

Wavelet-based Euclidean Distance for Image Quality Assessment

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1. d_{WED} vs SSIM : Einstein
image distortions

2. d_{WED} vs DMOS

3. d_{WED} improved : $J = 5$

4. d_{WED} improved : Best basis
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4. Conclusion

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- Full reference image QA,
- User-centric Quality Experiments \implies mimic HVS,
- Wavelet analysis : CDF9_7 bases,
- Only keep the coefficients of subbands of details (minus the high frequencies) as suggested by Watson et al. [WYSV97].

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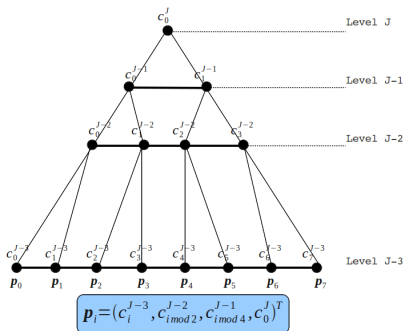
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An implicit localization is provided by the wavelet analysis

⇒ pixel descriptors.

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- The wavelet analysis provides the **pixel descriptors** of the **distorted image** and of the **reference image** independently.
- **But, how to measure the distance between the two images ?**
The wavelet properties must be taking account.

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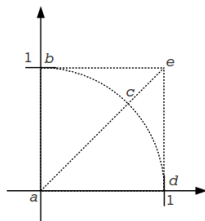
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2. Wavelet-based distance



Remark

- b and d share common coefficients with a ,
- b and d must be closer than c to a [CH07].

$$\text{Euclidean distance } d^2(x, y) = \sum_{i=1}^n (x_i - y_i)^2$$

$$b \quad c \quad d \quad \dots \quad a$$

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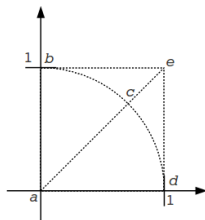
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- b and d share common coefficients with a ,
- b and d must be closer than c to a [CH07].
- Euclidean distance : $d_E^2(x, y) = \sum_{i=1}^n (x_i - y_i)^2$
- b , c and d are at the same distance from point a . .

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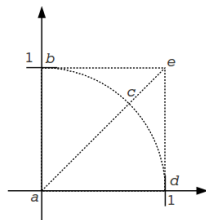
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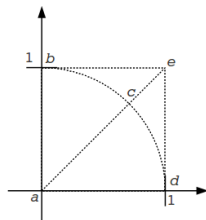
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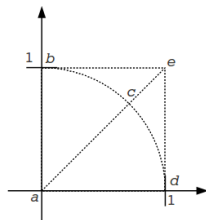
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Definition

Following the image distance definition introduced by Wang et al. [LYJ05], we defined an **anisotropic Euclidean distance** which takes into account common coefficients :

$$d_{WED}^2(x, y) = \sum_{i=1}^n \sum_{j=1}^n g_{i,j} (x_i - y_i)(x_j - y_j)$$

Example

$$g_{i,j} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(i-j)^2}{2\sigma^2}}$$

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Example

With $\sigma = 1$ and neglecting the normalization constant :

$$g_{i,j} \approx e^{-\frac{(i-j)^2}{2}}$$

The descriptors sharing identical subband coefficients, are close together :

$$\left\{ \begin{array}{l} d_{WED}^2(a, b) = d_{WED}^2(a, d) = 1 \\ d_{WED}^2(a, c) = 1 + e^{-\frac{1}{2}} \\ d_{WED}^2(a, e) = 2 \left(1 + e^{-\frac{1}{2}} \right) \end{array} \right.$$

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- No color information is taken into account,
And the maximum level of decomposition, $J = 3$,
- The First experiment exhibits the relationship
between SSIM and D_{WED} ,
- The following ones analyse the relationship between
 D_{WED} and DMOS,

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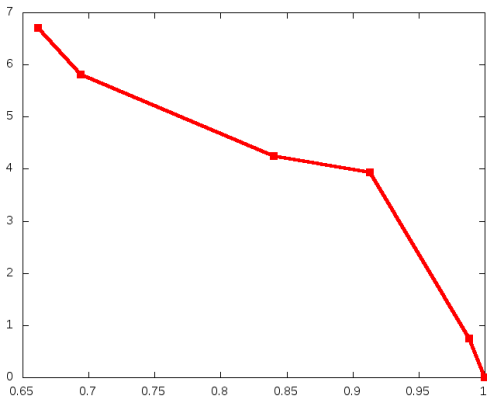
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1. d_{WED} vs SSIM : Einstein image distortions

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d_{WED} tends to be inversely proportional to SSIM responds.

Type	MSE	SSIM	d_{WED}
Ref.	0	1	0
Mean shift	144	0.988	0.743432
Contrast	144	0.913	3.93144
Impulse	144	0.840	4.23846
Blur	144	0.694	5.81087
JPEG	142	0.662	6.70539



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2. d_{WED} vs DMOS

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- Subset of the LIVE DB [SWCB] : *Caps, CoinsInFountain, Carnivaldolls, Plane, Sailing2, Parrots, Buildings, LightHouse, Monarch, Bikes and Statue,*
- Wed distance increases almost monotonically with the DMOS increasing.
- Few outliers appear within these tests,

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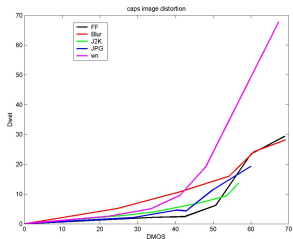
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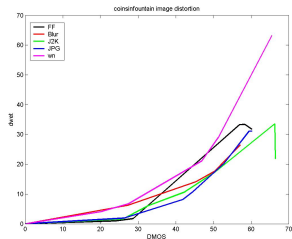
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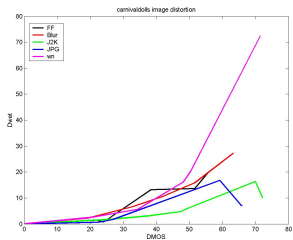
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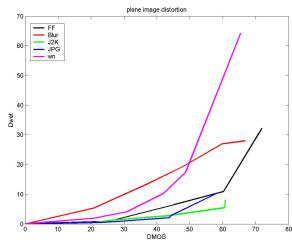
Caps



CoinsInFountain



CarnivalDolls



Plane

2. d_{WED} vs DMOS

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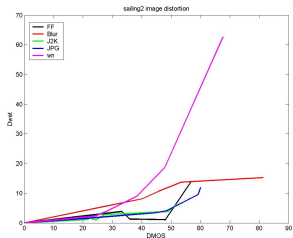
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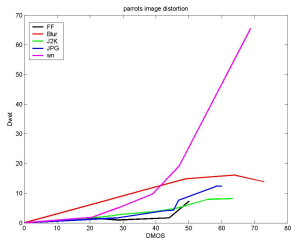
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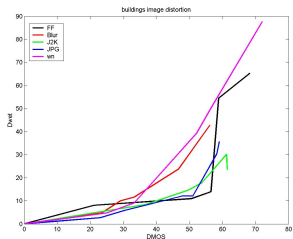
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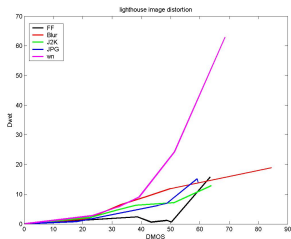
Sailing2



Parrots



Buildings



LightHouse

2. d_{WED} vs DMOS

3. Experimentations

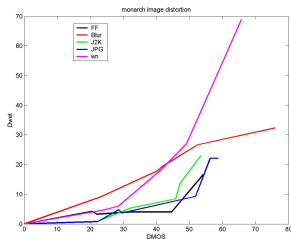
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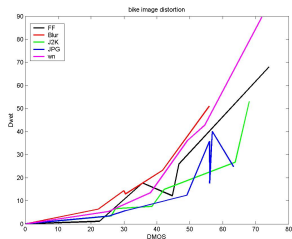
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Monarch



Bikes

2. d_{WED} vs DMOS

3. Experimentations

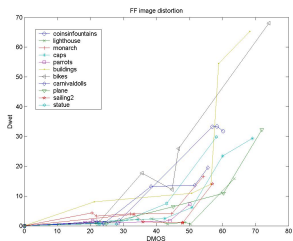
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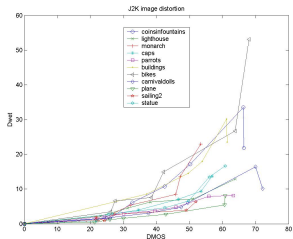
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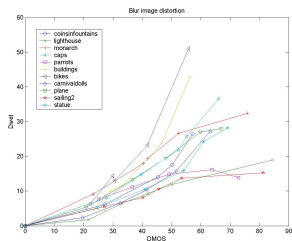
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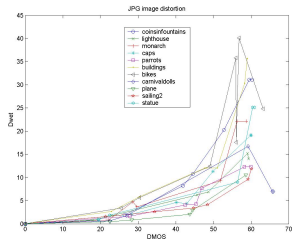
Fast fading



JPEG2000



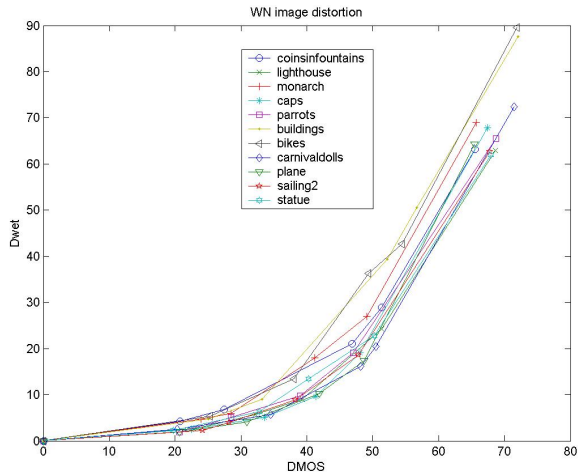
Gaussian blur



JPEG

2. d_{WED} vs DMOS

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White noise !

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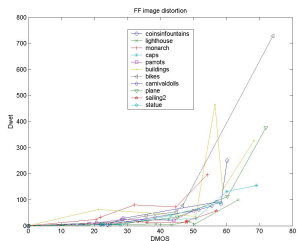
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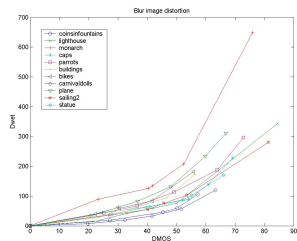
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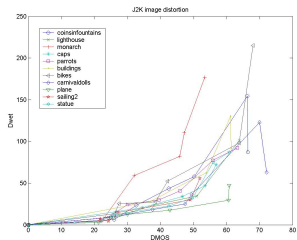
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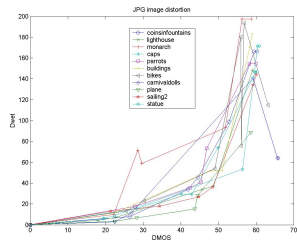
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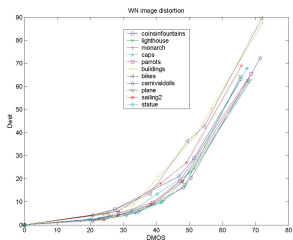
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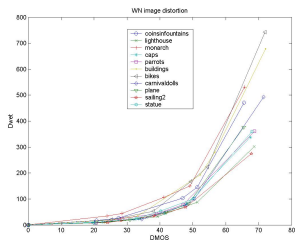
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4. Conclusion



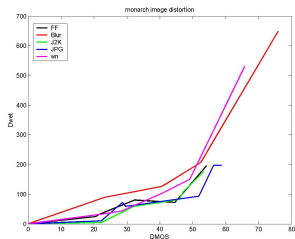
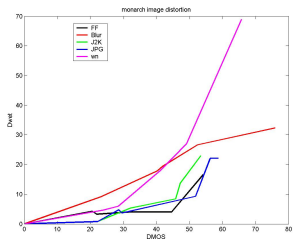
White noise with $J = 3$



White noise with $J = 5$

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Entropy-based selection applied on distortions of Monarch image [CW92].

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2. Wavelet-based distance

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1. d_{WED} vs SSIM : Einstein image distortions

2. d_{WED} vs DMOS

3. d_{WED} improved : $J = 5$

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Current study :

- Automatic selection of the maximum level of decomposition J ,
- Best bases selection adapted to image QA,
- What about the color ?
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- Region based image QA (since d_{WED} is pixel-based),
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