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# Multimode Time-Constrained Scheduling Problems with Generalized Temporal Constraints and Labor Skills

Tamara Borreguero Sanchidrián · Christian Artigues · Álvaro García Sánchez · Miguel Ortega Mier · Pierre Lopez

## 1 Introduction

Scheduling problems have been the subject of continuous research since the early days of operations research. They are NP-hard optimization problems and, in practice, among the most intractable classical ones. Henceforth, most of research works on scheduling are focused on one of the most difficult problems that is the Resource Constrained Project Scheduling Problem (RCPSP), which consists in scheduling several tasks subject to resource and precedence constraints [1]. Brucker *et al.* [2] provided a classification for this kind of problems together with an overview on existing solution methods. There have been a wide range of studies on both heuristic and metaheuristic methods for solving the RCPSP, as well as different Mixed-Integer Linear Programming (MILP) models [3], [4], [5], [6], [7]. Recently, Koné *et al.* [8] proposed the use of event-based formulations that, despite a poor LP relaxation have the advantage to be able to tackle instances having large time horizons. However, the event-based formulations proposed

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by Koné *et al.* are suitable for the standard RCPSP, which includes some assumptions that are too restrictive for many applications [5]. Therefore, it is of great interest to improve this kind of formulations so that they can be used on more practical contexts.

Furthermore, the RCPSP focuses on minimizing the project makespan given a set of available resources. Although this is appropriate for many real problems there are other wide range of cases where the main objective is to minimize the resource consumption.

This is the case in industries, such as aircraft or ships manufacturing, where the total makespan is usually fixed by the expected production rate or the client demand [9]. It can also be applied to the field of project scheduling, where the makespan is fixed and the objective is to engage a minimum number of resources. Those problems are Time-Constrained Scheduling Problems (TCSP), which have been very little treated in the literature. Although Möhring introduced them in 1984 [10], few recent references address this type of problems: [11], [12], [13] and [9]. Neither of them provide with an exact method for this problem and, moreover, the first two address some very specific mono-mode cases.

In this work we have solved a TCSP related to the scheduling of tasks to be done on a platform from an aircraft final assembly line. It is a multimode TCSP with generalized temporal constraints and several labor skills. Actually, our contribution can be stated as follows. First, on the modeling side, it includes the allowance of multiple modes per task, not only linked to the number of workers but also to their skill. Furthermore, it uses general temporal constraints, including some that are not commonly addressed for neither the TCSP nor the RCPSP. Moreover, we have developed two MILP event-based formulations that provide us with an exact MILP formulation for the TCSP. Finally, we are currently working on the use of constraint programming to solve the problem, in order to be able to compare the performance of both types of paradigms and, ultimately, to hybridize them.

## 2 Problem Statement

Aeronautical assembly lines consist of a series of platforms where different works are executed. Each product has to go through all the platforms. At the same time, the line is synchronized, which means that the time that each product remains on a platform is always the same and equal to the rate at which the assembly line produces its output. Each platform has a fixed makespan and a number of tasks to be performed. The scheduling decision consists on establishing the order in which the tasks will be done along with the resources allocated to each of them, given the line *takt time* (pace of production) and a set of workers per platform.

In accordance with the classification  $\alpha|\beta|\gamma$  introduced by Brucker *et al.* [2], the scheduling of the tasks from an aeronautical platform is denoted by:

$MPS_m, \sigma, \rho | temp | \sum c_k \max r_{kt}$ , with:

- $\alpha = MPS_m, \sigma, \rho$ . Each activity can be processed in several alternative modes and there exists a set of renewable resources available for each time period during the project execution: the number of operators (each of them belonging to a profile) and the space in each of the platform's working areas. Also, each mode for an activity defines a combination of operator profile, number of operators and durations. All the operators assigned to an activity must be from the same profile and the range of possible numbers of allocated operators per task is independent of the chosen profile.

- $\beta = temp$ . There are precedence constraints (task  $w'$  cannot start until task  $w$  has been completed), non-parallel (also called disjunctive) constraints (tasks  $w$  and  $w'$  cannot be in progress at the same time, but there is no predefined precedence relation between them), and maximal time lags between tasks (task  $w'$  must start within a maximal time after  $w$  has been completed). All the temporal constraints are independent from the mode in which a task is executed.
- $\gamma = \sum c_k \max r_{kt}$ . The objective function is to minimize the resource investment, which is in this case equivalent to the minimization of the labor cost of the assembly, as the operators once assigned to a platform stay working on it for all the takt time.

### 3 Solving Techniques: Two MILP Formulations and CP

Different MILP formulations have been proposed to solve the RCPSP. The first ones were *Discrete time formulations*, proposed by Pritsker *et al.* [14]. Afterwards, *Continuous time formulations* were proposed by Alvarez-Valdés and Tamarit (Forbidden sets formulations, [15]) and Artigues *et al.* (Flow-based continuous time formulations, [16]). More recently, *Event-based formulations* were developed by Koné *et al.* in 2011 [8] from a model introduced by Zapata *et al.* [17]. They provided different methods (including MILP exact methods and a heuristic) and concluded that event-based formulations outperformed the previous MILP models and performed even better than the heuristic for some highly cumulative instances, as it is our case.

However, Koné *et al.*'s event-based formulations are suitable for the standard RCPSP, which includes some assumptions that are too restrictive for many applications [5], including ours. Extension of previously existing event-based models to our problem is not straightforward. The main variables  $x_{we}$ ,  $y_{we}$  and  $z_{we}$  (related to start, finish, or in-process for activity  $w$  at event  $e$ , respectively) have been modified with two new sub-indices in order to deal with the multiple modes per task. As well as this, the original formulations included only general precedence constraints. Thus, new constraints and variables have been added in order to take into account the maximal time lags and non-parallel constraints. Finally, the objective function has been modified to tackle with the time constraint approach.

For the performance analysis, we have created a new set of instances of up to 11 tasks each, due to the uncommon structure of the problem with respect to the previous existing instance libraries. Throughout the computational results we have solved the instances up to optimality with both event-based formulations and compared the performance of each of them. The results of these comparisons are consistent with the ones reported by Koné *et al.* for the single mode RCPSP with only precedence constraints [8]: one of the formulations (so-called *On/Off*) outperforms the other on most of the instances. Finally, we have identified the number of events as one of the major factors that have an impact on the instance hardness (see Figure 1).

Our next step is to improve these results by incorporating alternative approaches in our method. In particular, Constraint Programming (CP) has been proven to be an efficient method on several combinatorial optimization problems, especially scheduling problems. Therefore, our current works are on the proposition of a global scheduling constraint, and associated filtering techniques, to solve the RCPSP under study. CP solution scheme encompasses also constraint propagation techniques that can be used as pre-processing to calculate the relevant number of events; this issue would lead to major performance improvements. A promising research direction is to propose a

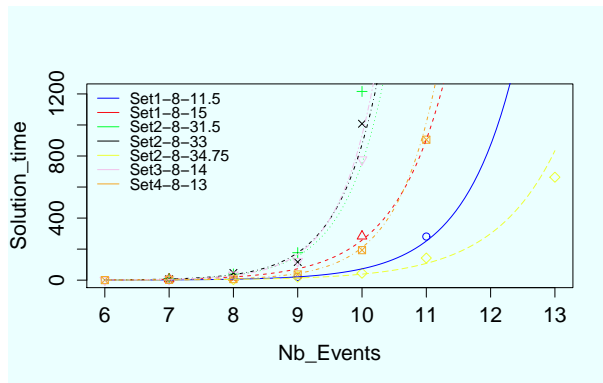


Fig. 1 Solution time of event-based formulations *vs.* number of events

hybrid MILP/CP large neighborhood search heuristic, as the one proposed for the MISTA challenge 2013 on the multi-mode RCPSP [18].

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