



HAL
open science

A state feedback input constrained control design for a 4-semi-active damper suspension system: a quasi-LPV approach

Manh Quan Nguyen, João Manoel Gomez da Silva Jr, Olivier Sename, Luc Dugard

► To cite this version:

Manh Quan Nguyen, João Manoel Gomez da Silva Jr, Olivier Sename, Luc Dugard. A state feedback input constrained control design for a 4-semi-active damper suspension system: a quasi-LPV approach. ROCOND 2015 - 8th IFAC Symposium on Robust Control Design, Jul 2015, Bratislava, Slovakia. 10.1016/j.ifacol.2015.09.467 . hal-01226754

HAL Id: hal-01226754

<https://hal.science/hal-01226754>

Submitted on 10 Nov 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A state feedback input constrained control design for a 4-semi-active damper suspension system: a quasi-LPV approach

M.Q. Nguyen* J.M. Gomes da Silva Jr** O. Sename* L. Dugard*

* Univ. Grenoble Alpes, GIPSA-lab, F-38402 Grenoble Cedex, France
CNRS, GIPSA-lab, F-38402 Grenoble Cedex, France.

E-mail: {manh-quan.nguyen, olivier.sename,
luc.dugard}@gipsa-lab.grenoble-inp.fr

** UFRGS, Department of Electrical Engineering, Porto Alegre-RS
90035-190, Brazil. E-mail: jmgomes@ece.ufrgs.br

Abstract: This paper addresses a semi-active suspension control of the full vehicle equipped with 4 controlled semi-active dampers by using a full 7 degree of freedom (DOF) vertical model. First, the dissipativity conditions of the semi-active dampers are recast as saturation conditions on the control inputs. Then, the suspension controller is derived by solving a state feedback control design problem for a class of linear parameter-varying (LPV) system in the presence of actuator saturation. To this aim, a generalized sector condition for LPV system is applied to treat the nonlinearity, caused by the input saturation and to relax the stability condition. The proposed control law ensures the disturbance attenuation by reducing the \mathcal{L}_2 gain from the disturbance to the controlled output. This controller, derived in the LPV/ H_∞ framework, is based on the LMI solution for polytopic systems. Some realistic simulation results are presented in order to illustrate the effectiveness of the proposed approach.

Keywords: Input saturation, Generalized sector condition, Semi-active suspension, State Feedback, LPV/ H_∞ control.

1. INTRODUCTION

The suspension system plays a key role in vehicle dynamic system. The semi-active suspensions are today widely used in automobile industry thanks to their low price and low energy consumption. However, the control design for semi-active suspension systems has always to face the challenge of the dissipativity constraints. Several control design problems for semi-active suspension systems have then been tackled with many different approaches during the last decades. In the works of (Savaresi et al. (2010), Poussot-Vassal et al. (2012)), the authors presented several control strategies for semi-active suspensions (based on the Skyhook, Groundhook, ADD). Moreover, to cope with the dissipativity constraints of semi-active dampers, some control approaches using the LPV techniques have been presented. In (Poussot-Vassal et al. (2008)), a kind of LPV gain scheduling anti-windup strategy was proposed by using a scheduling parameter which represents in some sense the excess of active control. More specifically, in some recent works, the dissipativity constraint of the semi-active damper has been recast as an actuator saturation problem. In (Do et al. (2010, 2012)), the nonlinearities of the semi-active suspension (including the saturation of the control input) are taken into account and written in an LPV form. In (Do et al. (2011a)), another output feedback LPV control with input saturation and state constraints was designed. Nevertheless, the results are valid only for the quarter car model equipped with one semi-active damper which is not enough to express the full dynamic of the vehicle. This motivates the investigations of the suspension control for the full vehicle equipped with 4 semi active dampers, i.e for a MIMO system.

On the other hand, nowadays, in many practical control applications, the actuator saturation is a challenge for the control system designer because it induces a nonlinear behavior for the closed-loop system even if the plant is linear. Actually, the input saturation is source of instability in control and loss in performance. In more recent years, researchers have focused on the problem of input saturation control. First, several models of the saturation constraint were proposed. In (Hu and Lin (2001)) and (Tarbouriech et al. (2011)), a full discussion about the saturation modeling based on the use of the polytopic differential inclusions is given. Tarbouriech et al. (2011) presents also a generalized sector condition approach where the saturation term is replaced by a dead-zone nonlinearity function. Then, these models for the saturation are used to treat the stability and stabilization problems for the class of LTI system. In addition, for the class of LPV system subject to input saturation, in (Cao et al. (2002)), and in (Cao and Lin (2006)), an anti-windup controller was proposed for polytopic LPV systems. Another anti-windup synthesis for LPV systems under the Linear Fractional Representation form is presented in (Prempain et al. (2009)). Moreover, regarding the works that cope not only with the input saturation but also with the disturbance attenuation problems in control design, we can cite for instance (Fang et al. (2004)), which uses a polytopic approach and (Castelan et al. (2006)) that uses a sector condition and Finsler's lemma to give a solution to the \mathcal{L}_2 stabilization problem.

In this work, a state feedback control design problem for LPV systems subject to actuator saturation, disturbances and state constraints is applied to the semi-active suspension control, specifically for the first time using a full vehicle equipped

with 4 semi-active dampers. Such a solution is an interesting alternative to dynamic output feedback control since it reduces the complexity of the implementation. It assumes that the state variables are known, or estimated as proposed in (Dugard et al. (2012)). The damper model used is a linear model and the semi-active constraint is recast as an input saturation. This saturation is tackled by a generalized sector condition modified for LPV systems. Then, conditions following to compute a parameter dependent state feedback control law that ensures the satisfaction of semi-active constraints while minimizing the effects of road induced disturbances on the roll motion are proposed.

The paper is organised as follows. Section 2 describes full car model and the motivation of the proposed work. Section 3 presents the control problem of LPV system subject to input saturation. Section 4 gives the different steps to design the controller and it is applied to a full vehicle model equipped with 4 semi-active suspensions. Finally, some conclusions are drawn in the section 5.

2. PROBLEM STATEMENT

2.1 Full car model

A full car vertical model is used for the analysis and control of the vehicle dynamic behaviors. This is a 7 degree of freedom (DOF) suspension model, obtained from a nonlinear full vehicle model (referred in Poussot-Vassal et al. (2011), Gillespie (1992), Kiencke and Nielsen (2000)). This model not only involves the chassis dynamics (vertical (z_s), roll (θ) and pitch(ϕ)), but it also figures out ($z_{us_{ij}}$) the vertical displacements of the wheels at the front/rear ($i = (f, r)$)-left/right corner ($j = (l, r)$). The dynamic equations of this 7 DOF model are given as follows:

$$\begin{cases} m_s \ddot{z}_s &= -F_{sfl} - F_{sfr} - F_{srl} - F_{srr} + F_{dz} \\ I_x \ddot{\theta} &= (-F_{sfr} + F_{sfl})l_f + (-F_{srr} + F_{srl})l_r \\ &\quad + mha_y + M_{dx} \\ I_y \ddot{\phi} &= (F_{srr} + F_{srl})l_r - (F_{sfr} + F_{sfl})l_f \\ &\quad - mha_x + M_{dy} \\ m_{us_{ij}} \ddot{z}_{us_{ij}} &= -F_{s_{ij}} + F_{tz_{ij}} \end{cases} \quad (1)$$

where I_x (resp. I_y) is the moment of inertia of the sprung mass around the longitudinal (resp. lateral) axis, h : is the height of center of gravity (COG), z_s is the vertical displacement of COG, θ the roll angle of the sprung mass, ϕ is the pitch angle of the sprung mass and $z_{us_{ij}}$ is the vertical displacements of wheels. Assuming that the chassis body is rigid, the characteristics of spring and damping are linear.

$F_{tz_{ij}}$ are the vertical tire forces, given as:

$$F_{tz_{ij}} = -k_{t_{ij}}(z_{us_{ij}} - z_{r_{ij}}) \quad (2)$$

where $k_{t_{ij}}$: is the stiffness coefficient of the tire.

The vertical suspension forces $F_{s_{ij}}$ at the 4 corners of the vehicle are modeled by a spring and a damper (see Zin et al. (2008)) with non linear characteristics for simulation and linear ones for control design. The equation (3) allows to model the suspension force used in the control design step:

$$F_{s_{ij}} = k_{ij}(z_{s_{ij}} - z_{us_{ij}}) + F_{damper_{ij}} \quad (3)$$

where k_{ij} is the nominal spring stiffness coefficient and $z_{s_{ij}}$ is the chassis position at each corner.

$F_{damper_{ij}}$ is the semi-active controlled damper force:

$$F_{damper_{ij}} = c_{ij}(\cdot)\dot{z}_{def_{ij}} = c_{ij}(\cdot)(\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}) \quad (4)$$

$c_{ij}(\cdot)$ the damping coefficient assumed to be varying for control purpose. To ensure the dissipativity constraint of the semi-active damper, the following constraint must be considered:

$$0 \leq c_{min_{ij}} \leq c_{ij}(\cdot) \leq c_{max_{ij}} \quad (5)$$

Now, let us rewrite the above semi-active damper force as follows:

$$\begin{aligned} F_{damper_{ij}} &= (c_{nom_{ij}} + u_{ij}^{H_\infty})\dot{z}_{def_{ij}} \\ F_{damper_{ij}} &= c_{nom_{ij}}\dot{z}_{def_{ij}} + u_{ij}^{H_\infty}\rho_{ij} \end{aligned} \quad (6)$$

where $c_{nom_{ij}} = (c_{max_{ij}} + c_{min_{ij}})/2$ is the nominal damping coefficient, $u_{ij}^{H_\infty}$ is the control input, and $\rho_{ij} = \dot{z}_{def_{ij}}$ are considered as the scheduling parameters (in this case, one has 4 semi-active dampers, hence 4 scheduling parameters are used).

Then the equation (3) becomes:

$$F_{s_{ij}} = k_{ij}(z_{s_{ij}} - z_{us_{ij}}) + c_{nom_{ij}}(\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}) + u_{ij}^{H_\infty}\rho_{ij} \quad (7)$$

By substituting the tire force equations (2) and the suspension force equations (7) into the vehicle equations (1) and assuming that the roll and pitch angles are small enough, the state-space representation of the dynamical equation (1) is given by (Sammier (2001)):

$$\dot{x}_g(t) = A_g x_g(t) + B_{1g} w(t) + B_{2g}(\rho) u \quad (8)$$

where $A_g = \begin{pmatrix} 0_{7 \times 7} & I_{7 \times 7} \\ -M_z^{-1} K_z & -M_z^{-1} B_z \end{pmatrix}$; $B_{1g} = \begin{pmatrix} 0_{7 \times 4} \\ M_z^{-1} K_r \end{pmatrix}$;

$B_{2g}(\rho) = \begin{pmatrix} 0_{7 \times 4} \\ M_z^{-1} T_a \end{pmatrix}$; $M_z = \begin{pmatrix} M_s & 0_{3 \times 4} \\ 0_{4 \times 3} & M_u \end{pmatrix}$;

$B_z = \begin{pmatrix} LB_s L^T & -LB_s \\ -B_s L^T & B_s \end{pmatrix}$; $K_z = \begin{pmatrix} LK_s L^T & -LK_s \\ -K_s L^T & K_s + K_t \end{pmatrix}$;

$K_r = \begin{pmatrix} 0_{3 \times 4} \\ K_t \end{pmatrix}$; $T_a(\rho) = \begin{pmatrix} L\Omega \\ -\Omega \end{pmatrix}$; $L = \begin{bmatrix} 1 & 1 & 1 & 1 \\ t_f & -t_f & t_r & -t_r \\ -l_f & -l_f & l_r & l_r \end{bmatrix}$;

$M_s = \text{diag}[m_s, I_x, I_y]$; $M_u = \text{diag}[m_{us_{fl}}, m_{us_{fr}}, m_{us_{rl}}, m_{us_{rr}}]$;

$B_s = \text{diag}[c_{fl}, c_{fr}, c_{rl}, c_{rr}]$; $K_s = \text{diag}[k_{s_{fl}}, k_{s_{fr}}, k_{s_{rl}}, k_{s_{rr}}]$;

$K_t = \text{diag}[k_{t_{fl}}, k_{t_{fr}}, k_{t_{rl}}, k_{t_{rr}}]$; $\Omega = \text{diag}[\rho_{fl}, \rho_{fr}, \rho_{rl}, \rho_{rr}]$;

and $x_g = [z_s, \theta, \phi, z_{us_{fl}}, z_{us_{fr}}, z_{us_{rl}}, z_{us_{rr}}, \dot{z}_s, \dot{\theta}, \dot{\phi}, \dot{z}_{us_{fl}}, \dot{z}_{us_{fr}}, \dot{z}_{us_{rl}}, \dot{z}_{us_{rr}}]^T$ is the state vector of the full car model, $w = [z_{r_{fl}}, z_{r_{fr}}, z_{r_{rl}}, z_{r_{rr}}]^T$ corresponds to the road profiles and $u = [u_{fl}^{H_\infty}, u_{fr}^{H_\infty}, u_{rl}^{H_\infty}, u_{rr}^{H_\infty}]^T$ is the control input vector.

Remark 1: The manipulations to obtain the state state representation of the 7 DOF vertical model of the vehicle is omitted here. The interested reader can refer to Park and Kim (1998), Sammier (2001) for more details.

2.2 Input and State constraints

In this part, the dissipativity conditions of the semi-active damper given in (5) will be transformed into input constraints. Note that from (5,6), it follows that:

$$\begin{aligned} c_{min_{ij}}\dot{z}_{def_{ij}} \leq F_{damper_{ij}} \leq c_{max_{ij}}\dot{z}_{def_{ij}} &\quad \text{if } \dot{z}_{def_{ij}} > 0 \\ c_{max_{ij}}\dot{z}_{def_{ij}} \leq F_{damper_{ij}} \leq c_{min_{ij}}\dot{z}_{def_{ij}} &\quad \text{if } \dot{z}_{def_{ij}} \leq 0 \end{aligned} \quad (9)$$

The dissipativity constraint is now recast into:

$$\begin{aligned} c_{\min_{ij}} \dot{z}_{def_{ij}} &\leq c_{\text{nom}_{ij}} \dot{z}_{def_{ij}} + u_{ij}^{H_\infty} \dot{z}_{def_{ij}} \leq c_{\max_{ij}} \dot{z}_{def_{ij}} & \text{if } \dot{z}_{def_{ij}} > 0 \\ c_{\max_{ij}} \dot{z}_{def_{ij}} &\leq c_{\text{nom}_{ij}} \dot{z}_{def_{ij}} + u_{ij}^{H_\infty} \dot{z}_{def_{ij}} \leq c_{\min_{ij}} \dot{z}_{def_{ij}} & \text{if } \dot{z}_{def_{ij}} \leq 0 \end{aligned}$$

Because of $c_{\text{nom}_{ij}} = \frac{(c_{\max_{ij}} + c_{\min_{ij}})}{2}$, then we must have:

$$|u_{ij}^{H_\infty}| \leq \frac{(c_{\max_{ij}} - c_{\min_{ij}})}{2} \quad (10)$$

Hence the dissipativity conditions (5) have been recast into the *input constraints* given in (10).

It is worth noting that (8) is actually a quasi-LPV system since the 4 scheduling parameters are defined by $\rho_{ij} = \dot{z}_{def_{ij}}$ ($i = (f, l); j = (l, r)$), and then depend on the system state. We suppose that the absolute deflection velocity $|\dot{z}_{def_{ij}}|$ is smaller than 1 m/s, hence $\rho_{ij} \in [-1, 1]$.

It should be noted that $|\rho_{ij}| = |\dot{z}_{def_{ij}}| = |\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}| \leq 1$. Thus, to ensure the constraints on the scheduling parameter $|\rho_{ij}| \leq 1$, we must ensure also a *state constraint* which will be rewritten later as:

$$|H \cdot x| \leq 1 \quad (11)$$

where x being the generalized system state (see (46)) and H is state constraint matrix.

2.3 Control problem

Based on the full car model and the semi-active suspension constraints detailed in previous sections, the control problem we are interested in solving in this work is the following:

Problem Statement: Design a suspension control in order to reduce the roll motion of the vehicle equipped with 4 semi-active dampers. The suspension control must satisfy the input saturation constraints (10) and the state constraint (11).

To tackle this problem, we consider an LPV approach detailed in the sequence.

3. LPV CONTROL IN THE PRESENCE OF INPUT SATURATION

3.1 System description

Consider the following quasi-LPV system \mathfrak{S}_ρ with input saturation and disturbance:

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1(\rho)w + B_2u \\ z &= C_1(\rho)x + D_{11}(\rho)w + D_{12}u \end{aligned} \quad (12)$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^m$ the control vector, $z \in \mathcal{R}^p$ the controlled output vector and $w \in \mathcal{R}^q$ the input disturbance signals. $\rho = (\rho_1, \dots, \rho_k)$ is the varying parameter vector. Assume that ρ is bounded to be able to apply the polytopic approach for LPV system:

$$\rho \in \Omega = \{\rho_i \mid \underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i, i = 1, \dots, k\}$$

and the matrices $A(\rho), B_1(\rho), C_1(\rho), D_{11}(\rho)$ depend affinely on the parameter $\rho = (\rho_1, \dots, \rho_k)$, that is:

$$\mathcal{A}(\rho) = \mathcal{A}_0 + \rho_1 \mathcal{A}_1 + \dots + \rho_k \mathcal{A}_k$$

where \mathcal{A} stands for matrices A, B_1, C_1, D_{11} . Then, provided that ρ is bounded in a polytope, the system \mathfrak{S}_ρ can be written as

a convex combination of the vertices \mathfrak{S}_ρ^j of the polytope as follows: $\mathfrak{S}_\rho = \sum_{j=1}^{2^k} \alpha_j(\rho) \mathfrak{S}_\rho^j$ where $\sum_{j=1}^{2^k} \alpha_j(\rho) = 1$ and $\mathfrak{S}_\rho^j = [A_j, B_{1j}, B_2, C_{1j}, D_{11j}, D_{12}]$, $j = 1, \dots, 2^k$.

Let us consider the following assumptions:

- The applied control signal u takes value in the compact set:

$$\mathcal{U} = \{u \in \mathcal{R}^m \mid -u_{0i} \leq u_i \leq u_{0i}, i = 1, \dots, m\} \quad (13)$$

- The input disturbances w are supposed to be bounded in amplitude i.e w belongs to a set \mathcal{W} :

$$\mathcal{W} = \{w \in \mathcal{R}^q \mid w^T w < \delta\} \quad (14)$$

- The state vector is assumed to be known (measured or estimated). Moreover, from (11), the trajectories of system must belong to a region \mathcal{X} defined as follows:

$$\mathcal{X} = \{x \in \mathcal{R}^n \mid |H_i x| \leq h_{0i}, i = 1, \dots, k\} \quad (15)$$

In this work, a state feedback control law is considered (Fig.1) and the control signal $v(t)$ computed by the state feedback controller is given by:

$$v(t) = K(\rho)x(t)$$

where $K(\rho) \in \mathcal{R}^{m \times n}$ is a parameter dependent state feedback matrix gain:

$$K(\rho) = \sum_{j=1}^{2^k} \alpha_j(\rho) K_j$$

Then, by virtue of the input constraints (13), the applied control u to system (12) is a saturated one, i.e:

$$u(t) = \text{sat}(v(t)) = \text{sat}(K(\rho)x(t)) \quad (16)$$

where the saturated function $\text{sat}(\cdot)$ is defined by:

$$\text{sat}(v_i(t)) = \begin{cases} u_{0i} & \text{if } v_i(t) > u_{0i} \\ v_i(t) & \text{if } -u_{0i} \leq v_i(t) \leq u_{0i} \\ -u_{0i} & \text{if } v_i(t) < -u_{0i} \end{cases} \quad (17)$$

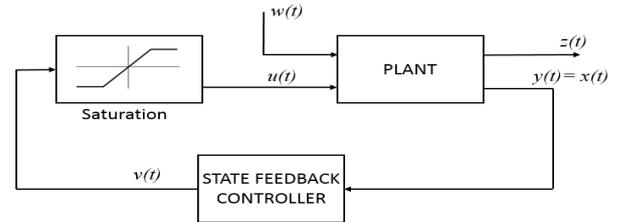


Fig. 1. State feedback control with input saturation

The closed-loop system obtained from the application of (16) in (12) reads as follows:

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1(\rho)w + B_2 \text{sat}(K(\rho)x) \\ z &= C_1(\rho)x + D_{11}(\rho)w + D_{12} \text{sat}(K(\rho)x) \end{aligned} \quad (18)$$

Let us define now the vector-valued dead-zone function $\phi(K(\rho)x)$:

$$\phi(K(\rho)x) = \text{sat}(K(\rho)x) - K(\rho)x \quad (19)$$

From (19), the closed-loop system can therefore be re-written as follows:

$$\dot{x} = (A(\rho) + B_2 K(\rho))x + B_2 \phi(K(\rho)x) + B_1(\rho)w \quad (20)$$

3.2 Problem definition

It should be noticed that under the input saturation, the state may become unbounded for large disturbances (Tarbouriech et al. (2011)). Hence, in this work, we propose the design of a state feedback $K(\rho)$ for the LPV system (18) in order to satisfy the following conditions:

- When the control input signal is saturated, the nonlinear behavior of the closed-loop system must be considered and the stability has to be guaranteed both internally as well as in the context of input to state, that is:
 - for $w \in \mathcal{W}$, the trajectories of the closed-loop system must be bounded.
 - if $w(t) = 0$ for $t > t_1 > 0$ then the trajectory of the system converge asymptotically to the origin.
- The control performance objective consists in minimizing the upper bound for the L_2 gain from the disturbance w to the controlled output z , i.e $\text{Min } \gamma > 0$, such that:

$$\sup \frac{\|z\|_2}{\|w\|_2} < \gamma \quad (21)$$

In order to reduce the conservatism, it is worth noting that in this work, the L_2 performance problem is solved only in the case that the input saturation is not activated. Actually, this is appropriated in reality because in the presence of actuator saturation, the main concern is to guarantee that the trajectories are bounded and the state constraints are not violated.

4. CONTROLLER DESIGN

4.1 Stability analysis

The system (18) has the input disturbance $w \in \mathcal{W}$ and its state variables must belong to the state region \mathcal{X} . Moreover, due to the saturation function, it induces an extra nonlinear behavior in the closed-loop system. Hence we will take into account these facts by using a regional (local) stability approach. To this aim, a modification of the generalized sector condition which uses a parameter dependant matrix $T(\rho)$ is proposed and applied for the LPV system.

Let us first define the following polyhedral set:

$$\mathcal{S}_\rho(K, G, u_0) = \{x \in \mathcal{R}^m \mid -u_0 \leq (K(\rho) - G(\rho))x \leq u_0\}$$

where this inequality stands for each input variable.

Lemma 1. If $x \in \mathcal{S}_\rho(K, G, u_0)$, then the deadzone function ϕ satisfies the following inequality:

$$\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] \leq 0 \quad (22)$$

for any diagonal and positive definite matrix $T(\rho) \in \mathcal{R}^{m \times m}$.

Proof: The proof of the lemma can be inferred easily from (Gomes da Silva Jr and Tarbouriech (2005))

Because of the boundness of the disturbance $w \in \mathcal{W}$, we consider the W-invariance concept (Blanchini (1999)):

Definition: The set $\mathcal{E} \subset \mathcal{R}^n$ is said to be W-invariant if $\forall x(t_0) \in \mathcal{E}, \forall w(t) \in \mathcal{W}$ implies that the trajectory $x(t) \in \mathcal{E}$ for all $t \geq t_0$.

As known, the quadratic stability can be interpreted in term of an invariant ellipsoids (Boyd et al. (1994)). In fact, considering

a quadratic Lyapunov function: $V = x^T P x$ with $P = P^T > 0$, the ellipsoid associated to this Lyapunov function is given by :

$$\mathcal{E}(P) = \{x \in \mathcal{R}^n : x^T P x < 1\} \quad (23)$$

Then, the idea is to ensure that $\mathcal{E}(P)$ is W-invariant for the closed-loop system (20). This can be achieved if $\dot{V}(t) < 0$ in the boundary of $\mathcal{E}(P)$. Thus, it suffices to ensure that $\dot{V}(t) < 0 \forall x \notin \text{int}\mathcal{E}(P)$ (the interior of $\mathcal{E}(P)$) i.e $x^T P x > 1$ and for any $w \in \mathcal{W}$ i.e $w^T w < \delta$. By using the S-procedure, this condition can be satisfied if there exist scalars $\lambda_1 > 0$ and $\lambda_2 > 0$, such that:

$$\dot{V} + \lambda_1(x^T P x - 1) + \lambda_2(\delta - w^T w) < 0 \quad (24)$$

Then, the following theorem regards a stabilization condition for the system (18):

Theorem 1. If there exist a matrix Q -positive definite, a matrix $S(\rho)$ -diagonal positive definite, matrices $\bar{K}(\rho), \bar{G}(\rho)$ of appropriate dimensions and positive scalar λ_1 such that the following conditions are verified:

$$\begin{bmatrix} \bar{M}(\rho) & (B_2 S(\rho) - \bar{G}(\rho)^T) & B_1(\rho) \\ (S(\rho) B_2^T - \bar{G}(\rho)) & -2S(\rho) & 0 \\ B_1(\rho)^T & 0 & -\lambda_2 I \end{bmatrix} < 0 \quad (25)$$

where $\bar{M}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T) + (QA(\rho)^T + \bar{K}(\rho)^T)^T + \lambda_1 Q$.

$$\left[\frac{Q}{\bar{K}_i(\rho) - \bar{G}_i(\rho)} \mid \frac{(\bar{K}_i(\rho) - \bar{G}_i(\rho))^T}{u_{0i}^2} \right] \geq 0, i = 1, \dots, m \quad (26)$$

where $\bar{K}_i(\rho), \bar{G}_i(\rho)$ are i^{th} line of $\bar{K}(\rho), \bar{G}(\rho)$ respectively.

$$\left[\frac{Q}{H_i Q} \mid \frac{QH_i^T}{h_{0i}^2} \right] \geq 0, i = 1, \dots, k \quad (27)$$

$$\lambda_2 \delta - \lambda_1 < 0 \quad (28)$$

Then, with $K(\rho) = \bar{K}(\rho)Q^{-1}$:

a) For any $w \in \mathcal{W}$ and $x(0) \in \mathcal{E}(P)$ the trajectories do not leave $\mathcal{E}(P)$, i.e. $\mathcal{E}(P)$ is an W-invariant domain for the system (18).

b) If $x(0) \in \mathcal{E}(P)$ and $w(t) = 0$ for $t > t_1$, then the corresponding trajectory converge asymptotically to the origin, i.e. $\mathcal{E}(P)$ (with $P = Q^{-1}$) is included in the region of attraction of the closed-loop system (18).

Proof: As mentioned previously, $\mathcal{E}(P)$ is W-invariant if:

$$\dot{V} + \lambda_1(x^T P x - 1) + \lambda_2(\delta - w^T w) < 0 \quad (29)$$

Now, from (22) and (29), provided that $x \in \mathcal{S}_\rho(K, G, u_0)$ one obtains:

$$\begin{aligned} & \dot{V} + \lambda_1(x^T P x - 1) + \lambda_2(\delta - w^T w) \leq \\ & \dot{V} + \lambda_1(x^T P x - 1) + \lambda_2(\delta - w^T w) - \\ & 2\phi(K(\rho)x)^T T(\rho)[\phi(K(\rho)x) + G(\rho)x] < 0 \end{aligned} \quad (30)$$

For the sake of simplicity, the argument ρ is omitted here, then:

$$\begin{aligned} & x^T M x - \lambda_2 w^T w - 2\phi^T T \phi + w^T B_1^T P x + \phi^T (B_2^T P - T G)x + \\ & x^T P B_1 w + x^T (P B_2 - G^T T) + \lambda_2 \delta - \lambda_1 < 0 \end{aligned} \quad (31)$$

where $M(\rho) = (A(\rho) + B_2 K(\rho))^T P + P(A(\rho) + B_2 K(\rho)) + \lambda_1 P$.

Then the condition (31) is guaranteed if both following inequalities hold:

$$\left[\begin{array}{c|c|c} M(\rho) & (PB_2 - G(\rho)^T T(\rho)) & PB_1(\rho) \\ \hline (PB_2 - G(\rho)^T T(\rho))^T & -2T(\rho) & 0 \\ \hline B_1(\rho)^T P & 0 & -\lambda_2 I \end{array} \right] < 0 \quad (32)$$

$$\lambda_2 \delta - \lambda_1 < 0 \quad (33)$$

Pre and post-multiplying (32) by $\text{diag}(P^{-1}, T^{-1}(\rho), I)$, and denoting $K(\rho)P^{-1} = \bar{K}(\rho)$, $G(\rho)P^{-1} = \bar{G}(\rho)$, $P^{-1} = Q$, $T(\rho)^{-1} = S(\rho)$, one obtains the LMI (25), i.e:

$$\left[\begin{array}{c|c|c} \bar{M}(\rho) & (B_2 S(\rho) - \bar{G}(\rho)^T) & B_1(\rho) \\ \hline (S(\rho) B_2^T - \bar{G}(\rho)) & -2S(\rho) & 0 \\ \hline B_1(\rho)^T & 0 & -\lambda_2 I \end{array} \right] < 0$$

where $\bar{M}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T) + (QA(\rho)^T + \bar{K}(\rho)^T)^T + \lambda_1 Q$.

Finally, to ensure that $x(t)$ belongs effectively to $\mathcal{S}_\rho(K, G, u_0)$ and that the state constraints are not violated, we must ensure that $\mathcal{E}(P) \subset \mathcal{S}_\rho(K, G, u_0) \cap \mathcal{X}$, i.e $\mathcal{E}(P) \subset \mathcal{S}_\rho(K, G, u_0)$ and $\mathcal{E}(P) \subset \mathcal{X}$.

To ensure $\mathcal{E}(P) \subset \mathcal{S}_\rho(K, G, u_0)$, we should satisfy:

$$\left[\begin{array}{c|c} P & (K_i(\rho) - G_i(\rho))^T \\ \hline K_i(\rho) - G_i(\rho) & u_{0i}^2 \end{array} \right] \geq 0, i = 1, \dots, m \quad (34)$$

Pre and post-multiplying (34) by $\text{diag}(P^{-1}, I)$, we have:

$$\left[\begin{array}{c|c} Q & (\bar{K}_i(\rho) - \bar{G}_i(\rho))^T \\ \hline \bar{K}_i(\rho) - \bar{G}_i(\rho) & u_{0i}^2 \end{array} \right] \geq 0, i = 1, \dots, m \quad (35)$$

To ensure $\mathcal{E}(P) \subset \mathcal{X}$, the following should be verified:

$$\left[\begin{array}{c|c} P & H_i^T \\ \hline H_i & h_{0i}^2 \end{array} \right] \geq 0, i = 1, \dots, k \quad (36)$$

Pre and post-multiplying (36) by $\text{diag}(P^{-1}, I)$, we have:

$$\left[\begin{array}{c|c} Q & QH_i^T \\ \hline H_i Q & h_{0i}^2 \end{array} \right] \geq 0, i = 1, \dots, k \quad (37)$$

Thus, if inequalities (25, 26, 27, 28) are satisfied then it follows that the ellipsoid $\mathcal{E}(P)$ is an W -invariant set.

Now, let us consider the case $w(t) = 0$, from (29), it follows:

$\dot{V}(x(t)) \leq -\lambda_1 x^T P x$, or equivalently:

$$\left[\begin{array}{c|c} \bar{M}(\rho) & (B_2 S(\rho) - \bar{G}(\rho)^T) \\ \hline (S B_2^T - \bar{G}(\rho)) & -2S(\rho) \end{array} \right] < 0 \quad (38)$$

where $\bar{M}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T) + (QA(\rho)^T + \bar{K}(\rho)^T)^T + \lambda_1 Q$. This LMI can be inferred directly from (25) thanks to Schur's Lemma. Thus, $\dot{V}(x(t)) \leq -\lambda_1 V(x(t)) < 0$ i.e $V(x(t)) \leq e^{-\lambda_1 t} V(x(0))$, it means that the trajectories of the system converge asymptotically to the origin.

4.2 Disturbance attenuation

As mentioned before, in this work, we consider a control objective regarding the disturbance attenuation for the the unconstrained closed-loop system, i.e. when the saturation is not

activated or $\text{sat}(K(\rho)x) = K(\rho)x$.

It is well known that relation (21) is verified if the following condition holds:

$$\dot{V}(x(t)) + \frac{1}{\gamma} z^T z - \gamma w^T w < 0 \quad (39)$$

Without the input saturation, the closed loop system (18) becomes:

$$\dot{x} = (A(\rho) + B_2 K(\rho))x + B_1(\rho)w \quad (40)$$

$$z = (C_1(\rho) + D_{12} K(\rho))x + D_{11}(\rho)w$$

Then, condition (39) holds if the following inequality is satisfied (Scherer et al. (1997)):

$$\left[\begin{array}{c|c|c} N(\rho) & PB_1(\rho) & (C_1(\rho) + D_{12} K(\rho))^T \\ \hline B_1(\rho)^T P & -\gamma I & D_{11}^T \\ \hline C_1(\rho) + D_{12} K(\rho) & D_{11} & -\gamma I \end{array} \right] < 0 \quad (41)$$

where $N(\rho) = (A(\rho) + B_2 K(\rho))^T P + P(A(\rho) + B_2 K(\rho))$.

Pre and post-multiplying (41) by $\text{diag}(P^{-1}, I, I)$, one obtains:

$$\left[\begin{array}{c|c|c} \bar{N}(\rho) & B_1(\rho) & (QC_1(\rho)^T + \bar{K}(\rho)^T) D_{12}^T \\ \hline B_1(\rho)^T & -\gamma I & D_{11}^T \\ \hline C_1(\rho)Q + D_{12} \bar{K}(\rho) & D_{11} & -\gamma I \end{array} \right] < 0 \quad (42)$$

where $\bar{N}(\rho) = (QA(\rho)^T + \bar{K}(\rho)^T) + (QA(\rho)^T + \bar{K}(\rho)^T)^T$

4.3 Controller computation

The state feedback gain $K(\rho)$ that satisfies the stability condition for the saturated system (section 4.1) and the disturbance attenuation for the unsaturated system (section 4.2) can be derived by solving the following optimization problem:

$$\begin{aligned} \min_{Q, S, \bar{K}, \bar{G}, \lambda_1} \quad & \gamma \\ \text{subject to} \quad & (25, 26, 27, 28, 42), \\ & Q, S > 0, \lambda_1 > 0. \end{aligned} \quad (43)$$

Then the state feedback gain matrix $K(\rho)$ can be computed by:

$$K(\rho) = \bar{K}(\rho)P = \bar{K}(\rho)Q^{-1} \quad (44)$$

It is worth noting that the above optimization problem has an infinite number of LMIs to solve because the varying parameter ρ varies in the set Ω . To relax this problem, the LMI framework for the polytopic system is used, i.e we will solve the optimization problem at each vertex \mathcal{E}_ρ^j of the polytope defined by the bounds of the varying parameters to obtain the state feedback matrix gain K_j at each vertex. Then, considering the measured value of ρ , the parameter dependent state feedback matrix $K(\rho)$ is computed as follows:

$$K(\rho) = \sum_{i=j}^{2^k} \alpha_j(\rho) K_j, \quad \sum_{i=j}^{2^k} \alpha_j(\rho) = 1.$$

5. APPLICATION OF THE LPV APPROACH TO THE FULL VEHICLE

In this work, the output signal used to test the proposed approach is the roll motion of the vehicle and the objective is

to reduce this motion. To do this, we aim at reducing the effect of the disturbance by minimizing the gain level γ of the closed loop transfer function from the road disturbance w to the controlled output z (in this case, the roll motion θ is chosen: $z = \theta$) while taking into account the actuator saturation. The H_∞ framework is used to solve this objective and we add the weighting function W_θ on θ , that can be chosen as (Do et al. (2011b)):

$$W_\theta = k_\theta \frac{s^2 + 2\xi_{11}\Omega_1 s + \Omega_1^2}{s^2 + 2\xi_{12}\Omega_1 s + \Omega_1^2}. \quad (45)$$

Noting that 7 DOF vertical model (8):

$$\dot{x}_g(t) = A_g x_g(t) + B_{1g} w(t) + B_{2g}(\rho) u$$

has parameter dependent input matrix $B_{2g}(\rho)$, so in order to apply the polytopic approach, one adds a low pass filter on the control input as in (Do et al. (2012)) to obtain the parameter independent input matrix. Then according to the interconnection between the 7 DOF vertical model, the weighting function W_θ , and the low pass filter, the following parameter dependent suspension generalized plant ($\Sigma_{gv}(\rho)$) is obtained:

$$\Sigma_{gv}(\rho) : \begin{cases} \dot{x} = A(\rho)x + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \end{cases} \quad (46)$$

where $x = [x_g^T \ x_{wf}^T \ x_f^T]^T$, x_g , x_{wf} , x_f are the vertical model, weighting function and filter states respectively.

$z = \theta$ are the controlled output vector, w , u are as defined in (8) and $\rho = [\rho_1 \ \rho_2 \ \rho_3 \ \rho_4]$: the vector of varying parameter, $\rho_i \in [-1 \ 1], i = 1, 2, 3, 4$.

This generalized plant (46) is a case of the system \mathfrak{E}_ρ defined in (12), so the LPV approach presented in the previous section can be applied. Actually, (46) depends on 4 varying parameters ($\rho_1, \rho_2, \rho_3, \rho_4$), hence we have a polytopic system which is computed by a convex combination of the systems defined at $N=16$ vertices $\omega_j, (j = 1 \dots 16)$ of the polytope. From the *Control computation* section, solving problem (43), the state feedback controller $K(\rho)$ is given by:

$$K(\rho) = \sum_{j=1}^{16} \alpha_j(\rho) K_j \quad (47)$$

where $\alpha_j(\rho) := \frac{\prod_{k=1}^4 |\rho_k - \text{Compl}(\omega_j)_k|}{\prod_{k=1}^4 (\overline{\rho_k} - \underline{\rho_k})}$, $\rho_k \in [\underline{\rho_k} \ \overline{\rho_k}]$ and

$$\sum_{j=1}^{16} \alpha_j(\rho) = 1, j = 1, \dots, 16;$$

$$\text{Compl}(\omega_j)_k := \{\overline{\rho_k} \text{ if } (\omega_j)_k = \underline{\rho_k} \text{ or } \underline{\rho_k} \text{ if } (\omega_j)_k = \overline{\rho_k}\}.$$

Then, the control input vector for the system (46) is computed as follows:

$$u^{H_\infty} = \sum_{j=1}^{N=16} \alpha_j(\rho) K_j x \quad (48)$$

5.1 Simulation results

To assess the proposed controller strategy, simulations are performed on a full non linear vehicle model (Poussot-Vassal et al. (2011)) that includes non linear suspension forces and validated upon a Renault M3gane Coup3. According to section 3, the previous assumptions are:

- The varying parameter $\rho_{ij} = \dot{z}_{def_{ij}} \in [-1 \ 1]$
- The damping coefficients vary as follows: for the front dampers, $c_{min_f} = 660 \text{ Ns/m}, c_{max_f} = 3740 \text{ Ns/m}$ and for the rear dampers, $c_{min_r} = 1000 \text{ Ns/m}, c_{max_r} = 8520 \text{ Ns/m}$. Thus, the input constraints (10) lead to:

$$[|u_{fl}^{H_\infty}| \ |u_{fr}^{H_\infty}| \ |u_{rl}^{H_\infty}| \ |u_{rr}^{H_\infty}|] \leq [1540 \ 1540 \ 3760 \ 3760]$$

- The road profile is chosen in the set \mathcal{W} subject to (14) with $\delta = 0.01 \text{ m}^2$.
- The state constraint in (15) is the constraint on suspension deflection speed: $|\dot{z}_{def_{ij}}| = |\dot{z}_{s_{ij}} - \dot{z}_{us_{ij}}| = |H_g \cdot x_g| = |[H_g \ 0_{wf} \ 0_f]x| = |Hx| \leq 1$. where H_g is the matrix that allows to calculate $\dot{z}_{def_{ij}}$ from x_g and $0_{wf}, 0_f$ are zero matrices.

The following scenario is used to demonstrate the effectiveness of the proposed LPV/ H_∞ State feedback control:

- The vehicle runs at 90 km/h in a straight line on a dry road ($\mu = 1$, where μ stands for the adherence to the road).
- A 5 cm bump occurs on the left wheels (from $t = 0.5 \text{ s}$ to $t = 1 \text{ s}$). A lateral wind disturbance occurs also in this time to excite the roll motion.
- Moreover, a line change that causes also the roll motion is performed from $t = 4 \text{ s}$ to $t = 7 \text{ s}$.

The road profile and steering angle are shown in the Fig. 2 and Fig. 3.

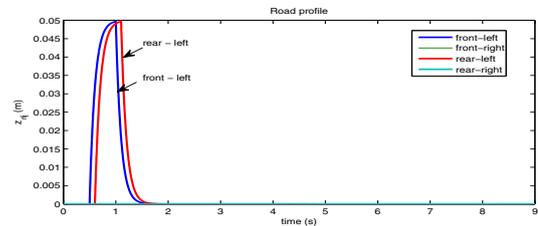


Fig. 2. Road profile

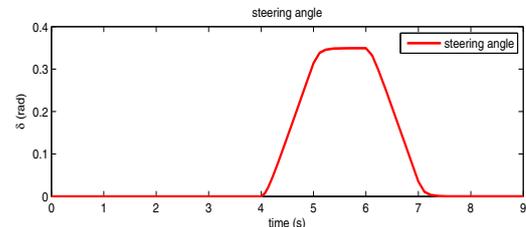


Fig. 3. Steering angle

Fig. 4 illustrates the varying parameters, i.e the suspension deflection speeds. We can see that the state constraints in (11) are respected.

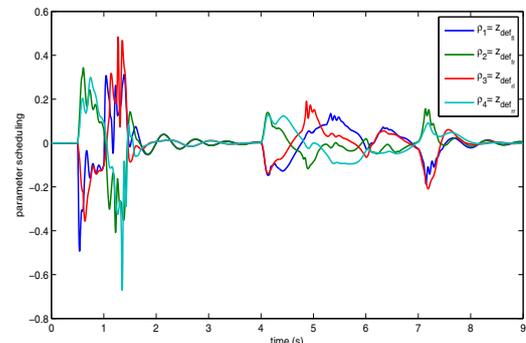


Fig. 4. Scheduling parameters satisfy the condition $|\rho_{ij}| \leq 1$

Fig. 5 shows the comparison between the roll motion of closed-loop system with the proposed LPV/H_∞ State Feedback and the open-loop case (Nominal Damper i.e $u_{ij}^{H_\infty} = 0$). The proposed methodology gives better performances. Indeed, with the proposed method, the roll motion is reduced with respect to the case of nominal damper. That proves the efficiency of the proposed LPV/H_∞ State Feedback.

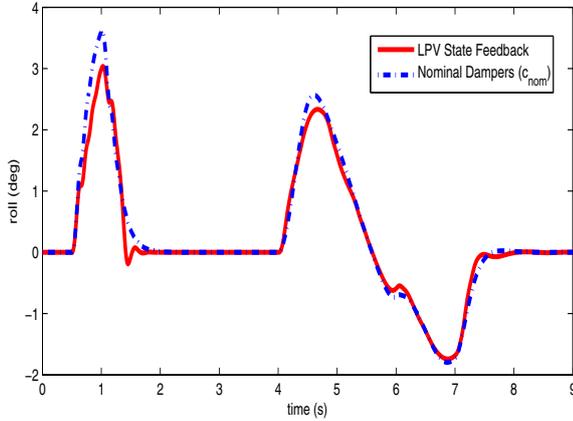


Fig. 5. Comparison of roll motion

Fig. 6 shows the relation between the damper force and suspension speed. The figure demonstrates that the damper force satisfies the dissipativity constraint:

$$\begin{aligned} c_{\min}\dot{z}_{def} \leq F_{damper} \leq c_{\max}\dot{z}_{def} & \text{ if } \dot{z}_{def} > 0 \\ c_{\max}\dot{z}_{def} \leq F_{damper} \leq c_{\min}\dot{z}_{def} & \text{ if } \dot{z}_{def} \leq 0 \end{aligned}$$

Moreover, it can be seen that the actuator saturation occurred when the damper forces are saturated by the bounds $c_{\max}\dot{z}_{def}$, $c_{\min}\dot{z}_{def}$, and thanks to the proposed LPV state feedback, the stability is kept.

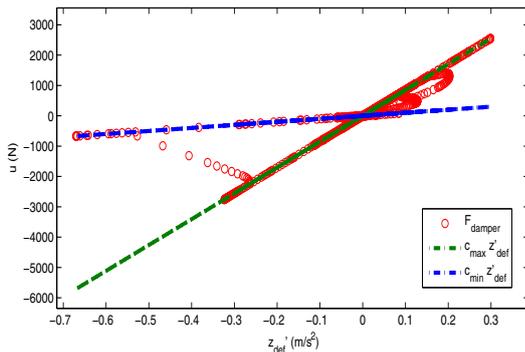


Fig. 6. Force/deflection speed

6. CONCLUSION

This article presented the application of an LPV/H_∞ State Feedback approach subject to input saturation to the problem of semi-active suspension control for a full vehicle equipped with 4 semi-active dampers. The simulation results have shown the effectiveness of the proposed approach. The next step of this work will be the implementation of this strategy on a test benchmark, available at Gipsa-lab Grenoble, developed in

collaboration with a high-tech start-up "SOBEN". It consists of a vehicle equipped with four controllable Electro-Rheological dampers, and of 4 DC motors generating separately different road profiles on each wheel. First experimental results on the test-bed are presented in Sename et al. (2014).

REFERENCES

- Blanchini, F. (1999). Survey paper: Set invariance in control. *Automatica*, 35(11), 1747–1767.
- Boyd, S.P., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*, volume 15. SIAM.
- Cao, Y.Y. and Lin, Z. (2006). An anti-windup design for polytopic systems by a parameter-dependent lyapunov function approach. *International Journal of Systems Science*, 37(2), 129–139.
- Cao, Y.Y., Lin, Z., and Shamash, Y. (2002). Set invariance analysis and gain-scheduling control for lpv systems subject to actuator saturation. *Systems & Control Letters*, 46(2), 137–151.
- Castelan, E., Tarbouriech, S., Gomes da Silva, J.M., and Queinnec, I. (2006). L2-stabilization of continuous-time linear systems with saturating actuators. *International Journal of Robust and Nonlinear Control*, 16(18), 935–944.
- Do, A.L., Sename, O., and Dugard, L. (2010). An LPV control approach for semi-active suspension control with actuator constraints. In *Proceedings of the IEEE American Control Conference (ACC)*, 4653 – 4658. Baltimore, Maryland, USA.
- Do, A.L., Sename, O., and Dugard, L. (2012). LPV modelling and control of semi-active dampers in automotive systems. In J. Mohammadpour and C. Scherer (eds.), *Control of Linear Parameter Varying Systems with Applications*, chapter 15. Springer.
- Do, A.L., Gomes da Silva Jr, J.M., Sename, O., and Dugard, L. (2011a). Control design for lpv systems with input saturation and state constraints: An application to a semi-active suspension. In *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*, 3416–3421. IEEE.
- Do, A.L., Sename, O., Dugard, L., Boussaad, S., et al. (2011b). Multi-objective optimization by genetic algorithms in h/lpv control of semi-active suspension. In *Proceedings of the 18th IFAC World Congress*.
- Dugard, L., Sename, O., Aubouet, S., and Talon, B. (2012). Full vertical car observer design methodology for suspension control applications. *Control Engineering Practice*, 20(9), 832–845.
- Fang, H., Lin, Z., and Hu, T. (2004). Analysis of linear systems in the presence of actuator saturation and l2-disturbances. *Automatica*, 40(7), 1229–1238.
- Gillespie, T.D. (1992). *Fundamentals of vehicle dynamics*.
- Gomes da Silva Jr, J.M. and Tarbouriech, S. (2005). Anti-windup design with guaranteed regions of stability: an lmi-based approach. *IEEE Transactions on Automatic Control*, 50(1), 106–111.
- Hu, T. and Lin, Z. (2001). *Control systems with actuator saturation: analysis and design*. Birkhuser.
- Kiencke, U. and Nielsen, L. (2000). Automotive control systems: for engine, driveline, and vehicle. *Measurement Science and Technology*, 11(12), 1828.
- Park, J.H. and Kim, Y.S. (1998). Decentralized variable structure control for active suspensions based on a full-car model.

- In *Control Applications, 1998. Proceedings of the 1998 IEEE International Conference on*, volume 1, 383–387. IEEE.
- Poussot-Vassal, C., Sename, O., Dugard, L., Gaspar, P., Szabo, Z., and Bokor, J. (2008). A new semi-active suspension control strategy through lpv technique. *Control Engineering Practice*, 16(12), 1519–1534.
- Poussot-Vassal, C., Sename, O., Dugard, L., Gaspar, P., Szabo, Z., and Bokor, J. (2011). Attitude and handling improvements through gain-scheduled suspensions and brakes control. *Control Engineering Practice*, 19(3), 252–263.
- Poussot-Vassal, C., Spelta, C., Sename, O., Savaresi, S.M., and Dugard, L. (2012). Survey and performance evaluation on some automotive semi-active suspension control methods: A comparative study on a single-corner model. *Annual Reviews in Control*, 36(1), 148–160.
- Prempain, E., Turner, M.C., and Postlethwaite, I. (2009). Co-prime factor based anti-windup synthesis for parameter-dependent systems. *Systems & Control Letters*, 58(12), 810–817.
- Sammier, D. (2001). *Sur la modélisation et la commande de suspension de véhicules automobiles*. Ph.D. thesis, Institut National Polytechnique de Grenoble-INPG.
- Savaresi, S.M., Poussot-Vassal, C., Spelta, C., Sename, O., and Dugard, L. (2010). *Semi-active suspension control design for vehicles*. Elsevier.
- Scherer, C., Gahinet, P., and Chilali, M. (1997). Multiobjective output-feedback control via lmi optimization. *IEEE Transactions on Automatic Control*, 42(7), 896–911.
- Sename, O., Basset, M., Talon, B., D’Andrea-Novet, B., Koenig, D., and Martinez, J. (2014). The inove anr 2010 blan 0308 project: Integrated approach for observation and control of vehicle dynamics. In *ECC*, 2340–2345. Strasbourg, France.
- Tarbouriech, S., Garcia, G., Gomes da Silva Jr, J.M., and Queinnec, I. (2011). *Stability and stabilization of linear systems with saturating actuators*. Springer.
- Zin, A., Sename, O., Gaspar, P., Dugard, L., and Bokor, J. (2008). Robust lpv/ \mathcal{H}_∞ control for active suspensions with performance adaptation in view of global chassis control. *Vehicle System Dynamics*, 46(10), 889–912.