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Decentralized Multi-cell Beamforming with QoS Guarantees via Large System Analysis

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Abstract—This work focuses on the power minimization problem while ensuring target rates in the downlink of a multi-cell multi-user MIMO system wherein L base stations (BSs) of N antennas serve in total K single-antenna user equipments. We assume that the transmit antennas at each BS are correlated and propose a decentralized solution to compute an approximation of the optimal beamforming vectors. The analysis is conducted in the asymptotic regime in which N and K grow large with a given ratio K/N . In particular, the proposed solution relies on the exchange of intercell interference terms whose large system approximations are computed at each BS using knowledge of local transmit correlation matrices and non-local pathlosses. Numerical results are used to evaluate the performance loss of the proposed solution compared to the optimal one and to investigate its accuracy in system of finite size.

I. INTRODUCTION

The road forward for satisfying the increasing number of users and high rates expectations in 5G systems is very high spatial utilization. Among the different technologies in this context, massive MIMO is considered as one of the most promising [1]. Under the assumption of uncorrelated channels, if the number of antennas N goes to infinity and the number of UEs K is maintained fixed, the performance of massive MIMO systems become limited only by pilot contamination and simple matched filter (MF) and maximum ratio transmission (MRT) with no cooperation among cells can entirely eliminate the uplink and downlink multicell interference. However, the maximum number of antennas at each base station (BS) is limited in practice. In such a case, interference-aware precoder design with cooperation among cells should be applied for optimal handling of the remaining inter-cell interference (ICI).

Coordinated multi-cell resource allocation is generally formulated as optimization problems in which a desired network utility is maximized subject to some requirements [2], [3]. In particular, the coordinated multi-cell minimum power beamforming approach aims to satisfy a given signal-to-interference-plus-noise ratio (SINR) for all users while minimizing the total transmitted power [2]. In the last years, different centralized and decentralized solutions have been suggested to tackle this problem [2]–[4]. All these solutions require some exchange of information that depends on the actual channel conditions. This makes them unsuited for scenarios with highly varying channels or with large values of N and K (as envisioned in 5G networks).

A possible way out to the above issue relies on conducting the analysis in the large system regime in which N and K grow large with a non-trivial ratio K/N . In these cir-

cumstances, tools from random matrix theory allow to get explicit expressions, that depend only on the large-scale fading components, for (most) performance metrics. The large system analysis of the minimum power beamforming problem was first addressed in [5] for a simple two-cell network with perfect channel state information (CSI). Some more recent results can be found in [6]–[10]. A common drawback of these works is that the target rates are not guaranteed to be achieved when N is finite and relatively small. This is because the approximation errors translate into fluctuations in the resulting SINR values.

To overcome this issue, in [11] the authors consider the case of uncorrelated antennas at the BSs and extend the results in [3] to decompose the multicell beamforming problem into different subproblems (one for each BS) coupled only by the intercell interference among BSs. The large dimension approximation for the ICI terms is then used to derive an approximately optimal distributed algorithm that relies only on the exchange, among nearby BSs, of the large-scale fading components. Compared to the centralized solution, [11] requires lower backhaul exchange and processing load but a slightly higher transmit power. If the per-user correlation matrix model is used [12], then correlation matrices must be also exchanged via the backhaul links. This might be a potential problem when the dimension of the network is large.

To avoid such a potential issue, this work proposes a heuristic simplification of the algorithm presented in [12]. In particular, we assume that only partial knowledge of the non-local statistics is available at each BS. More specifically, each BS b is assumed to estimate the correlation matrices for all local channels (from BS b to all users) while the correlation matrices from all other BSs $i \neq b$ to all users are not known locally at the given BS b . Only the large-scale attenuation (due to pathloss and fading) are available for the non-local channels. The latter are then sent to the corresponding BSs for computing powers and beamforming vectors so as to meet SINR constraints. Numerical results show that the penalty from using partial non-local statistics is slightly higher transmission power than in [12].

II. SYSTEM MODEL

Consider the downlink of a multi-cell multi-user MIMO system composed of L cells where each BS has N antennas. A total number of K single-antenna UEs are randomly dropped in the coverage area. We assume that each UE is attached to a single BS while receiving interfering data from other BSs. We call \mathcal{U}_b the set of UEs served by BS b and denote b_k the

index of the BS associated to UE k . The set of all UEs is denoted by \mathcal{U} whereas \mathcal{B} collects all BS indexes. Under this convention, we define $\mathbf{h}_{b,k} \in \mathbb{C}^N$ as the channel from BS b to UE k . The per-user channel correlation model is adopted [13]. Then, we have that $\mathbf{h}_{b,k}$ is given by $\mathbf{h}_{b,k} = \mathbf{\Theta}_{b,k}^{1/2} \mathbf{z}_{b,k}$ where $\mathbf{z}_{b,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}/N\mathbf{I}_N)$ is the small-scale fading channel and $\mathbf{\Theta}_{b,k} \in \mathbb{C}^{N \times N}$ accounts for the correlation among antennas at the BS. The matrices $\mathbf{\Theta}_{b,k}$ are assumed to change slowly compared to the channel coherence time. The above per-user channel correlation model is quite general and allows to model different propagation environments that naturally arise in cellular networks [13]. In the simple i.i.d case, the correlation matrix $\mathbf{\Theta}_{b,k}$ reduces to $a_{b,k} \mathbf{I}_N$ where $a_{b,k}$ is the large-scale attenuation from BS b to UE k due to pathloss and shadowing.

Denoting by $\mathbf{w}_k \in \mathbb{C}^N$ the precoding vector of UE k from its intended BS, the received signal can be written as

$$y_{b_k,k} = \mathbf{h}_{b_k,k}^H \mathbf{w}_k s_k + \sum_{i \in \mathcal{U}_{b_k} \setminus k} \mathbf{h}_{b_k,k}^H \mathbf{w}_i s_i + \sum_{b \in \mathcal{B} \setminus b_k} \sum_{l \in \mathcal{U}_b} \mathbf{h}_{b,k}^H \mathbf{w}_l s_l + n_k \quad (1)$$

where s_i is the symbol transmitted to UE i with $\mathbb{E}\{|s_i|^2\} = 1$, and n_k is the thermal noise that follows $\sim \mathcal{CN}(0, \sigma^2)$.

III. PROBLEM FORMULATION

The optimization problem for attaining the optimal beamforming vectors $\{\mathbf{w}_k; k = 1, 2, \dots, K\}$ as proposed by [3] can be stated as

$$\begin{aligned} \min_{\{\mathbf{w}_k\}, \{\epsilon_{b,k}\}} & \sum_{k=1}^K \mu_k \|\mathbf{w}_k\|^2 \\ \text{subject to} & \text{SINR}_k \geq \gamma_k \quad \forall k \\ & \sum_{l \in \mathcal{U}_b} |\mathbf{h}_{b,k}^H \mathbf{w}_l|^2 \leq \epsilon_{b,k} \quad \forall b, \forall k \notin \mathcal{U}_b \end{aligned} \quad (2)$$

where the SINR takes the form

$$\text{SINR}_{jk} = \frac{|\mathbf{h}_{b_k,k}^H \mathbf{w}_k|^2}{\sum_{i \in \mathcal{U}_{b_k} \setminus k} |\mathbf{h}_{b_k,k}^H \mathbf{w}_i|^2 + \sum_{b \in \mathcal{B} \setminus b_k} \epsilon_{b,k} + \sigma^2} \quad (3)$$

and $\epsilon_{b,k}$ accounts for the interference experienced by UE k from BS b . Also, $\{\gamma_k\}$ represent the target SINRs whereas the weights $\{\mu_k\}$ reflect the different power budget at each BS. As mentioned in the Introduction, the above problem can be solved using different approaches based for example on centralized SOCP formulation [3] and standard dual or primal decomposition techniques [3], [14]. Next, we first revise the uplink-downlink duality approach and then provide the main results from its large system analysis that will be instrumental to derive a distributed beamforming algorithm.

A. Solution via uplink-downlink duality

Using the uplink-downlink duality [2] the optimal \mathbf{w}_k is found to be $\mathbf{w}_k = \sqrt{\delta_k} \mathbf{v}_k$ with

$$\mathbf{v}_k = \left(\sum_{l \in \mathcal{U} \setminus k} \lambda_l \mathbf{h}_{b_k,l} \mathbf{h}_{b_k,l}^H + \mu_k \mathbf{I}_N \right)^{-1} \mathbf{h}_{b_k,k} \quad (4)$$

where $\{\lambda_k\}$ correspond to the dual uplink powers and $\{\delta_k\}$ are weighting coefficients from which the downlink powers $\{p_k\}$ are obtained as $p_k = \delta_k \|\mathbf{v}_k\|^2$. The values of $\{\delta_k\}$ can be computed as

$$\boldsymbol{\delta} = \sigma^2 \mathbf{G}^{-1} \mathbf{1}_K \quad (5)$$

where the (i, j) th element of $\mathbf{G} \in \mathbb{C}^{K \times K}$ is

$$[\mathbf{G}]_{i,k} = \begin{cases} \frac{1}{\gamma_k} |\mathbf{h}_{b_k,k}^H \mathbf{v}_k|^2 & \text{for } i = k \\ -|\mathbf{h}_{b_k,i}^H \mathbf{v}_k|^2 & \text{for } i \neq k. \end{cases} \quad (6)$$

The dual uplink powers $\{\lambda_k\}$ can be calculated iteratively as follows

$$\lambda_k(t+1) = \lambda_k(t) \frac{\gamma_k}{\Gamma_k(t)} \quad (7)$$

where t is the iteration index and $\Gamma_k(t)$ is given by

$$\Gamma_k(t) = \lambda_k(t) \mathbf{h}_{b_k,k}^H \left(\sum_{l \in \mathcal{U} \setminus k} \lambda_l(t) \mathbf{h}_{b_k,l} \mathbf{h}_{b_k,l}^H + \mu_k \mathbf{I}_N \right)^{-1} \mathbf{h}_{b_k,k} \quad (8)$$

which corresponds to the SINR at iteration t for the set of uplink powers $\{\lambda_k(t)\}$. The above set of equations defines an algorithm which gives the optimal precoding vectors $\{\mathbf{w}_k\}$. However, the evaluation of (5) requires a global channel state information, which makes its distributed implementation a difficult task, especially when N and K are large.

B. Solution via large system analysis

To overcome the above issues, assume that $N, K \rightarrow \infty$ with $K/N \in (0, L)$ so as to use the large system analysis to compute the so-called deterministic equivalents of $\{\lambda_k\}$ and $\{\delta_k\}$. For technical reasons, the following assumption is made.

Assumption 1. *The spectral norm of $\{\mathbf{\Theta}_{b,k}\}$ is uniformly bounded:*

$$\limsup_{N, K \rightarrow \infty} \max_{\forall b, k} \{\|\mathbf{\Theta}_{b,k}\|\} < \infty.$$

Then, the following result is found.

Theorem 1. *Let Assumption 1 hold. Then, as $N, K \rightarrow \infty$ with $K/N \in (0, L)$ we have that $\lambda_k - \bar{\lambda}_k \rightarrow 0$ almost surely where $\bar{\lambda}_k$ is given by*

$$\bar{\lambda}_k = \frac{\gamma_k}{e_{b_k,k}} \quad (9)$$

and can be computed through the following recursive procedure. Let t be the iteration index, then $\lim_{t \rightarrow \infty} \bar{\lambda}_k(t) = \bar{\lambda}_k$ and $\lim_{t \rightarrow \infty} e_{b,k}(t) = e_{b,k}$ where

$$\bar{\lambda}_k(t+1) = \bar{\lambda}_k(t) \frac{\gamma_k}{e_{b_k,k}(t)} \quad (10)$$

with $\{e_{b_k,i}(t)\}$ being obtained as the unique nonnegative solution of the following system of equations:

$$e_{b_k,i}(t) = \frac{\bar{\lambda}_i(t)}{N} \text{Tr} \left(\Theta_{b_k,i} \left(\frac{1}{N} \sum_{l \in \mathcal{U}} \frac{\bar{\lambda}_l(t) \Theta_{b_k,l}}{1 + e_{b_k,l}(t)} + \mu_k \mathbf{I}_N \right)^{-1} \right) \forall i. \quad (11)$$

Then, we have the following result.

Theorem 2. *Let Assumption 1 hold. Then, as $N, K \rightarrow \infty$ with $K/N \in (0, L)$ we have that $[\mathbf{G}]_{i,k} - [\bar{\mathbf{G}}]_{i,k} \rightarrow 0$ almost surely with*

$$[\bar{\mathbf{G}}]_{i,k} = \begin{cases} \frac{\gamma_k}{(\bar{\lambda}_k)^2} & \text{for } i = k \\ -\frac{1}{N} \frac{e'_{b_k,i}}{\bar{\lambda}_k(1+e_{b_k,i})^2} & \text{for } i \neq k \end{cases} \quad (12)$$

where $\{\bar{\lambda}_i\}$ and $\{e_{b,i}\}$ are given by Theorem 1. The vector e'_{b_k} is computed as $e'_{b_k} = (\mathbf{I}_N - \mathbf{L}_{b_k})^{-1} \mathbf{u}_{b_k}$ where

$$[\mathbf{L}_{b_k}]_{j,i} = \frac{\bar{\lambda}_j \bar{\lambda}_i}{N^2} \frac{\text{tr}(\Theta_{b_k,j} \mathbf{T}_{b_k} \Theta_{b_k,i} \mathbf{T}_{b_k})}{(1 + e_{b_k,i})^2} \quad (13)$$

and

$$\mathbf{u}_{b_k} = \left[\frac{\bar{\lambda}_1}{N} \text{tr}(\Theta_{b_k,1} \mathbf{T}_{b_k} \Theta_{b_k,k} \mathbf{T}_{b_k}), \dots, \right. \quad (14)$$

$$\left. \frac{\bar{\lambda}_K}{N} \text{tr}(\Theta_{b_k,K} \mathbf{T}_{b_k} \Theta_{b_k,k} \mathbf{T}_{b_k}) \right]^T \quad (15)$$

with \mathbf{T}_b being the solution of

$$\mathbf{T}_b = \left(\frac{1}{N} \sum_{l \in \mathcal{U}} \frac{\bar{\lambda}_l \Theta_{b,l}}{1 + e_{b,l}} + \mu_b \mathbf{I}_N \right)^{-1}. \quad (16)$$

A known problem with any algorithm relying only on the asymptotic analysis is that the approximation errors translate into fluctuations in the resulting SINR values. Therefore, the target rates are not guaranteed to be achieved when N is finite and relatively small. This issue is addressed in the next section.

IV. DISTRIBUTED OPTIMIZATION

We begin by observing that any fixed ICI term in (2) results in a suboptimal solution that, however, always satisfies the SINR constraints. The price to pay, compared to the optimum, is an increase of the transmit power [11], [12]. Next, we look for a possible solution providing a good approximation of the ICI values by means of a distributed algorithm.

We begin by observing that the ICI from BS b to UE k can be rewritten as

$$\epsilon_{b,k} = \sum_{l \in \mathcal{U}_b} |\mathbf{h}_{b,k}^H \mathbf{w}_l|^2 = \sum_{l \in \mathcal{U}_b} \delta_l |\mathbf{h}_{b,k}^H \mathbf{v}_l|^2. \quad (17)$$

An approximation of the coefficients $\{\delta_l\}$ can be computed as

$$\bar{\delta} = \sigma^2 \bar{\mathbf{G}}^{-1} \mathbf{1}_K \quad (18)$$

where $\bar{\mathbf{G}}$ is obtained through Theorem 2. The terms $|\mathbf{h}_{b,k}^H \mathbf{v}_l|^2$ in (17) corresponds to the non-diagonal elements of $\bar{\mathbf{G}}$. Therefore, $\epsilon_{b,k}$ can be reasonably approximated as

$$\epsilon_{b,k} \approx - \sum_{l \in \mathcal{U}_b} \bar{\delta}_l [\bar{\mathbf{G}}]_{k,l}. \quad (19)$$

A close inspection of Theorems 1 and 2 reveals that the computation of $\{\bar{\lambda}_l\}$ and $\bar{\mathbf{G}}$ requires knowledge of the per-user correlation matrices $\{\Theta_{b,k}\}$. Although each $\Theta_{b,k}$ changes slowly in time compared to small-scale fading components and can be usually represented by simple mathematical models (depending on a small number of system parameters), the exchange of such information among coupled BSs via backhaul may not be practical when the network size becomes large.

To overcome this potential issue, we propose to compute an approximation of the ICI values using full knowledge of local statistics and only partial knowledge of the non-local ones. More specifically, we assume that each BS b is able to estimate (perfectly) the channel correlation matrices $\Theta_{b,k} \forall k$ while the correlation matrices $\Theta_{i,k} \forall k$ from all other BSs $i \neq b$ are not known locally at BS b . Only the large-scale attenuation (due to pathloss and fading) is available for the non-local channels. The first assumption relies on the observation that the correlation matrix remains constant for a sufficiently large number of reception phases to be accurately estimated at the BS. The second one is motivated by the observation that most current standards require the users to periodically report Received Signal Strength Indication (RSSI) values to their serving BSs (usually using orthogonal uplink resources). Under the assumption that nearby BSs are also able to receive such RSSI measurements, a partial knowledge of the non-local channel statistics can be obtained without any backhaul information exchange.

Under the above assumptions, each BS can locally compute (through Theorems 1 and 2) approximations for $\{\bar{\lambda}_l\}$ and $\bar{\mathbf{G}}$. The latter can thus be used in (19) to locally obtain ICI values. Finally, each BS must send the UE specific ICI values to the respective BSs. The solution is summarized in Algorithm 1.

Algorithm 1 Distributed optimization

- 1: **loop**
 - 2: **if** Any change in the user statistics or during the initial stage **then**
 - 3: Users broadcast the pathloss information to the nearby BSs using uplink resources.
 - 4: Each BS b gets the approximation for uplink and downlink powers using Theorems 1 and 2 where BS b locally assumes $\Theta_{i,k} = a_{i,k} \mathbf{I}_N \forall k$ for all $i \neq b$.
 - 5: ICI values are computed from (19) at each BS.
 - 6: Each BS b sends the ICI values $\epsilon_{b,k}, \forall k \notin \mathcal{U}_b$ to their corresponding serving BSs.
 - 7: **end if**
 - 8: Use the approximated ICI values as fixed $\epsilon_{b,k}$ in (2) and solve the sub-problems locally to get the approximately optimal downlink beamformers.
 - 9: **end loop**
-

The exchange rate of Algorithm 1 results to be significantly lower than that required by [12]. This is because only some UE specific scalars (ICI values) must be sent to their serving BSs at each update interval while correlation matrices must be exchanged in [12].

V. NUMERICAL ANALYSIS

Monte-Carlo simulations are now used to validate the performance of the proposed solution. We consider a network with $L = 7$ cells and assume that UEs are equally distributed among cells. The pathloss function is obtained as $a_{b,k} = (d_0/d_{b,k})^{2.5}$ where $d_{b,k}$ represents the distance between BS b and UE k and $d_0 = 1$ m is the reference distance. The pathloss from a generic BS to the boundary of the reference distance of the neighboring BS is fixed to 75 dB (which corresponds to a 1000 m distance between BSs). The correlation among channel entries is modeled by assuming a diffuse 2-D field of isotropic scatterers around the receivers [13], [15]. The waves arrive from an angular spread from φ_{\min} to φ_{\max} . The correlation matrix for antenna element spacing of Δ is given by [15].

$$[\Theta_{b,k}]_{j,l} = \frac{1}{\varphi_{b,k}^{\max} - \varphi_{b,k}^{\min}} \int_{\varphi_{b,k}^{\min}}^{\varphi_{b,k}^{\max}} e^{i \frac{2\pi}{w} \Delta |j-l| \cos(\varphi)} d\varphi \quad (20)$$

where w is the wavelength and $\varphi_{b,k}^{\min} = -\pi$ whereas $\varphi_{b,k}^{\max}$ is given by the position of UE k with respect to BS b . The antenna element spacing is fixed to half the wavelength $\Delta = 1/2w$.

Fig. 1 shows the transmit SNR in dB (defined as the transmit power normalized to the noise variance σ^2) for the proposed algorithm labelled as “partial statistical information” when $K/N = 1/2$ and $\gamma_k = 0$ dB $\forall k$. Comparisons are made with simple ZF beamforming, optimal centralized solution [3] and decentralized solution with full statistical information [12]. It can be seen that the proposed solution almost overlaps with the full statistical information algorithm. Alternatively, a fully i.i.d decentralized solution is also proposed, which relies only on reported pathloss information (ignoring also the local correlation information). In this case the ICI values calculated at all BSs are the same globally, thus, there is no need to exchange the ICI values on the backhaul. However, the full decentralization is attained with at least 2dB loss compared to the proposed solution.

VI. CONCLUSION

This work focuses on the power minimization problem in multicell MU-MIMO networks and proposes a heuristic distributed beamforming algorithm that inherits the benefits of [12] without requiring the exchange of correlation matrices. Numerical results indicate that the proposed solution achieve the same performance as the case in which correlation matrices are exchanged. This is achieved even for small system dimensions with a reasonable performance loss with respect to the optimal one.

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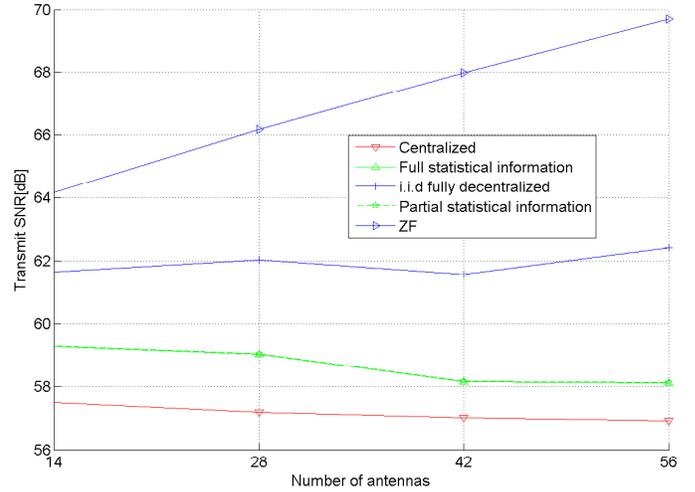


Fig. 1. Transmit SNR versus number of antennas with $K/N = 1/2$ and $\gamma = 0$ dB.

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