PGD driven by the Constitutive Relation Error - Minimal CRE/PGD

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PGD driven by the CRE

Minimal CRE/PGD

Pierre-Éric Allier, Ludovic Chamoin, Pierre Ladevèze

**1. Problematic**
- Numerical models for engineering
  - simulate to design and decide
  - resolution of PDEs with
    - a huge number of degrees of freedom (dofs)
    - a huge number of parameters
    - even nonlinearities …
  - direct approaches impossible due to large computation times and large data sets!

**2. PGD: model reduction**
- Principle
  - capture the main features of the behavior
  - separation of variables: approximation of the solution on a reduced basis
  - without prior knowledge of this reduced basis

PDE: \( \mathcal{L}(u(M,t)) = 0 \) \rightarrow \( u(M,t) \approx \sum_{i=1}^{m} \psi_i(M)\lambda_i(t) \)

- alternative methods: POD / RB
  - drawback: require partial knowledge of the reduced basis

**3. Reduction error**
- Different PGD strategies
  - [Ladevèze 99, Ammar 10, Nouy 10]
- Versus a reference reduced solution
  - SVD decomposition of a FEM solution
- Measure of the deviation
  - highly refined space-time meshes: discretization error ignored
  - classic
  - update of \( \lambda \) (LMT method)

**4. Project goal**
- Mix of two tools
  - the PGD model reduction
  - the Constitutive Relation Error (CRE) estimation
- Idea: minimization of the error indicator

\[
\text{argmin} \| \sigma - \mathcal{K}\epsilon(u) \|_E
\]

**5. Feasibility study**
- Model problem: transient thermal

\[
\begin{align*}
\frac{\partial u}{\partial t} - \nu \Delta u &= \delta(x - vt) \\
\frac{\partial u(0,t)}{\partial t} = u(L,t) = u(x,0) = 0
\end{align*}
\]

- Minimizing the CRE indicator
  - checking the equilibrium implies

\[
\sigma(M,t) \simeq \sum_{i=1}^{m} \frac{\partial \psi_i(t)}{\partial t} Z_i(M) \quad \& \quad \int_{\Omega} Z_i \frac{\partial w}{\partial x} + c \psi_i w \, dx = 0, \forall w \in \Omega
\]

- Greedy algorithm
  - \( u_{m+1}, \sigma_{m+1} \) known, computation of a \( m \)th mode (i.e. \( \psi_m, \lambda_m, Z_m \))
- Fixed point algorithm
  - computes alternately \( \lambda_m \) and \( (\psi_m, Z_m) \)

**6. First results**
- A PGD strategy with its own error indicator

**7. Conclusion**
- A new PGD strategy with
  - improved PGD solution
  - reliable error reduction estimation
- Outlook: extension to nonlinear problems