



Moritz Schlick's reading of Poincaré's theory of relativity

Scott Walter

► To cite this version:

Scott Walter. Moritz Schlick's reading of Poincaré's theory of relativity. Fynn Ole Engler, Mathias Iven. Moritz Schlick: Ursprünge und Entwicklungen seines Denkens, Parerga Verlag, pp.191-203, 2010. hal-01235724

HAL Id: hal-01235724

<https://hal.science/hal-01235724>

Submitted on 30 Nov 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Moritz Schlick's reading of Poincaré's theory of relativity

Scott A. Walter*

Fynn Ole Engler and Mathias Iven, eds,
Moritz Schlick: Ursprünge und Entwicklungen seines Denkens,
Schlickiana 5, Berlin, Parerga, 2010, pp. 191–203.

Henri Poincaré's conventionalist philosophy of geometry looms large in Moritz Schlick's early writings on the theory of relativity. But as Tom Ryckman (2005, p. 52) points out, Poincaré's star begins to pale for Schlick in the early 1920s, when his philosophy takes a decidedly empiricist turn. This turn of events may naturally be viewed as a by-product of the much-celebrated confirmation of Einstein's general theory of relativity by the British eclipse expeditions of 1919. While the significance of the empirical success of Einstein's general theory of relativity on Schlick's philosophy can hardly be doubted, Schlick's defection from Poincaré's conventionalist philosophy of space may have an additional source: dissatisfaction with Poincaré's theory of relativity.

My discussion of Schlick's view of Poincaré's theory begins with a review of the difference between Einstein's and Poincaré's theories, that turns on the form of light-waves as judged by observers in inertial frames of reference. I summarize the evolution of Poincaré's philosophy of geometry in the early years of relativity theory, which Schlick ignored throughout his life, and in the second section of the paper, I recall Schlick's discussion of Poincaré's views on the relativity of space, in which Schlick focused on similitude relations, at the expense of relations of covariance. I then take up Lindemann's preface to the English translation of Schlick's *Raum und Zeit in der gegenwärtigen Physik*, where the issue of the shape of light-shells in inertial frames is raised. It is this very issue, I suggest, which distinguished most sharply Poincaré's theory of relativity from Einstein's special theory of relativity, and may have prompted Schlick to move away from

*scott.walter [at] univ-nantes.fr, University of Nantes,
Centre François Viète

Poincaré’s neo-Kantian conventionalism towards an Einsteinian empiricism with constitutive principles.

1 Einstein’s light-sphere, Poincaré’s light-ellipsoid

The compatibility of Einstein’s postulates of relativity and light-speed invariance followed for Einstein (1905) from an argument which may be summarized as follows.¹ Let a spherical light-wave propagate from the common coordinate origin of two inertial frames designated k and K at time $t = \tau = 0$. In system K the wave spreads with velocity c such that the wavefront is expressed as:

$$x^2 + y^2 + z^2 = c^2 t^2. \quad (1)$$

To obtain the equation of the wavefront in frame k moving with velocity v with respect to K , we apply a certain transformation of coordinates from K to k to the equation (1) and find:

$$\xi^2 + \eta^2 + \zeta^2 = c^2 \tau^2. \quad (2)$$

Since (1) goes over to (2), Einstein observed, the light-wave that is spherical in K is likewise spherical in k , it propagates with the same velocity c , and consequently, “our two basic principles are mutually compatible” (Einstein 1905, § 3, p. 901).

Henri Poincaré was quick to grasp the idea that the principle of relativity could be expressed mathematically by transformations that form a group. This fact had several immediate consequences for Poincaré’s understanding of relativity. Notably, Poincaré identified invariants of the Lorentz transformation directly from the fact that the transformation may be construed as a rotation about the coordinate origin in four-dimensional space (with one imaginary axis). Any transformation of the Lorentz group, he noted further, may be decomposed into a dilation and a linear transformation leaving invariant the quadratic form $x^2 + y^2 + z^2 - t^2$, where light velocity is rationalized to unity (Poincaré 1906, § 4).

Somewhat curiously, for one who had contributed to discussions on the so-called Riemann-Helmholtz-Lie problem of space, Poincaré avoided drawing consequences for the foundations of geometry from the “new mechanics” of the Lorentz group, with one exception. He observed that measurement of length had implied the displacement of solids considered to be rigid, and yet:

[T]hat is no longer true in the current theory, if we admit the Lorentzian contraction. In this theory, two equal lengths are, by definition, two

¹The notation has been modernized. For technical details, see my study of light-spheres in relativity, to appear in a forthcoming volume of *Einstein Studies*, edited by David Rowe. The content of this section draws largely on the latter study.

lengths that are spanned by light in the same lapse of time. (Poincaré 1906, p. 132).

Light-waves, in other words, constituted the new standard of both temporal and spatial measurement. But how was one to go about measuring lengths with Lorentz-contracted rods?

Poincaré’s measurement problem called for a solution, and shortly, Poincaré provided one. In lectures delivered at the Sorbonne in 1906–1907, he interpreted the Lorentz transformation with respect to a geometric figure representing the surface of a light-wave, which I will refer to as a “light-ellipsoid”, following Darrigol’s coinage (1995). The light-ellipsoid is characteristic of Poincaré’s approach to kinematics, illustrating it on four separate occasions, with minor variations, during the final six years of his life, from 1906 to 1912.²

Like Einstein, Poincaré considered electromagnetic radiation to be the only physical phenomenon not subject to Lorentz-contraction. In his first philosophical commentary on relativity theory, he drew a series of consequences for the philosophy of phenomenal space, during which he invoked a thought-experiment, borrowed from Delbœuf, which proceeded as follows. Let all objects undergo the same expansion overnight; in the morning, the unsuspecting physicist will not notice any change. Poincaré likened his fantasy of an overnight spatial expansion to the relativity of moving bodies in contemporary physics, in that Lorentz’s theory admitted a contraction of bodies in their direction of motion with respect to the ether. In the latter case, Poincaré similarly disallowed detection of the contraction, due to compensating effects on measuring instruments (Poincaré 1908, pp. 96–100).

Also like Einstein, Poincaré admitted the principle of observational equivalence among inertial observers. He retained, however, a semantic distinction between true and apparent quantities, corresponding respectively to quantities measured in a frame at absolute rest, and those measured in a frame in uniform motion with respect to the absolutely-resting frame.³ To convey his meaning, Poincaré called up an observer in uniform motion with respect to a frame K , considered to be at rest. The observer in motion is at rest with respect to a frame k , in which all measuring rods of length ℓ' are contracted in the direction of their motion with respect to the ether, according to Lorentz contraction:

$$\ell' = \gamma^{-1} \ell, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

²A description of the light-ellipsoid appears in *Science et méthode* (1908, p. 239).

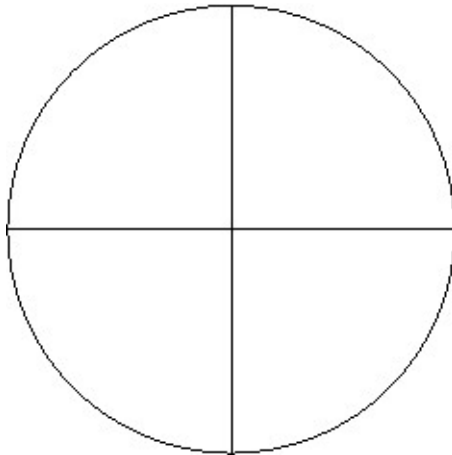
³The notion of an absolutely-resting frame remained abstract for Poincaré, who later embraced the conventionality of spacetime, preferring Galilei spacetime over Minkowski spacetime (Walter 2009).

where ℓ designates the length of the rod in frame K , v is the velocity of frame k with respect to K , and c is the velocity of light, a universal constant. Physicists at rest in k can correct for the Lorentz-contraction of their rulers due to their velocity v with respect to K ; the correction factor for terrestrial observers was calculated by Poincaré to be on the order of $5 \cdot 10^{-9}$.

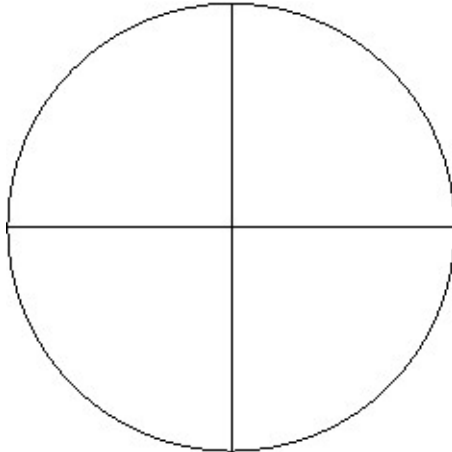
Provided we neglect any motion of the Sun with respect to the ether, Poincaré's measurement protocol allows us to ascertain the "true" dimensions of objects in motion with respect to the ether frame K , measured by co-moving observers. To see better how this protocol might work in practice, imagine a material sphere, clamped to a workbench. When measured at rest with respect to K , an orthogonal projection of the sphere is a circle (Fig. 1a). Since measuring rods are contracted in the sense of the observer's motion, the "true" form of this sphere is a flattened ellipsoid. We multiply the measured diameter by the above-mentioned correction factor to find the length of the ellipsoid's minor axis. In a direction orthogonal to the sphere's motion with respect to K , neither measuring rod nor sphere is contracted, such that the measured diameter of the sphere is equal to the length of the major axis of the ellipsoid. When projected orthogonally onto a plane parallel to the motion of the sphere, the flattened ellipsoid has the shape of an ellipse, as shown in Fig. 1c, where the eccentricity is greatly exaggerated for the sake of illustration.

Not all objects in motion are subject to Lorentz-contraction, the unique exception being electromagnetic waves. Light-waves propagate isotropically in the ether, according to Poincaré, and consequently, if instead of a material sphere we had a spark generator at rest in K , the resulting light shell has the form of a sphere, as measured by observers at rest with respect to K (Fig. 1b).

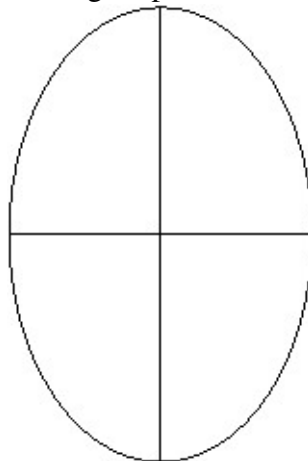
When the same light shell is measured in the moving frame k with concrete rods, at a certain moment of apparent time t' determined by light-synchronized clocks at rest in k , the shell is naturally found to be spherical. Knowing that the measuring rods are actually Lorentz-contracted, we correct for the contraction in the same manner as in the case of the material sphere, and realize that the light shell has the form of an ellipsoid of revolution, the major-axis of which is aligned with the direction of motion of k with respect to K . Orthogonal projection of the corrected form of the light-shell on a plane parallel to the motion of k is shown in Fig. 1d (again, with eccentricity corresponding to a frame velocity approaching that of light).



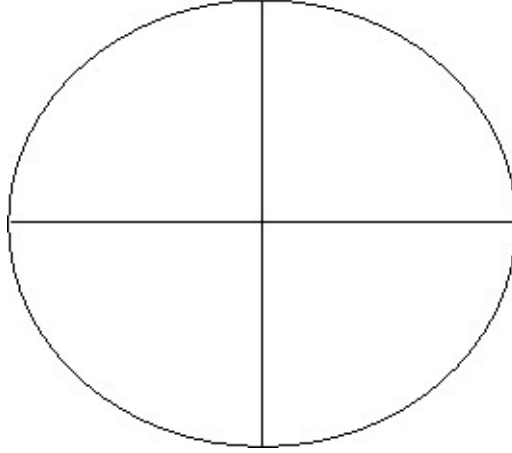
a. Matter-sphere in K.



b. Light-sphere in K.



c. Matter-sphere in k.



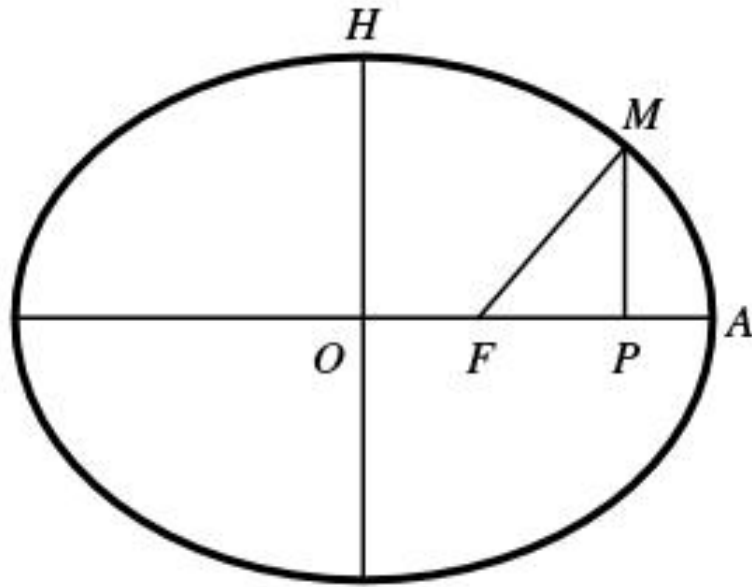
d. Light-sphere in k .

FIGURE 1. Illustration of Poincaré's view of the "true" form of a fast-moving material sphere and a light shell in frames K and k , at rest (a, b), and in uniform motion (c, d), respectively, with respect to the quiescent ether. Measurements are performed by observers at rest with respect to frame K (a, b), or k (c, d).

In his Sorbonne lectures, Poincaré employed the light-ellipsoid in pursuit of two objectives. First, he showed that length and time measurements are transitive for observers in uniform motion, by imagining a light source in uniform motion v , that passes through the coordinate origin O at time $t_0 = 0$. At a later time t , the light-wave originating at t_0 and propagating in all directions with speed c has a spherical wavefront of radius $OH = ct$.

At an instant of apparent time t' , an observer at rest in frame k measures the light-sphere's radius with a concrete rod, that she knows to be *contracted* in her direction of motion, and finds the light shell to be perfectly spherical. She corrects her measurements by a Lorentz factor, and realizes that the "true" locus of light coincides with an ellipsoid of rotation *elongated* in her direction of motion with respect to K . The exact dimensions of the light-ellipsoid depend on the moment of apparent time t' at which the length measurements are performed. However, the *form* of the light-ellipsoid is the same for all apparent times ($t' > 0$), in that the ellipsoidal eccentricity e is a constant function of frame velocity v : $e = v/c$.

In Poincaré's hands, the light-ellipsoid was a powerful tool. From the suppositions of Lorentz contraction and invariance of light-waves in inertial frames, he demonstrated that the apparent time of moving observers is a linear function of apparent displacement, and consequently, that space and time measurements are transitive among frames moving uniformly with respect to the ether. It allowed



contraction factor	$\gamma = 1/\sqrt{1 - v^2/c^2}$
semimajor axis	$a = OA = \gamma ct$
semiminor axis	$b = OH = ct$
eccentricity	$e = \sqrt{1 - b^2/a^2} = v/c$
focal distance	$f = OF = \gamma vt$
apparent time	$t' = FM/c$
app. displacement	$x' = FP$

Figure 2: A section of Poincaré's light ellipsoid (with modern notation).

him to derive the Lorentz transformation, making his light-ellipsoid section the first graphical illustration of kinematic relations in the new mechanics.⁴

The source of the stark contradiction between Einstein's and Poincaré's views of the form of a light-wave lies in their variant protocols for length measurement. Instead of considering all inertial frames to be equivalent with respect to space and time measurement, as recommended by Einstein, Poincaré employed a privileged frame. In any non-privileged inertial frame, bodies in motion are contracted, and time intervals are dilated with respect to the privileged frame.

Like Einstein, Poincaré considered light-waves to be the only objects not subject to Lorentz contraction. In his first philosophical commentary on relativity theory, he proposed a thought-experiment, which proceeds as follows: let all objects undergo the same expansion overnight; in the morning, the unsuspecting physicist will not notice any change. The worlds of last night and this morning are then, as Poincaré writes, "indiscernible".

Up to this point in Poincaré's parable, there is no link to the principle of relativity, since all objects are at relative rest in his imaginary universe. In what follows, however, Poincaré likens the overnight expansion to the relativity of moving bodies:

In both cases, there can be no question of absolute magnitude, but [only] of the measurement of magnitude by means of some instrument; this instrument may be a meter-stick, or a segment spanned by light; we measure only the relation of magnitude to instrument; and if this relation is altered, we have no means of knowing whether it is the magnitude or the instrument that has changed. (Poincaré 1908, p. 100)

According to Lorentz's electron theory, all bodies contract in their direction of motion with respect to the ether, but the contraction escapes detection in principle, because of compensating effects on our measuring instruments.⁵

Schlick was impressed with Poincaré's argument, which he rehearsed in his philosophical treatise on the theory of relativity, *Raum und Zeit in der gegenwärtigen Physik* (1917, reed. Engler and Neuber, eds., 2006). What Schlick focused upon was the objectivity of length measurement:

If we, for instance, assumed that the dimensions of all objects are lengthened or shortened in one direction only, say that of the Earth's

⁴Details are supplied in a forthcoming article in *Einstein Studies*. My reconstruction of the light-ellipsoid differs from that of Darrigol (2006), in that length measurement is carried out by comoving observers at an instant of apparent time t' , instead of true time t . [Note added 2013-12-19: My views on this topic have changed due to archival finds in 2012 (see Walter 2014).]

⁵For an insightful review of Lorentz's electron theory, see Darrigol (1994).

axis, we should again not notice this transformation, although the shape of bodies would have changed completely, spheres becoming ellipsoids of rotation, cubes becoming parallelepipeds, and indeed perhaps very elongated ones. (Schlick 1920b, p. 26; cf. Engler and Neuber, eds., 2006, p. 202)

Where Poincaré took care to distinguish between the principle of similitude and the principle of relativity, Schlick wiped out any such distinction. We recall that for Poincaré, just as for Einstein, light-waves are not subject to Lorentzian contraction.

In fact, according to Poincaré, if it were not for the invariance of the form of light-waves with respect to uniform motion, there would be no talk of Lorentz contraction at all:

If the wave-surfaces of light were subject to the same deformations as material bodies, we would never have noticed the Lorentz-FitzGerald deformation. (Poincaré 1908, p. 100)

This is a crucial observation for Poincaré, but one that Schlick ignores in 1917. Why did Schlick not engage with the fundamental question of kinematics raised by Poincaré's text? Certainly by 1917, the relevance of Einstein's special theory of relativity to Poincaré's discussion of the objectivity of Lorentz contraction would have been apparent to Schlick.

If Schlick elided discussion of Poincaré's theory of relativity in 1917, it may well be due to a change in status of the Lorentz contraction. Most notably in this respect, Hermann Minkowski, in his celebrated lecture in Cologne, "Space and time" (Minkowski 1909), stigmatized the low evidensory status of the contraction in the Lorentz's electron theory as a "gift from above". As Minkowski's space-time theory would have it, the phenomenon of Lorentz contraction is a simple manifestation of the spacetime metric.

Such an evolution in Schlick's understanding appears all the more likely, in light of the fact that Poincaré's own views on the relativity of space and time evolved after 1907. In 1912, for example, he formulated a response of sorts to Minkowski's Cologne lecture, entitled, quite naturally, "Space and time". Most notably, Poincaré allowed the symmetry group of mechanics to define the concepts of space and time. What this amounted to, in Poincaré's conventionalist scheme, was the adoption of *spacetime* as a convention, where earlier, he considered the geometry of phenomenal *space* to be conventional. According to Minkowski, empirical and theoretical considerations forced scientists to adopt Minkowski spacetime. Poincaré retorted, in effect, that scientists are free to choose between *two* spacetime conventions, and that his own preference was for Galilei spacetime,

rather than Minkowski spacetime (Walter 2009). Like other philosophers, however, Schlick did not acknowledge Poincaré's twelfth-hour embrace of spacetime conventionalism.

2 Poincaré's philosophy of relativity and Schlick's empiricist turn

One consequence of adopting the convention of either Galilei or Minkowski spacetime is that the geometry of phenomenal space is fixed by this choice, and the spatial geometry in both cases is that of Euclid. And while Poincaré recognized this consequence, he did not acknowledge that it mooted his pre-relativist philosophy of geometry. As for Schlick, the immediate consequence of his neglect of Poincaré's switch to conventional spacetime is that his version of Poincaré's philosophy of space remained that of the author of *La science et l'hypothèse* (1902), and not "*L'espace et le temps*" (1912).

In his essay, "*Die philosophische Bedeutung des Relativitätsproblems*," for instance, Schlick rehearsed Poincaré's pre-relativist doctrine of space, according to which there is no fact to the matter of the geometry of phenomenal space (Schlick 1915, 150). Among the readers of Schlick's paper was Albert Einstein. Schlick sent him a copy of his paper, prompting a quick reply on 14 December, 1915. Einstein fully approved the view of relativity presented by Schlick, remarking that "There is nothing in your exposition with which I find fault."⁶ Further correspondence between Schlick and Einstein bears witness to Einstein's early admiration for Schlick's conventionalist reading of general relativity, which Schlick defended in opposition to the neo-Kantian interpretations of the Marburg School. For example, in the original edition of *Raum und Zeit in der Gegenwärtigen Physik*, Schlick limited the role of empirical facts to that of a helpful guide, informing us only "whether it is more practical to employ Euclidean geometry or a non-Euclidean geometry in physical descriptions of nature" (Engler and Neuber, eds., 2006, p. 211).

The first inkling in Schlick's published work that something is not right about Poincaré's theory of relativity comes not from Schlick himself, but from the scientist who introduced his work to English-language readers, the Oxford physicist Frederick A. Lindemann. The key philosophical achievement of the special theory of relativity, Lindemann observed, was that the descriptions of events depend on the state of motion of the observer. This spatio-temporal relativity was not just another philosopher's dream, or theorist's convention for Lindemann, but a view forced upon scientists:

⁶Cited by Howard (1984, p. 618).

The reasons which force this conclusion upon the physicist may be made clear by considering what will be the impression of two observers passing one another who send out a flash of light at the moment at which they are close together. The light spreads out in a spherical shell, and it might seem obvious, since the observers are moving relatively to one another, that they cannot both remain at the center of this shell. The celebrated Michelson-Morley experiment proves that each observer will conclude that he *does* remain at the center of the shell. The only explanation for this is that the ideas of length and time of the one observer differ from those of the other. (Lindemann, in Schlick 1920b, p. iv.)

Contrary to Lindemann's assertion, an alternative explanation for the Michelson-Morley experiment had been proposed by Poincaré, as mentioned above. Lindemann's observation that light-spheres remain light-spheres in special relativity was something of a commonplace among relativists by 1910. It may, however, have led Schlick to realize that Poincaré and Einstein differed over the shape of light-waves for inertial observers, and that this difference was significant for the philosophy of space and time.

Beyond the latter conflict between Poincaré's and Einstein's theories of relativity, the question remains of what relation, if any, Schlick noticed between these theories of physics and Poincaré's philosophy of geometry. Lindemann's preface notwithstanding, there is no compelling evidence that Schlick was aware of the difference between Poincaré's theory of relativity and Einstein's special theory of relativity. As we saw above, Schlick was quite familiar with Poincaré's discussion of the relation between the new mechanics and conventionalist philosophy of geometry, but did not see fit to comment.

Some of Schlick's contemporaries drew such conclusions, beginning with Minkowski. In 1908, Minkowski argued that only the four-dimensional world is real, thereby implicitly contradicting Poincaré's geometric conventionalism. Twelve years later, Schlick agreed wholeheartedly with Minkowski's spacetime realism, paraphrasing the mathematician in the pages of *Die Naturwissenschaften*:

Everything real is four-dimensional; three-dimensional bodies are just as much mere abstractions as are lines or surfaces.⁷

Schlick's awareness of the problem posed by Poincaré's theory of relativity for Poincaré's conventionalist philosophy is similarly suggested by an observation made the following year, to the effect that the special theory of relativity is "irreconcilable" with Galilean kinematics, just as general relativity is irreconcilable

⁷"Alles Wirkliche ist vierdimensional; dreidimensionale Körper sind genau so gut bloße Abstraktionen wie Linien oder Flächen." Schlick (1920a, p. 474).

with Euclidean geometry.⁸ These remarks of Schlick's are not, however, explicitly tied to Poincaré. As mentioned above, in 1912 Poincaré explained in essence that the theory of relativity is wholly compatible with Galilean kinematics, and that furthermore, Galilei spacetime is more convenient than Minkowski spacetime. Schlick's remarks on the subject in 1921 suggest a disagreement with Poincaré's view; they do not establish that Poincaré's conventionalism was his target.

As for general relativity, Poincaré died a full year before Einstein began work on a relativistic theory of gravitation based on variably-curved (pseudo-Riemannian) spacetime. In a much-commented lecture delivered to the Prussian Academy of Science in 1921, Einstein (1921) engaged with Poincaré's philosophy of geometry. On this occasion, Einstein acknowledged that Poincaré's philosophy of geometry, according to which the facts can never decide the question of the geometry of phenomenal space, is correct, "sub specie æterni". He also opined that the current state of theoretical physics was such that there was little choice but to admit notions such as rigid rods and ideal clocks, which have no exact referent in reality. Einstein's remark applies to both the special and general theories of relativity, and would appear to constitute strong support for the conventionalist interpretation of these theories, as laid out in Schlick's writings of the time.

At about this time, however, Schlick began to move away from conventionalism, toward a new empiricism. It appears unlikely that Einstein was the source of Schlick's dissatisfaction with conventionalism in the early 1920s, in light of his remarks to the Prussian Academy. Perhaps Lindemann's praise of an Einsteinian version of special relativity led Schlick to reconsider the relation between Poincaré's new mechanics and his philosophy. Whatever the source of Schlick's change of heart may be, his embrace of a Minkowskian spacetime ontology marks a turning point in his relation to Poincaré's conventionalist philosophy of geometry.

References

- Darrigol, O. The electron theories of Larmor and Lorentz: a comparative study. *Historical Studies in the Physical and Biological Sciences* 24(2) (1994): 265–336. URL <http://www.jstor.org/stable/27757722>.
- . Henri Poincaré's criticism of fin de siècle electrodynamics. *Studies in History and Philosophy of Modern Physics* 26 (1995): 1–44.
- . The genesis of the theory of relativity. In *Einstein, 1905–2005*. Edited by

⁸"Nun ist die Spezielle Relativitätstheorie mit den Sätzen der Galileischen Kinematik, die Allgemeine außerdem noch mit den Sätzen Euklids unvereinbar." Schlick (1921, p. 99).

- T. Damour, O. Darrigol, B. Duplantier, and V. Rivasseau, 1–32. *Progress in Mathematical Physics* 47. Basel: Birkhäuser, 2006.
- Einstein, A. Zur Elektrodynamik bewegter Körper. *Annalen der Physik* 322 (1905): 891–921. URL <http://gallica.bnf.fr/ark:/12148/bpt6k2094597/f896.image.langEN>.
- . *Geometrie und Erfahrung*. Berlin: Springer, 1921. URL http://alberteinstein.info/vufind1/images/einstein/ear01/view/1/1011_000000592.pdf.
- Engler, F. O. and Neuber, M., eds. *Moritz Schlick Gesamtausgabe, Abteilung I, Veröffentlichte Schriften, Band 2: Über die Reflexion des Lichtes in einer inhomogenen Schicht; Raum und Zeit in der gegenwärtigen Physik*. Berlin: Springer, 2006.
- Howard, D. Realism and conventionalism in Einstein's philosophy of science: the Einstein-Schlick correspondence. *Philosophia Naturalis* 21 (1984): 616–629.
- Minkowski, H. Raum und Zeit. *Jahresbericht der deutschen Mathematiker-Vereinigung* 18 (1909): 75–88. URL <http://www.digizeitschriften.de/dms/resolveppn/?PPN=GDZPPN002122049>.
- Poincaré, H. Sur la dynamique de l'électron. *Rendiconti del Circolo Matematico di Palermo* 21 (1906): 129–176. URL <http://henripoincarepapers.univ-lorraine.fr/chp/hp-pdf/hp1906rp.pdf>.
- . *Science et méthode*. Paris: Flammarion, 1908. URL http://ia700403.us.archive.org/0/items/scienceetmethod00poin/scienceetmethod00poin_bw.pdf.
- Ryckman, T. A. *The Reign of Relativity*. Oxford: Oxford University Press, 2005.
- Schlick, M. Die philosophische Bedeutung des Relativitätsproblems. *Zeitschrift für Philosophie und philosophische Kritik* 159 (1915): 129–175.
- . *Raum und Zeit in der Gegenwärtigen Physik; Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie*. Berlin: Springer, 1917.
- . Naturphilosophische Betrachtungen über das Kausalprinzip. *Naturwissenschaften* 8(24) (1920a): 461–474.

- . *Space and Time in Contemporary Physics: An Introduction to the Theory of Relativity and Gravitation*. Oxford: Oxford University Press, 1920b. URL <http://ia331343.us.archive.org/0/items/spaceandtimeinco011187mbp/>.
- . Kritizistische oder empiristische Deutung der neuen Physik? *Kant-Studien* 26 (1921): 96–111. URL <http://ia360910.us.archive.org/2/items/kantstudienphilo2627kantuoft/>.
- Walter, S. A. Hypothesis and convention in Poincaré’s defense of Galilei spacetime. In *The Significance of the Hypothetical in the Natural Sciences*. Edited by M. Heidelberger and G. Schiemann, 193–219. Berlin: Walter de Gruyter, 2009. URL <http://scottwalter.free.fr/papers/2009-heidelberger-walter.html>.
- . Poincaré on clocks in motion. *Studies in History and Philosophy of Modern Physics* 47(1) (2014): 131–141. URL <http://scottwalter.free.fr/papers/2014-shpmp-walter.html>.