

# An experimental study and model determination of the stiffness of paper folds

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Workshop on Folding and Creasing of Thin Plate Structures



Advertising slide: CFM 2015!

French Congress on Mechanics is organized

- August 24th- 28th, 2015
- At Lyon

(first) call for abstract ends: April 13<sup>th</sup>

short abstract (2 pages)

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#### **Motivations**

- Folding / unfolding structures
  Deployable panels, adjustable architecture designs
  Medical devices, biomimicry studies
- Paper models
  - Prototyping other material structures Changes in scale and material
- Beyond kinematics (merely pseudo-rigidity assumption)
- Local (macro) paper crease behavior (straight or curved)





# Outlines

- Experiments and results In-plane tensile test and anisotropy Crease behavior (monotonous loading)
- Advanced model identification
  Raw data and PCA
  Deterministic vs. variability
- Discussion
  - Anisotropy
  - Variability
- Conclusions and perspectives



Paper standard characteristics

Arjowiggins Conqueror CX22<sup>™</sup> (120 g/m<sup>2</sup> ECF woodfree pulp uncoated paper) Thanks to J.-M. Baumlin



- Preliminary measurements (ambient temperature 25°C, RH 40%)
  - Mean thickness e = 12 mm
    - ( $\Delta e/e$  variability 3.9%, measurement precision 4.8%)
  - Grammage  $\rho_s = 116 \text{ g/m}_2$

( $\Delta \rho_s / \rho_s$  variability 2.1%, measurement precision 1.7%)

- Tensile in-plane traction test, with DIC for strain field (roughly a factor 2 anisotropy, max shift / max stress < 2.6%)</p>
- No significant dispersion



# Crease model

- Macroscopic and phenomenological
  - Quasi-static
  - Monotonic folding path (at a first step)
  - Local behavior
  - Hinge joint with a torsional spring (possibly non linear)

Could be, for the torque C, folding angle  $\alpha$ , length L and for a given fold orientation:  $C = f(\alpha)L$ 



# Experimental protocol

Single straight crease samples
 Several orientations (longitudinal, transverse, 45°)
 Several lengths (30 mm, 40 mm, 50 mm)

### Creasing process

Mark with thumbnail

Open with a roller (5 kg)

Fold again with the same roller

Unfolding path measures
 By increasing weights
 Opening angle α, torque C

Temperature 20.5-21.5°C, RH 38-47%

Face bending neglected





#### Experimental results

With different orientations (longitudinal, transverse, 45°), lengths  $(n_L = 3)$ , 43 specimens and 611 experimental points









- For a given fold orientation
  - Fold torque function of opening angle  $\alpha$  and fold length *L* expected to be  $C(\alpha, L) = k(\alpha)f(L) + \varepsilon$

no a priori knowledge on functions k, f nor on random variable  $\varepsilon$ 

Single-shot identification

Sets of specimens of same length  $L_{ji}$ ,  $j = 1...n_L$ with somehow unstructured data  $(\alpha_{jii}, C_{ji})$ ,  $i = 1...m_j$ Residuals  $\varepsilon_{ji} = C_{ji} - N(\alpha_{ji})\kappa f_j$ 

With  $N(\alpha_{ji})$  the FE-like shape functions,  $\kappa$  the dof vector for the discretization of  $k(\alpha)$ 



For a given fold orientation

Identification is the optimization problem  $\min_{k,f} e^2 = \|\varepsilon^2\|_F = \sum_{k,f} e^k$ 



• Can be iterated on  $C(\alpha, L) - k(\alpha)f(L) \approx \overline{k}(\alpha)\overline{f}(L)$ 

A generalization of the svd, providing a cross-orthogonality of modes and the singular value (contribution of each mode)  $\sigma$ Using a fixed point algorithm to find the best rank-1 approximations

• 1 user parameter:  $n_{\alpha}$  number of points for the discretization of  $k(\alpha)$ 



- For a given fold orientation
  - Identified modes  $(k(\alpha), f(L))$
  - Model validation
    - 1 predominant mode: no angle-length coupling Quasi-linear f(L): local behavior assumption

$n_{\alpha}$	$\frac{\sigma_1}{\sqrt{m}}$	$\frac{\sigma_2}{\sqrt{m}}$	$\frac{\sigma_3}{\sqrt{m}}$	$\frac{\sigma_2}{\sigma_1}$	e <sub>RMS</sub>
2	1.72	0.040	0.006	2.3%	2.4%
5	1.73	0.030	0.013	1.7%	2.1%
11	1.73	0.033	0.022	1.9%	2%
31	1.73	0.046	0.032	2.7%	2%



 $\alpha$  / degree





- For a given fold orientation
  - Influence of user parameter  $n_{\alpha}$ 
    - Hierarchical modelling Cut-off for the deterministic / random

splitting





#### For a given fold orientation

- Variability issue
  - Mean of random variable ε is zero (constant field belongs to the FE space)





#### For a given fold orientation

Variability issue

Dispersion due to the complex folding process and the protocol Part of the phenomenological model

 $k(\alpha) / N$ 

- For all the fold orientations (crease anisotropy)
  - No neat dependence on the orientation

(N.B. in-plane anisotropy of factor 2)

■ Due to the complex fold process zone? Width ≈ 2.3 thickness





Single identification for all the fold orientations





Single identification for all the fold orientations





Single identification for all the fold orientations





# Outlooks and prospects

# Anisotropy

Large influence on in-plane tension

Small influence on crease behavior

(first order model: no crease orientation dependence)

# Variability

Highly controlled paper making process: reproducibility of tensile tests Uncertainties in the folding process: taking the variability into account Identification of a deterministic part and a variable component Could / should the process be improved?



# Outlooks and prospects

#### Model identification & validation

Locality (linear length dependence, no length-angle cross dependence) One-shot identification procedure with 1 user parameter ( $n_{\alpha}$ ): hierarchical approach

## Model limitations

Limited angle range

Monotonic evolution

No internal variable for damage memory / fatigue / cyclic loading

Applicable to other thin material characterization



# Thank you for your attention



#### Contacts and Structural Mechanics Laboratory



## In-plane anisotropy

#### Tensile tests

Temperature 22-23.5°C, RH 40-45% Lloyd LF Plus machine with 1 kN load cell, 1 mm/min velocity Samples 30 mm wide, 125 mm long Camera & DIC for strain field 8 samples in each direction





# In-plane anisotropy

- Tensile tests
  - Roughly a factor 2 anisotropy
  - Max shift / max stress < 2.6%</p>

Only small data dispersion reproducibility controlled process





# In-plane anisotropy

Microstructure & manufacturing process weakly entangled and in-plane layered fiber arrangement component, within a softer matrix



Tomograph of a sample [Bloch et al, ESRF beamline ID19 Grenoble]

> Example of fiber orientation distribution for machine-made paper [Sampson, Materials properties of paper as influenced by its fibrous architecture, Int. Mat. Rev. 54(3), 2009]





• Optimization problem • Leads to:  $\forall j \in \{1 \dots n_L\}, \quad (\kappa^T M_j \kappa) f_j = \kappa^t c_j$   $M\kappa = c$ • With  $M_j = \sum_{i=1}^{m_j} N(\alpha_{ji})^T N(\alpha_{ji}) \quad c_j = \sum_{i=1}^{m_j} N(\alpha_{ji})^T C_{ji}$   $M = \sum_{j=1}^{n_L} f_j M_j f_j \quad c = \sum_{j=1}^{n_L} f_j c_j$ 

Fixing underdeterminacy with  $\sum_{j=1}^{n_L} f_j^2 = 1$ .



Can be iterated on  $C(\alpha, L) - k(\alpha)f(L) \approx \bar{k}(\alpha)\bar{f}(L)$ Cross-orthogonality of modes  $\sum_{j=1}^{n_L} f_j[\bar{\kappa}^T M_j\bar{\kappa}]\bar{f}_j = 0 \quad \text{and} \quad \kappa^T[\sum_{j=1}^{n_L} \bar{f}_j M_j\bar{f}_j]\bar{\kappa} = 0$ 

Singular value (mode contribution)

$$\sigma = \sqrt{\kappa^T M \kappa} = \sqrt{\kappa^T c}$$



#### For a given fold orientation

- Variability issue
  - Dispersion due to the complex folding process and the protocol Part of the phenomenological model

Example of a pre-crease cardboard industrial process



Industrial process for crease predefined path (cardboard) [Giampieri et al, A constitutive model for the mechanical response of the folding of creased paperboard, Int. J. Solids Struct. 48(16-17), 2011]



#### For a given fold orientation

- Variability issue
  - Mean of random variable  $\varepsilon$  is zero (constant field belongs to the FE space)

#### Variance

$n_{\alpha}$	$\frac{\sigma_1}{\sqrt{m}}$	$\frac{\sigma_2}{\sqrt{m}}$	$\frac{\sigma_3}{\sqrt{m}}$	$\frac{\sigma_2}{\sigma_1}$	e <sub>RMS</sub>	$\sqrt{V}$
2	1.72	0.040	0.006	2.3%	2.4%	0.196
5	1.73	0.030	0.013	1.7%	2.1%	0.177
11	1.73	0.033	0.022	1.9%	2%	0.176
31	1.73	0.046	0.032	2.7%	2%	0.170



#### Crease anisotropy

- For all the fold orientations
  - No neat dependence on the orientation
  - Due to the complex fold process zone? Width ≈ 2.3 thickness







[Rolland du Roscoat et al, Estimation of microstructural properties from synchrotron X-rays microtomography and determination of the REV in paper materials, Acta. Mater. 55(8), 2007]



#### Creasing process



Sectional view of crease

[Nagasawa et al, Effect of crease depth and crease deviation on folding deformation

characteristics of coated paperboard, J. Mat. Proc. Tech. 140(1–3), 2003]



### Crease anisotropy

- Single identification for all the fold orientations
  - Representative model
  - Increase in variability

