



Algebraic representations of ergodic actions

Bruno Duchesne

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Arbeitsgemeinschaft: Superrigidity

Organised by
Uri Bader, Haifa
Alex Furman, Chicago
Jesse Peterson, Nashville

30 March – 5 April 2014

ABSTRACT. The purpose of the Arbeitsgemeinschaft was to review old and new phenomena of rigidity in mathematics. The broad spectrum of such results was covered, such as Margulis-Zimmer superrigidity, cocycle and character rigidity.

Mathematics Subject Classification (2010): 20G05, 22E, 22A26, 28D05, 47C15.

Introduction by the Organisers

The Superrigidity Arbeitsgemeinschaft was attended by about 50 participants. Most of them were young researchers or PhD students representing different branches of mathematics and with different backgrounds. Rigidity problems were studied since decades and (as it is visible in the program below) are still very attractive and dynamical areas of research. Rigidity is an interdisciplinary area of mathematics which combines elements of ergodic theory, Lie groups, actions of groups, von Neumann algebras and others. To deal with such a variety of material to cover the talks were divided into sections. Usually in each section first we discussed an appropriate classical rigidity results followed by recent ones. When it was possible, the focus was put on particular examples which illustrated the general theorems. The short description of each section with reference to the abstracts is given below.

It is a pleasure to thank the institute and the organizers for their effort for providing a pleasant and fruitful meeting.

B: Background talks. The purpose of this section was to review some classical material essential in the incoming sections. The starting point was the notion of amenable actions on measured spaces (à la Zimmer), which is an indispensable tool in most rigidity results (*B1*). Then we passed to the strong negation of amenability, that is Kazhdan property (T), (FH) and its consequences (*B2*). On the analytic side, we recalled some basic theory of von Neumann algebras (*B3*). To facilitate later talks on superrigidity of von Neumann algebras the notion of measured equivalence relation was introduced (*B4*).

C: Character superrigidity The introduction to characters and basic facts as correspondence between extremal characters and finite factors representation were given. Then we investigated character rigidity phenomena of Thompson's group F (*C1*). The next examples we covered were recent results on characters rigidity of $PSL(n, \mathbb{Z})$ (by B. Bekka) and $PSL(2, k)$ for k infinite field (Peterson, Thom). The applications to freeness of ergodic actions and factor representations were discussed (*C2*). Following Creutz and Peterson we proved character rigidity for Λ an irreducible lattice in a product $G \times H$, where G is a non-compact simple Lie group with (T) and H is a totally disconnected non-discrete simple group with the Howe-Moore property (*C3 and C4*).

D: Deformation and rigidity techniques. Cocycle superrigidity. In this sections we dealt with Popa and Ioana Cocycle Superrigidity (*D1*) as well as basic applications to Orbit Equivalence rigidity (*D2*). The analytic counterpart was the rigidity phenomenon for crossed product von Neumann algebras associated with group actions (*D3 and D4*).

S: Margulis-Zimmer superrigidity for higher rank Lie groups and their lattice. Here we concentrated on the new approach to celebrated Margulis-Zimmer superrigidity due to U. Bader and A. Furman. After introducing the language of algebraic representations and birepresentations and proving crucial facts (*S1 and S2*) the superrigidity theorem of Margulis for lattices in $SL_3(\mathbb{R})$ is demonstrated (*S3*). In the last lecture it was showed how classical Margulis normal subgroup theorem can be deduced from Factor Theorem (*S4*).

A: From Superrigidity to Arithmeticity of lattices It was showed how Margulis Arithmeticity theorem follows from Superrigidity theorem (*A1*). Subsequently we discussed Margulis Commensurator Superrigidity and how it implies arithmeticity criterion (*A2*).

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Arbeitsgemeinschaft: Superrigidity**Table of Contents**

Carlos A. De la Cruz Mengual	
<i>B1: Amenability of Groups and Group Actions</i>	961
Stephan Tornier	
<i>B2: Property (T), (FH) and cohomology</i>	961
Tim de Laat	
<i>B3: Von Neumann algebras</i>	962
Daniel Hoff	
<i>B4: Measured Equivalence Relations</i>	962
Niels Meesschaert and Yash Lodha	
<i>C1: Character superrigidity: Thompson's group</i>	963
Chenxu Wen	
<i>C2: Character rigidity for special linear groups (after Peterson-Thom)</i> .	963
Michael Cantrell	
<i>D1: Deformation rigidity methods for cocycles</i>	964
Steven Deprez	
<i>D2: Deformation rigidity methods for cocycles</i>	964
Bruno Duchesne	
<i>S1: Algebraic representations of ergodic actions</i>	965
Thierry Stulemeijer	
<i>S2: bi-algebraic representations of bi-ergodic actions</i>	966
Jean Lecureux	
<i>S3: Higher rank Lattices Super-rigidity</i>	967
Holger Kammeyer	
<i>S4: Factor and normal subgroup theorems for lattices in products</i>	967
Remi Boutonnet and Peter Verraedt	
<i>D3 & D4: Unique group-measure space decomposition</i>	968
F. Le Maître and S. Raum	
<i>C3 & C4: Character rigidity for commensurators</i>	968
Gregor Masbaum	
<i>A1: How Superrigidity implies Arithmeticity</i>	969
Alin Galatan	
<i>A2: Commensurator Superrigidity</i>	969

Abstracts

B1: Amenability of Groups and Group Actions

CARLOS A. DE LA CRUZ MENGUAL

This is a background talk on amenability of locally compact groups and of their actions. For groups, we present a characterization of amenability in terms of the existence of fixed points of their affine actions, as in [1], and discuss some examples. Then, we introduce the notion of amenable action: an action $G \curvearrowright (S, \mu)$ [where S is a Borel space and μ is a G -quasi-invariant measure on S] is amenable if for every compact metric G -space X , there exists a measurable map $S \rightarrow \text{Prob}(X)$ that is G -equivariant up to a null set. We conclude by showing that if Γ is a lattice of G and $P \leq G$ is closed and amenable, then the action $\Gamma \curvearrowright \frac{G}{P}$ is amenable; in particular, this holds if G is a semisimple Lie group and P is minimal parabolic. This fact in the latter situation is relevant when proving Margulis' superrigidity theorem.

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B2: Property (T), (FH) and cohomology

STEPHAN TORNIER

In this talk we gave a definition of Property (T) for locally compact groups, examples and non-examples. We focused on the following material. Hereditary properties: Morphisms with dense image, extensions, lattices. Relation to amenability: A locally compact amenable groups has Property (T) if and only if it is compact. Consequences of Property (T): Unimodularity, compact Hausdorff abelianization, compact generation. Property (FH) for locally compact groups. Continuous affine isometric actions and continuous cohomology. Property (T) and reduced continuous cohomology. Dependence of Property (T) and (FH) on whether $\mathbb{R}$ and/or $\mathbb{C}$ are allowed as fields in the respective definition. Equivalence of Property (T) and (FH) for locally compact σ -compact groups. We gave a proof of $(\text{FH}) \Rightarrow (\text{T})$.

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- [1] B. Bekka, P. de la Harpe, and A. Valette, *Kazhdan's property (T)*, New Mathematical Monographs, vol. **11**, Cambridge University Press, Cambridge, 2008.

B3: Von Neumann algebras

TIM DE LAAT

In this talk, we recalled some basic theory on von Neumann algebras that was needed as background for the talks (C1)-(C4) and (D1)-(D4). Starting from the Bicommutant Theorem, we introduced the type decomposition, types of factors, conditional expectations, and normal faithful traces on finite von Neumann algebras. After that, we discussed amenability and property (T) for von Neumann algebras, which are operator algebraic analogues of these well-known properties for groups. We finished with the notion of Cartan subalgebras and Popa's intertwining-by-bimodules theorem. In the talk, the group von Neumann algebra served as a motivating example.

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B4: Measured Equivalence Relations

DANIEL HOFF

The primary goal of this talk was to give some of the background necessary to motivate and facilitate later talks on superrigidity in von Neumann algebras. The group measure space von Neumann algebra arising from a non-singular action of a countable group on a countably separated measure space was constructed and the type classification for factors arising in this way was given. Restricting to the case of standard probability spaces, Zimmer's Theorem on injective von Neumann algebras coming from amenable actions was stated. Then, in order to motivate the study of measured equivalence relations and Cartan subalgebras, Singer's Theorem was stated and discussed. Measured equivalence relations were then defined, and the talk concluded with brief remarks about their associated von Neumann algebras.

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C1: Character superrigidity: Thompson's group

NIELS MEESSCHAERT AND YASH LODHA

In the first half of the talk, we give an introduction to characters on countable groups and prove that there exists a one-to-one correspondence between extremal characters and equivalence classes of finite factor representations. In the second half of the talk, we give a short introduction to Thompson's group F and present Dudko and Medynets result ([1]) stating: every extremal character of F is either the regular character or arises as a homomorphism of the abelianisation F/F' to the unit circle.

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C2: Character rigidity for special linear groups (after Peterson-Thom)

CHENXU WEN

A character on a group G is a positive definite function $\varphi : G \rightarrow \mathbb{C}$ which is invariant under conjugation and is normalized so that $\varphi(e) = 1$. Via the GNS-construction, characters are naturally related to the unitary representations of the group on finite von Neumann algebras.

Recently, Bekka proved in [1] that for $G = PSL(n, \mathbb{Z})$, $n \geq 3$, G has character rigidity. He also noticed that from this fact it follows that the only II_1 -factor representation for these groups must be the left regular representation. More recently, Peterson and Thom [2], proved the character rigidity for $PSL(2, k)$, where k is a infinite field. They also got some applications for this result to the question of freeness of ergodic actions of those groups.

In the talk I will go through the main idea of the proof of the result of Peterson-Thom, and compare it with the Bekka's proof.

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- [2] Jesse Peterson and Andreas Thom, *Character rigidity for special linear groups*, Journal für die reine und angewandte Mathematik (2014).

D1: Deformation rigidity methods for cocycles

MICHAEL CANTRELL

We prove Popa and Ioana Cocycle Superrigidity. Given a countable group Γ and a probability measure-preserving action $\Gamma \curvearrowright (X, \mu)$, we say $\Gamma \curvearrowright (X, \mu)$ is cocycle-superrigid if for every countable group Λ and every measurable cocycle $c : \Gamma \times X \rightarrow \Lambda$, c is measurably conjugate to a homomorphism, i.e. there exists a homomorphism $\rho : \Gamma \rightarrow \Lambda$ and a measurable map $\phi : X \rightarrow \Lambda$ such that for all $\gamma \in \Gamma$ and μ -a.e. $x \in X$, $c(\gamma, x) = \phi(\gamma \cdot x) \rho(\gamma) \phi(x)^{-1}$. A concrete version of Popa's Cocycle Superrigidity Theorem is that if Γ is a countable group with property T, then the Bernoulli action $\Gamma \curvearrowright ([0, 1], \text{lebesgue})^\Gamma$ is cocycle-superrigid. A concrete version of Ioana's Cocycle Superrigidity Theorem is that if Γ is a countable group with property T, then any profinite action of Γ is cocycle-superrigid, up to finite index.

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D2: Deformation rigidity methods for cocycles

STEVEN DEPREZ

Let $\Gamma \curvearrowright (X, \mu)$ be a free, ergodic, measure-preserving with cocycle superrigidity. Assume moreover that Γ does not have finite normal subgroups. We proved a theorem of Popa, that if such an action is orbit equivalent to some $\Lambda \curvearrowright (Y, \nu)$, then this orbit equivalence is actually a conjugation.

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S1: Algebraic representations of ergodic actions

BRUNO DUCHESNE

Given an ergodic action of a locally compact T on a standard probability space (X, μ) and a homomorphism $S \rightarrow G$ where G is a real algebraic group, we explain how to represent algebraically the action $S \curvearrowright X$. It was the first talk in a series of three talks on the new approach of U. Bader and A. Furman on Margulis-Zimmer superrigidity.

1. INTRODUCTION

Margulis superrigidity theorem had a very strong impact because of its main consequence: the proof of arithmeticity of lattices in algebraic groups over local fields with higher rank. It still continue to inspire actual research. Recently U. Bader and A. Furman [2] published a new paper explaining a quite simple and systematic approach of superrigidity phenomenon in higher rank based on original ideas of Margulis and Zimmer [3, 4].

The general philosophy is based on the fact that ergodic actions in the algebraic world are poorer than general ones since they reduce to the homogeneous ones. The key tool to see this property is *smoothness*. This property allows the previous authors to construct the category of algebraic representations of ergodic actions and prove that this category has an initial object which is unique up to isomorphism. In particular, this uniqueness property will lead to the notion of *birepresentations* for commuting actions.

This report is based on a talk that was the first one among three talks on [2]. The two others were given by Thierry Stulemeijer and Jean Lécureux. The main goal was to emphasize ideas instead of technicalities. In particular, we didn't speak about cocycles nor algebraic groups over general fields with an absolute value. We concentrated on the case of a lattice Γ in $\mathrm{SL}_n(\mathbb{R})$ with $n \geq 3$ and proved that any homomorphism from Γ to some simple real algebraic group extend to $\mathrm{SL}_n(\mathbb{R})$.

2. SMOOTHNESS OF ALGEBRAIC ACTIONS

Let T be locally compact second countable group acting measurably on some standard probability space (Y, μ) preserving the measure class of μ .

Definition. *The action $T \curvearrowright Y$ is smooth if $T \backslash Y$ is countably separated as measurable space.*

The main point about smooth actions is the fact that ergodic smooth actions are actually transitive (the measure is supported on one orbit). This is a direct consequence of the definition. Let us give two examples. The first one is an irrational rotation of the circle. The orbits space is ugly, in particular not countably separated. The second one is the linear action of the diagonal group $\left\{ \begin{bmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{bmatrix} \mid \lambda \in \mathbb{R} \right\}$ of $\mathrm{SL}_2(\mathbb{R})$ on the plane $\mathbb{R}^2$. The action is perfectly smooth as one can convince himself with a drawing.

The reason for this action to be smooth is the fact it is algebraic. More precisely, a group acting algebraically on an algebraic variety has locally closed orbits (in the Hausdorff topology) [4, Theorem 3.1.3] and thanks to Effros-Glimm theorem [4, Theorem 2.1.14], this is equivalent to say that the action is smooth.

3. ALGEBRAIC REPRESENTATIONS

Assume that there is a continuous homomorphism $T \rightarrow G$ where G is some connected non compact simple Lie group without center.

Definition. An algebraic representation of $T \curvearrowright Y$ is a measurable T -equivariant map $Y \rightarrow V$ where V is some algebraic variety on which G acts algebraically.

Algebraic representations form a category for which morphisms are G -equivariant maps. One main point in [2] is to show there is an initial object in that category which is unique up to isomorphism. Moreover, using smoothness, it is shown this initial object V_0 is a *coset representation* that is there is an algebraic subgroup H such that $V_0 = G/H$.

A priori, this category can be reduced to the trivial representation, that is V_0 is a point. In case, the action $T \curvearrowright Y$ is ergodic and amenable then it is guaranteed there is a non-trivial algebraic representation. Let us sketch the argument. Fix P some minimal parabolic of G . One knows that G/P is a compact metrizable space and thus amenability implies there is a T -map $Y \rightarrow \text{Prob}(G/P)$. Even if $\text{Prob}(G/P)$ is not an algebraic variety, the action of G on $\text{Prob}(G/P)$ is smooth and stabilizers of probability measures on G/P are algebraic [4, 1]. In particular from the map $Y \rightarrow \text{Prob}(G/P)$ one get another map $Y \rightarrow G/H$ where H is an algebraic subgroup of G . This shows that the considered category is not reduced to the trivial algebraic representation.

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S2: bi-algebraic representations of bi-ergodic actions

THIERRY STULEMEIJER

Building on the previous talk about algebraic representation of ergodic actions, we define the slightly more general category of bi-algebraic representation of bi-actions.

We prove that under strong ergodic assumptions, this category still has an initial object. The proof follows essentially the same lines than the corresponding statement for algebraic representation.

Finally, we end the talk by showing the invariance of this initial object for commuting actions.

The talk covered section 9 of [1].

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S3: Higher rank Lattices Super-rigidity

JEAN LECUREUX

In this talk we combine the tools developed in the previous talks (algebraic representations and bi-representations) and give a complete proof of the superrigidity theorem of Margulis for lattices in $SL_3(R)$: let G be such a lattice, and H be a simple, non-compact, center-free Lie group. Then any morphism from G to H with Zariski dense image extends to a morphism from $SL_3(R)$ to H .

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S4: Factor and normal subgroup theorems for lattices in products

HOLGER KAMMEYER

In this talk we outlined how the classical Margulis normal subgroup theorem can be deduced from a Factor Theorem. This theorem measurably identifies spaces, which are acted on by a higher rank lattice, with a flag variety. Building on this, we presented the more recent normal subgroup theorem for lattices in products of just noncompact groups due to Bader–Shalom. The proof has a similar outline. We proved the key step, the intermediate factor theorem, and stressed that it crucially relies on the uniqueness of Poisson boundary maps.

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D3 & D4: Unique group-measure space decomposition

REMI BOUTONNET AND PETER VERRAEDT

We illustrate deformation-rigidity techniques for von Neumann algebras by proving the following theorem.

Theorem ([1]). *Let Γ be a countable discrete group, and assume that*

- *there exists a nonamenable subgroup $H < \Gamma$ such that the pair (Γ, H) has relative property (T), and*
- *there exists a mixing orthogonal representation $\pi : \Gamma \rightarrow \mathcal{O}(H_{\mathbb{R}})$ and an unbounded 1-cocycle b for π .*

Let $\Gamma \curvearrowright (X, \mu)$ be a free ergodic p.m.p. action and put $M = L^\infty(X) \rtimes \Gamma$. Whenever $M = L^\infty(Y) \rtimes \Lambda$ for another free ergodic p.m.p. action $\Lambda \curvearrowright (Y, \nu)$, there exists a unitary $u \in \mathcal{U}(M)$ such that $uL^\infty(Y)u^ = L^\infty(X)$.*

This result describes some rigidity phenomenon for crossed product von Neumann algebras associated with group actions. It allows to retrieve the orbit equivalence relation from the von Neumann algebra. We present the approach introduced by S. Vaes, [2].

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C3 & C4: Character rigidity for commensurators

F. LE MAÎTRE AND S. RAUM

In these two talks we gave a proof of the following theorem of Creutz-Peterson [1]: if Λ is an irreducible lattice in a product $G \times H$, where G is a non-compact simple Lie group with (T) and H is a totally disconnected non-discrete simple group with the Howe-Moore property, then every finite factor representation of Λ is either the regular representation or finite dimensional. As an example, one may consider $\Lambda = PSl_n(\mathbb{Z}[1/p])$ embedded diagonally into $PSl_n(\mathbb{R}) \times PSl_n(\mathbb{Q}_p)$. Such a result generalizes the Margulis Normal Subgroup Theorem for Λ , whose proof consists in playing property (T) against amenability. The first talk was devoted to the property (T) part, and used another result of Creutz-Peterson on the absence of nontrivial finite factor representations for H . It was shown that under the hypothesis that a finite factor representation π of Λ satisfies $\pi(\Gamma)''$ amenable, such a representation has to be finite dimensional. So the second talk focused on proving that whenever π is not regular, $\pi(\Gamma)''$ needs to be amenable. It relied on the Poisson boundary of $\pi(\Gamma)''$, which is an amenable von Neumann algebra and, using the contractivity of the action of G on its Poisson boundary, turns out to be isomorphic to $\pi(\Gamma)''$ in our setting.

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- [1] D. Creutz and J. Peterson, *Character rigidity for lattices and commensurators*. ArXiv e-prints, 2013.

A1: How Superrigidity implies Arithmeticity

GREGOR MASBAUM

Margulis' Arithmeticity Theorem states that irreducible lattices in semisimple groups of higher rank are arithmetic. The goal of the talk was to explain what is an arithmetic lattice and to deduce Margulis' Arithmeticity Theorem from Margulis' Superrigidity theorem.

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A2: Commensurator Superrigidity

ALIN GALATAN

The aim of the lecture is to sketch a proof of the fundamental theorem of Margulis concerning lattices in semisimple Lie groups, under the assumption that the commensurator of the lattice is dense in the ambient group. This way, we can prove arithmeticity of these lattices in semisimple Lie groups, not necessarily of higher rank (for example, $SL_2(\mathbb{R})$). We will actually prove the following:

Let S be a non-discrete locally compact, second countable group, and $\Gamma \subset S$ a lattice, Λ a dense subgroup in S such that $\Gamma \subset \Lambda \subset \text{Commens}_S(\Gamma)$. Then for any connected, simple, center-free, non-compact, real Lie group, and any $\rho : \Lambda \rightarrow G$ a homomorphism with Zariski dense image, ρ extends to a continuous epimorphism $\hat{\rho} : S \rightarrow G$.

The proof of this theorem will assume the existence and uniqueness of the boundary map as a black box. More exactly, we will use the following:

Given a lsc group S , there exists a measurable S space (B, μ) with the following properties. Given any lattice $\Gamma \subset S$ and linear representation $\rho : \Gamma \rightarrow G$ in a simple real Lie group, there exists a measurable Γ equivariant map $\phi : B \rightarrow G/P$ where $P \subset G$ is a minimal parabolic subgroup. Moreover, such a map ϕ is unique.

Using this and the notion of quasi-projective transformations (pointwise limits of $PGL(d, \mathbb{R})$ acting say on a projective space) that were introduced by Furstenberg, we will prove the theorem, and say how the proof of arithmeticity for higher rank can be adapted for the commensurator case.

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Reporter: Michał Marcinkowski

Participants

Prof. Dr. Miklos Abert

Alfred Renyi Institute of Mathematics
Hungarian Academy of Sciences
P.O.Box 127
1364 Budapest
HUNGARY

Dr. Vadim Alekseev

Mathematisches Institut
Georg-August-Universität Göttingen
Bunsenstr. 3-5
37073 Göttingen
GERMANY

Dr. Hiroshi Ando

Department of Mathematical Sciences
University of Copenhagen
Universitetsparken 5
2100 Copenhagen
DENMARK

Dr. Michael Björklund

Department of Mathematics
Chalmers University of Technology
412 96 Göteborg
SWEDEN

Mohamed Bouljihad

Mathématiques
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Remi Boutonnet

Mathématiques
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Caterina Campagnolo

Section de Mathématiques
Université de Genève
Case postale 240
1211 Genève 24
SWITZERLAND

Michael Cantrell

Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
Chicago, IL 60607-7045
UNITED STATES

Alessandro Carderi

Mathématiques
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Marcus De Chiffre

Mathematisches Institut
Universität Leipzig
04103 Leipzig
GERMANY

Dr. Tim de Laat

Department of Mathematics
KU Leuven
Celestijnenlaan 200 B
P.O. Box 2400
3001 Leuven
BELGIUM

Carlos A. De La Cruz Mengual

Departement Mathematik
ETH-Zentrum
Rämistr. 101
8092 Zürich
SWITZERLAND

Prof. Dr. Christopher Deninger

Mathematisches Institut
Universität Münster
Einsteinstr. 62
48149 Münster
GERMANY

Dr. Steven Deprez

Dept. of Mathematical Sciences
University of Copenhagen
Universitetsparken 5
2100 Copenhagen
DENMARK

Rolando de Santiago

Department of Mathematics
College of Liberal Arts & Sciences
University of Iowa
14 MLH
Iowa City, IA 52242
UNITED STATES

Bruno Duchesne

Université Henri Poincaré
Nancy 1
Institut Elie Cartan
P.O. Box 239
54506 Vandoeuvre-les-Nancy
FRANCE

Dr. Vladimir Finkelshtein

Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
Chicago, IL 60607-7045
UNITED STATES

Prof. Dr. Alex Furman

Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
851 S. Morgan Street
Chicago, IL 60607-7045
UNITED STATES

Prof. Dr. Damien Gaboriau

Dept. de Mathématiques, U.M.P.A.
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Alin Galatan

Department of Mathematics
University of California at
Los Angeles
Los Angeles, CA 90095-1555
UNITED STATES

Daniel Hoff

Department of Mathematics
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0112
UNITED STATES

Sebastian Hurtado

Department of Mathematics
University of California, Berkeley
970 Evans Hall
Berkeley CA 94720-3840
UNITED STATES

Dr. Holger Kammeyer

Mathematisches Institut
Universität Bonn
Endenicher Allee 60
53115 Bonn
GERMANY

Prof. Dr. Benjamin Klopsch

Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
40225 Düsseldorf
GERMANY

Dr. Jean Lécureux

Laboratoire de Mathématiques
Université Paris Sud (Paris XI)
Batiment 425
91405 Orsay Cedex
FRANCE

Waltraud Lederle

Departement Mathematik
ETH-Zentrum
Rämistr. 101
8092 Zürich
SWITZERLAND

Dr. Francois Le Maitre

Dept. de Mathématiques, U.M.P.A.
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Yash Lodha

Department of Mathematics
Cornell University
Malott Hall
Ithaca, NY 14853
UNITED STATES

Michał Marcinkowski

Institute of Mathematics
University of Wrocław
50-384 Wrocław
POLAND

Prof. Dr. Gregor Masbaum

Institut de Mathématiques de Jussieu
Case 247
Université de Paris VI
4, Place Jussieu
75252 Paris Cedex 05
FRANCE

Niels Meesschaert

Department of Mathematics
KU Leuven
Celestijnenlaan 200 B
P.O. Box 2400
3001 Leuven
BELGIUM

Baptiste Olivier

Department of Mathematics
Technion - Israel Institute of
Technology
Haifa 32000
ISRAEL

Sandeepan Parekh

Department of Mathematics
Vanderbilt University
1326 Stevenson Center
Nashville TN 37240-0001
UNITED STATES

Dr. Henrik Densing Petersen

EPFL SB MATHGEOM EGG
MA C3 605 (Bâtiment MA)
Station 8
1015 Lausanne
SWITZERLAND

Hester Pieters

Section de Mathématiques
Université de Genève
2-4 rue du Lièvre, CP 64
1211 Genève 4
SWITZERLAND

Felix Pogorzelski

Friedrich-Schiller-Universität Jena
Institut für Mathematik
Ernst-Abbe-Platz 2
07743 Jena
GERMANY

Dr. Sanaz Pooya

Department of Mathematics
Ferdowsi University of Mashhad (FUM)
Azadi Sq.
P.O. Box 9177948974
Mashhad, Khorasan, Razavi
IRAN, ISLAMIC REPUBLIC OF

Maria Beatrice Pozzetti

Departement Mathematik
ETH-Zentrum
Rämistr. 101
8092 Zürich
SWITZERLAND

Dr. Piotr Przytycki

Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 21
ul. Sniadeckich 8
00-956 Warszawa
POLAND

Prof. Dr. Michael Puschnigg

Institut de Mathématiques de Luminy
UMR 6206
Université de la Méditerranée
163 Ave. de Luminy
13288 Marseille Cedex 9
FRANCE

Dr. Sven Raum

Unité de Mathématiques Pures &
Appliquées
École Normale Supérieure de Lyon
46, Allée d'Italie
69364 Lyon Cedex 07
FRANCE

Dr. Marcin Sabok

Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 21
ul. Sniadeckich 8
00-956 Warszawa
POLAND

Prof. Dr. Roman Sauer

Institut für Algebra & Geometrie
Fakultät für Mathematik (KIT)
Kaiserstr. 89-93
76133 Karlsruhe
GERMANY

Dr. Peter Schlicht

EPFL SB MATHGEOM EGG
MA C3 584 (Bâtiment MA)
Station 8
1015 Lausanne
SWITZERLAND

Thierry Stulemeijer

Département de Mathématique
Université Catholique de Louvain
Chemin du Cyclotron, 2
1348 Louvain-la-Neuve
BELGIUM

Prof. Dr. Andreas B. Thom

Mathematisches Institut
Universität Leipzig
Augustusplatz 10
04103 Leipzig
GERMANY

Stephan Tornier

Departement Mathematik
ETH-Zentrum
Rämistr. 101
8092 Zürich
SWITZERLAND

Peter Verraedt

Department of Mathematics
Katholieke Universiteit Leuven
BUS 2400
Celestijnenlaan 200 B
3001 Leuven
BELGIUM

Chenxu Wen

Department of Mathematics
Vanderbilt University
1326 Stevenson Center
Nashville TN 37240-0001
UNITED STATES

Prof. Dr. Jörg Winkelmann

Fakultät für Mathematik
Ruhr-Universität Bochum
44780 Bochum
GERMANY

