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On the robust guidance of users in road traffic networks

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Abstract. We present in this article a model for the guidance of users in road traffic networks. It is well known since decades that a path with a reliable travel time can be obtained by maximizing, over the different possible paths, the probability of realizing a travel time less than a given time budget. We propose here an adaptation of this approach, in order to introduce robustness in the selection of the optimal path. A robust path here is a path that is not likely to change during the travel, and that admits acceptable alternative detours in case of failure.

Keywords: robust guidance, reliable travel time, road networks, traffic modeling.

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INTRODUCTION

The complexity of the guidance of users in road traffic networks is due to the variability and stochasticity of the travel times through those networks. The problem is tackled with several approaches: shortest path - based ones; see for example [7], stochastic methods calculating the probability of arriving at time [6, 2, 9], optimal control approaches solving optimization problems in feedback of the state of the traffic in the network [8, 1], deterministic approaches based on the calculus of travel time guarantees; see for example [5, 4, 3], etc. We are interested here on the stochastic approaches that permit to guarantee reliable travel times and robust paths for the users.

The approach we adopt here is based on the idea of Frank [6] who proposed to derive reliable paths from a given origin to a given destination by maximizing the probability of realizing a travel time less than a given time budget; see also [2] and [9]. In order to give the main idea of the model we propose here, we consider the network of Figure 1. A user departing from node 0 to node 7 needs to be guided in his selection of an appropriate path. We assume that the user would like to minimize his travel time, maximize its reliability, and also optimize the robustness of the path to be selected at node 0. The question for us is how to determine such *optimal* path for that user.

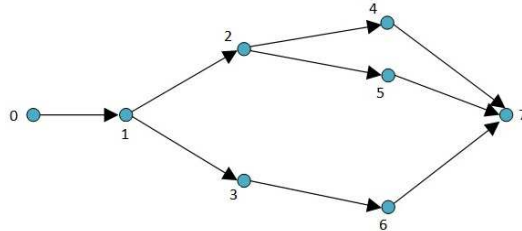


FIGURE 1. Illustration of model (3).

Solution 1 consists in calculating the shortest path in term of average travel time from node 0 to node 7. This solution is known to be non optimal in term of the reliability of travel time of the obtained shortest path. Solution 2 consists in calculating the maximum probability of arriving at node 7, departing from node 0, in a given time budget; see [6, 2, 9]. This solution is clearly better than solution 1. Indeed, solution 2 guarantees the travel time reliability of the selected path.

We will show here that we can still improve (or extend) solution 2 in term of robustness of the selected path. Suppose that the calculus of such maximum probability at the level of node 0 gives the optimal path passing through nodes 3 and 6. The travel times through the links of the network being variable in time, it is possible that, once a user is arrived to node 1, the path passing through nodes 3 and 6 is no more optimal (a posteriori). We can still calculate the

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maximum probability of arriving to node 7, departing from node 1, in the remaining time budget, and, by that, select another optimal path from node 1. However, some users (if not all of them) may be non comfortable with this process of changing the selected path at every node. Moreover, even if the user is not sensitive to the changing of path at every node, he shall require a detour that will not deteriorate the initially promised travel time. Furthermore, if the initially selected path at node 0 (the path passing through nodes 3 and 6) is optimal up to node 3, and becomes no more optimal from the time when the user arrives to node 3, the user will not have any alternative path. The calculus of the maximum probabilities of arriving at the destination node in a given time budget (by solution 2), as proposed in [6, 2, 9], does not guarantee the existence, neither the performance of such alternative detours.

We propose here an adaptation of solution 2 in such a way that the selection of the optimal path from node 0 to node 7 takes into account the robustness of the choice. We mean here by robustness, a kind of guarantee that the proposed path is not likely to change a lot during the travel; and in the case where the path needs to be changed, acceptable detours (that do not deteriorate the initial promised travel time, in its average and its reliability) exist.

THE MODEL

Franck (1969) [6] proposed to determine reliable paths in a road network by calculating the maximum probabilities to reach a given destination departing from a given origin, in a time less than or equal to a given time budget. Without loss of generality, we consider the case of a road network with only one destination node s , where the travel times through the links of the network are correlated. We use the following notations.

- \mathcal{N} : the set of nodes of the network.
- \mathcal{A} : the set of links (arcs) of the network.
- $i^+ = \{j \in \mathcal{N}, (i, j) \in \mathcal{A}\}$, $i^- = \{k \in \mathcal{N}, (k, i) \in \mathcal{A}\}$.
- $p_{ki}(t, y)$: probability that the travel time through link (i, j) is t , knowing that we come from the predecessor node k of i , and that the realized travel time on link (k, i) is y . That is the probability distribution of the travel time through link (i, j) , conditioned by the travel time on link (k, i) .
- $u_{ki}(t, y)$: maximum cumulative distribution of the travel time from node i to the destination node, knowing that we come from node k , and that the realized travel time through (k, i) is y . That is the maximum probability, departing from node i , of arriving to the destination in a time less than t , knowing that we come from node k upstream of node i , and that the realized travel time on (k, i) is y .

The probabilities $u_{ki}(t, y), \forall i \in \mathcal{N}, k \in i^-, t \in [0, T], y \geq 0$ are obtained by solving the following system.

$$u_{ki}(t, y) = \max_{j \in i^+} \int_0^t p_{kij}(w, y) u_{ij}(t - w, w) dw, \quad \forall i \in \mathcal{N} \setminus \{s\}, k \in i^-, 0 \leq t \leq T, 0 \leq y \leq T - t, \quad (1)$$

with $u_{ks}(t, y) = 1, \quad \forall k \in s^-, 0 \leq t \leq T, 0 \leq y \leq T - t$.

We notice that in the case of non-correlated travel times, system (1) is reduced to

$$u_i(t) = \max_{j \in i^+} \int_0^t p_{ij}(w) u_j(t - w) dw, \quad \forall i \in \mathcal{N} \setminus \{s\}, 0 \leq t \leq T, \quad (2)$$

with $u_s(t) = 1, \quad \forall 0 \leq t \leq T$.

Equation (1) tells that the probabilities $u_{ki}(t)$ satisfy a kind of backward dynamic programming system, with a maximization over the successors of node i . The implicit system can be solved, for example, with the successive approximation method; see for example [9]. In term of guidance, the arguments (nodes) of the maximizations in the calculus of the probabilities $u_{ki}(t)$ give the optimal path that guarantees the maximum probability of arriving to s in a time less than t . It is clear that the calculus of those probabilities permits the selection of reliable paths through the network.

We follow here the same idea in term of reliability of the travel time of the selected path, but by introducing a robustness guarantee of the selected path. We mean by robust path here, a path which is flexible, in the sense that, if the path fails before the users who selected it reach the destination node, the path admits acceptable detours (deviations with acceptable travel time). For that, we propose to calculate, for each node i , a probability z_{ki} (which we define below) of reaching the destination node, departing from node k upstream of i , where we take into account the case

where the selected path fails before the users who selected it reach the destination node; for which case, alternative neighboring paths are used.

The probabilities z_{ki} are defined as follows.

$$z_{ki}(t, y) = \sum_{j=1}^d \phi_{kij}(t) \int_0^t p_{kij}(w, y) z_{ij}(t - w, w) dw, \quad \forall i \in \mathcal{N} \setminus \{s\}, k \in i^-, 0 \leq t \leq T, 0 \leq y \leq T - t, \quad (3)$$

with $z_{ks}(t, y) = 1$, $\forall k \in s^-, 0 \leq t \leq T, 0 \leq y \leq T - t$, and where $\sum_{j=1}^d \phi_{kij}(t) = 1, \forall i \in \mathcal{N}, k \in i^-, 0 \leq t \leq T$, and d is the number of successors to be taken into account.

In the calculus of $z_{ki}(t, y)$, instead of maximizing the quantities

$$\int_0^t p_{kij}(w, y) z_{ij}(t - w, w) dw, \quad \text{for } j \in i^+, \quad (4)$$

we propose to take a mean of these quantities, with chosen weights $\phi_{kij}(t), i \in \mathcal{N}, k \in i^-, j = 1, 2, \dots, d, 0 \leq t \leq T$.

The optimal path is still determined by the sequence of the successor nodes which maximizes the quantities (4). We notice that even though the optimal paths are determined by taking the arg-max in both models (1) and (3), the results shall be different since $z_{ki}(t)$ and $u_{ki}(t)$ are calculated with different ways.

We may have $d < |i^+|$, in which case, only some successors of i are considered in the mean. We may also have $d > |i^+|$, in which case, we have $\sum_{j \in i^+} \phi_{kij}(t) < \sum_{j=1}^d \phi_{kij}(t) = 1$. In the latter case, nodes i with small number of successors are penalized (they get low values $z_{ki}(t)$). Therefore, paths passing through those nodes (ie. paths with small number of alternatives or detours) shall have low probabilities to be selected as optimal paths.

We note that d may depend on i and k , in which case, we write d_{ki} . The parameter d (or d_{ki}) is in fact dependent on the weights ϕ_{kij} . Indeed, d is simply the number of non null ϕ_{kij} .

In order that the calculus (3) has a meaning, the weights $\phi_{kij}(t), i \in \mathcal{N}, k \in i^-, j = 1, 2, \dots, d, 0 \leq t \leq T$ must be increasing with the quantities (4). This dependence of $\phi_{kij}(t)$ on those quantities makes the model (3) non trivial. We notice here that if $d = 1$, or if $d > 1$ but for some $\bar{j} \in \{1, \dots, d\}, \phi_{ki\bar{j}}(t) = 1$, and thus $\phi_{kij}(t) = 0, \forall j \neq \bar{j}$, the models (1) and (3) coincide.

By taking a mean in (3) rather than the maximum (as in (1)), we do not only take into account the path maximizing the probabilities $u_{ki}(t)$, but we also take into account the existence and the performance of alternative deviations at each node.

A NUMERICAL EXAMPLE

We take again the network of Figure 1 and consider the non-correlated travel times case. We calculate here the probabilities $z_i(t)$ by solving the system (3) using the method of successive approximations; see for example [9]. The travel times through the links are stochastic with Gamma distributions. The travel times on all the links of the network, except link (1,3), follow Gamma probability distribution with mean 10 and standard deviation 3. The travel time on link (1,3) follow the same probability distribution but with a mean 9 and standard deviation 3.

To reach the destination node (node 7), departing from node 1, we have three paths: path 1: $1 \rightarrow 2 \rightarrow 4 \rightarrow 7$, path 2: $1 \rightarrow 2 \rightarrow 5 \rightarrow 7$, and path 3: $1 \rightarrow 3 \rightarrow 6 \rightarrow 7$. As all the standard deviations of all the links are the same, it is trivial that path 3 is the optimal path by the model (1). Indeed, it is the path with the minimum average travel time, while the standard deviations on all the links are the same.

For this example, we take $d = 2$, and

$$\phi_{kij}(t) = \phi_j, \forall i, k, t, \quad j \in \{1, 2\}.$$

As $d = 2$, we simply denote $\phi = \phi_1$, and then $\phi_2 = 1 - \phi$. We apply model (3) by varying the constant ϕ in $[1/2, 1]$.

By taking $\phi < 1$, we take into account the fact that if we choose path 1 or path 2, we have an option to change the path at the level of node 2, while path 3 does not have such option.

The results of the calculus are presented in Table 1. As expected, we can see that the probabilities $z_i(t)$ are increasing with ϕ . However, as mentioned above, for a given $i \in \mathcal{N}$, the order of the $z_j(t)$ for $j \in i^+$ changes by varying the weights ϕ_j . This changing in the order of $z_j(t), j \in i^+$ induces modifications of the optimal paths.

We see in Table 1 that with $\phi = 1$, and for any budget time, the optimal path is path 3. For example, for a time budget equal to 28 time units, we have $z_1(28) = 0.63$. With $\phi = 0.9$, the same performance in term of reliability of

travel time (ie $z_1(t) = 0.63$) is obtained with a time budget of 30 time units, with path 2 as the optimal path. Therefore, the decision to take here in term of route choice depends on the users. With a time budget $T=30$ time units,

- if a user maximizes the travel time reliability of the paths, without taking into account their robustness (or their flexibility), then he shall choose path 3, because it is the one maximizing the probability of reaching the destination node in the considered time budget ($z_1(30) = 0.76$).
- If a user is satisfied with the value 0.63 for the travel time reliability of paths, and likes rather to optimise the robustness, then he shall choose path 2.

TABLE 1. The probabilities $z_i(t)$ and the optimal paths from node 0 to node 7 in the network of Figure 1, obtained by the model (3).

	time t	20	21	22	23	24	25	26	27	28	29	30
$\varphi_1 = 1$	$z_1(t)$	0.10	0.14	0.20	0.26	0.33	0.41	0.49	0.56	0.63	0.70	0.76
	$succ(1)$	3	3	3	3	3	3	3	3	3	3	3
$\varphi_1 = 0.9$	$z_1(t)$	0.08	0.11	0.16	0.21	0.27	0.33	0.39	0.45	0.51	0.57	0.63
	$succ(1)$	3	3	3	3	3	3	3	3	3	2	2
$\varphi_1 = 0.8$	$z_1(t)$	0.06	0.09	0.13	0.16	0.21	0.26	0.32	0.38	0.44	0.50	0.55
	$succ(1)$	3	3	3	3	3	2	2	2	2	2	2
$\varphi_1 = 0.7$	$z_1(t)$	0.05	0.07	0.1	0.13	0.18	0.22	0.27	0.32	0.37	0.41	0.46
	$succ(1)$	3	3	2	2	2	2	2	2	2	2	2

The value $0.14 = 0.76 - 0.63$ represents the price of robustness to pay in term of the travel time reliability. The time period $2 = 30 - 28$ represents the price of robustness to pay in term of travel time budget. Indeed, with a time budget of 28 time units, the most reliable path is path 3 with the value 0.63. In order to get the same value of travel time reliability, and assure a robust path, we need 2 more time units in our time budget. The selection of path 2 as the optimal path instead of path 3, in the case where $\varphi = 0.9$ is justified by the fact that path 2 is more flexible compaigng to path 3. That is, path 2 has the option of changing at the level of node 2 (change to take path 1).

CONCLUSION

We presented in this abstract a model for the robust guidance of users in road traffic networks. It is a modification of an existing model based on the calculus of the maximum probability to reach a destination node in a given time budget. Our modification permits the selection of an optimal path according to two criteria: the reliability of the path, in term of travel time, and the robustness of the path, in term of flexibility (existence and performance of alternative detours). The model shows how to do a compromise with the two criteria, and gives the prices of robustness in term of travel time reliability, and in term of travel time budget.

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