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PGD driven by the Constitutive Relation Error - Minimal CRE/PGD

Pierre-Eric Allier, Ludovic Chamoin, Pierre Ladevèze

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PGD driven by the CRE

Minimal CRE/PGD

Pierre-Éric Allier, Ludovic Chamoin, Pierre Ladevèze

1 - Problematic

Numerical models for engineering

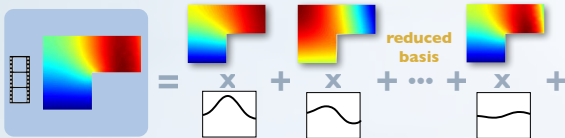
- simulate to design and decide
- resolution of PDEs with
 - a huge number of degrees of freedom (dofs)
 - a huge number of parameters
 - even nonlinearities ...
- direct approaches impossible due to large computation times and large data sets !

2 - PGD: model reduction

Principle

- capture the main features of the behavior
- separation of variables: approximation of the solution on a reduced basis
- without prior knowledge of this reduced basis

$$\text{PDE: } \mathcal{L}(u(M, t)) = 0 \quad \rightarrow \quad u(M, t) \approx \sum_{i=1}^m \psi_i(M) \lambda_i(t)$$



- alternative methods : POD / RB
 - drawback: require partial knowledge of the reduced basis

3 - Reduction error

Different PGD strategies

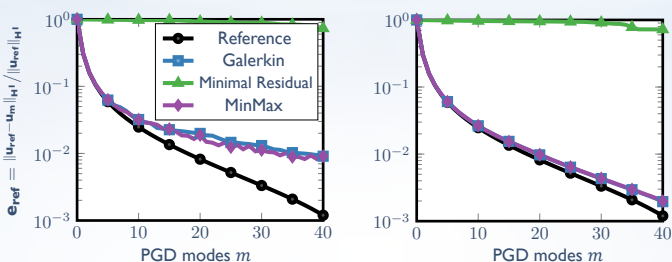
- [Ladevèze 99, Ammar 10, Nouy 10]

Versus a reference reduced solution

- SVD decomposition of a FEM solution

Measure of the deviation

- highly refined space-time meshes: discretization error ignored
- classic
 - update of λ_i (LMT method)



Towards a more robust and stable estimator

Minimal CRE - PGD

Steering the progressive algorithm with an immediate and accurate error estimation, in order to improve the PGD solution

4 - Project goal

Mix of two tools

- the PGD model reduction
- the Constitutive Relation Error (CRE) estimation

Idea: minimization of the error indicator

$$\operatorname{argmin}_{\mathcal{L}(u, \sigma)=0} \|\sigma - \mathcal{K}\epsilon(u)\|_E$$

5 - Feasibility study

Model problem: transient thermal

$$\begin{cases} c \frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial x} = \delta(x - vt) \\ u(0, t) = u(L, t) = u(x, 0) = 0 \end{cases}$$

Minimizing the CRE indicator

- checking the equilibrium implies

$$\sigma(M, t) \approx \sum_{i=1}^m \frac{\partial \lambda_i(t)}{\partial t} Z_i(M) \quad \& \quad \int_{\Omega} Z_i \frac{\partial w}{\partial x} + c \psi_i w \, dx = 0, \quad \forall w \, \forall i$$

Greedy algorithm

- u_{m-1}, σ_{m-1} known, computation of a m^{th} mode (i.e. ψ_m, λ_m, Z_m)

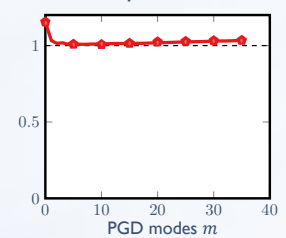
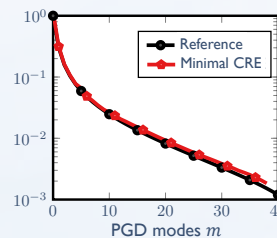
Fixed point algorithm

- computes alternately λ_m and (ψ_m, Z_m)

6 - First results

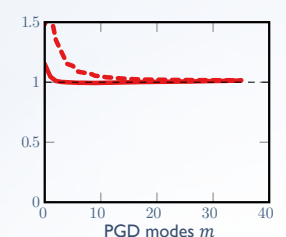
A PGD strategy with its own error indicator

- estimation
- effectivity



Tested for different loads

- time dependent velocity
- train of back & forth loading



7 - Conclusion

A new PGD strategy with

- improved PGD solution
- reliable error reduction estimation

Outlook: extension to nonlinear problems