 is obtained as follows.

$$P_{n_1} = P(N_1 = n_1) = \sum_{n_2=0}^{c_2} P_{n_1|n_2} = \sum_{n_2=0}^{c_2} \frac{(\lambda q_1^{\max})^{n_1}}{\prod_{i=1}^{n_1} f(i, n_2) i} P_{0|n_2}. \quad (4)$$

## TWO SECTIONS IN TANDEM

We consider two sections in tandem, where the flow supply of the second section is constrained by a fictive third section, as in Figure 3. As above, we assume that the boundary condition (flow supply) is given and known. That is, the stationary probability distribution of the number of cars in the fictive (third) section is given.



**FIGURE 3.** Three links in tandem.

As above, the question is to determine the stationary probability distribution of the number of cars in the two sections  $P(N_1 = n_1, N_2 = n_2)$ , which we denote by  $P_{(n_1, n_2)}$ . To do that, we proceed in two steps. We first consider the second

section, constrained by the supply of the third one. We do the same reasoning as the previous section, we obtain the stationary probability distribution of the number of cars in section 2 as follows.

$$P_{n_2}^{(2)} = P(N_2 = n_2) = \sum_{n_3=0}^{c_3} \frac{(\lambda q_2^{\max})^{n_2}}{\prod_{i=1}^{n_2} f(i, n_3) i} P_0^{(2)}, \quad \text{with} \quad P_0^{(2)} = \left( 1 + \sum_{n_2=1}^{c_2} \sum_{n_3=0}^{c_3} \frac{(\lambda q_2^{\max})^{n_2}}{\prod_{i=1}^{n_2} f(i, n_3) i} \right)^{-1}. \quad (5)$$

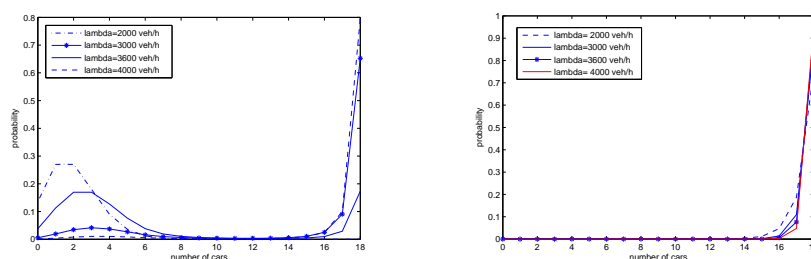
In the second step, we consider the first section constrained by the supply of the second section, which is now known. We obtain the following.

$$P_{(n_1, n_2)} = P(N_2 = n_2) P(N_1 = n_1 | N_2 = n_2) = P_{n_2}^{(2)} P_{n_1 | n_2}, \quad (6)$$

where  $P_{n_2}^{(2)}$  is given by (5) and  $P_{n_1 | n_2}$  is given by (3).

In Figure 4, we derived the stationary probability distribution of the number of cars in a road section, for the two cases of self-supplied section (formula 2) and downstream supplied section (formula 4). The parameters for the considered section and for the downstream section are given as follows.

Section	$L$ (km)	$v$ (km/h)	$w$ (km/h)	$\rho_j$ (veh/km)	$q_{\max}$ (veh/h)	$\rho_{cr}$ (veh/km)	$c$ (veh)
1	0.1	100	50	180	6000	60	18
2	0.1	50	25	180	3000	60	18



**FIGURE 4.** Stationary probability distribution of the number of cars (the traffic state). On the left side, the case of self-supplied section. On the right side, the case of downstream supplied section.

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