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Selma Belhadj Amor, Samir M. Perlaza, Ioannis Krikidis, H. Vincent Poor. Feedback Enhances Simultaneous Wireless Information and Energy Transmission in Multiple Access Channels. [Research Report] RR-8804, INRIA Grenoble - Rhône-Alpes. 2015. hal-01223586v3

**HAL Id: hal-01223586**

**<https://hal.science/hal-01223586v3>**

Submitted on 8 Feb 2016 (v3), last revised 25 Jul 2017 (v7)

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H. Vincent Poor

**RESEARCH  
REPORT**

**N° 8804**

November 2015

Project-Team Socrate





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Research Report n° 8804 — November 2015 — 37 pages

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**Abstract:** In this report, the fundamental limits of simultaneous information and energy transmission in the two-user Gaussian multiple access channel (G-MAC) with and without feedback are fully characterized. All the achievable information and energy transmission rates (in bits per channel use and energy-units per channel use, respectively) are identified. Thus, the information-energy capacity region is defined in both cases. In the case without feedback, an achievability scheme based on power-splitting and successive interference cancelation is shown to be optimal. Alternatively, in the case with feedback (G-MAC-F), a simple yet optimal achievability scheme based on power-splitting and Ozarow's capacity achieving scheme is presented. Three of the most important observations in this work are: (a) The information-energy capacity region of the G-MAC without feedback is a proper subset of the information-energy capacity region of the G-MAC-F; (b) Feedback can at most double the energy rate for a fixed information rate; and (c) Time-sharing with power control is strictly suboptimal in terms of sum-rate in the G-MAC without feedback.

**Key-words:** Feedback, Gaussian multiple access channel, simultaneous information and energy transmission, RF energy harvesting, information-energy capacity region.

# L'utilisation de la voie de retour améliore la transmission simultanée d'information et d'énergie dans les canaux sans fils à accès multiple

**Résumé :** Dans le présent-rapport, les limites fondamentales de la transmission simultanée d'information et d'énergie dans le canal Gaussien à accès multiple (G-MAC) avec et sans voie de retour sont déterminées. L'ensemble des débits atteignables de transmission d'information et d'énergie (en bits par utilisation canal et en unités d'énergie par utilisation canal respectivement) est identifié. Ainsi, les régions de capacité d'information-énergie sont définies dans les deux cas. Dans le cas sans voie de retour, on démontre qu'un schéma d'atteignabilité, basé sur la division de puissance et sur l'annulation successive de l'interférence, est optimal. Alternativement, dans le cas avec voie de retour (G-MAC-F), un schéma d'atteignabilité, simple mais optimal, basé sur la division de puissance et sur le schéma d'Ozarow qui atteint la capacité, est présenté. Trois parmi les observations les plus importantes dans ce travail sont: (a) la région de capacité d'information-énergie du G-MAC sans voie de retour est un sous-ensemble propre de la région de capacité d'information-énergie du G-MAC-F; (b) l'utilisation de la voie de retour peut au plus multiplier par deux le débit d'énergie pour un débit d'information fixé; et (c) le partage du temps avec contrôle de puissance est strictement sous-optimal en termes de débit-somme pour le G-MAC sans voie de retour.

**Mots-clés :** Voie de retour, canal Gaussien à accès multiple (G-MAC), transmission simultanée d'information et d'énergie, collecte d'énergie RF, région de capacité d'information-énergie.

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## 1 Introduction

For decades, a traditional engineering perspective was to use radio frequency (RF) signals exclusively for information transmission. However, this approach has been shown to be suboptimal [2] since an RF signal carries both energy and information. From this standpoint, a variety of modern wireless systems question this conventional assumption and suggest that RF signals can be simultaneously used for information and energy transmission [3]. Typical examples of communications technologies already exploiting this principle are radio frequency identification (RFID) devices and power line communications. Beyond the existing applications, simultaneous transmission of both information and energy appears as a promising technology for a variety of emerging applications including low-power short-range communication systems, sensor networks, machine-to-machine networks and body-area networks, among others [4]. Nevertheless, information and energy transmission are often conflicting tasks and thus subject to a trade-off between the information transmission rate (bits per channel use) and the energy transmission rate (energy-units per channel use). This trade-off is evidenced in finite constellation schemes, as highlighted in Popovski *et al.*'s [5]. Consider the noiseless transmission of a 4-PAM signal over a point-to-point channel in the alphabet  $\{-2, -1, 1, 2\}$ . If there is no received energy rate constraint, one can clearly convey 2 bits/ch.use by choosing all available symbols with equal probability. However, if one requires the received energy rate to be for instance the maximum possible, the maximum transferable information rate is 1 bit/ch.use (since one is forced to communicate only with the largest energy symbols). From this simple example, it is easy to see how additional energy rate constraints may change the overall performance of the network.

Despite the vast existing literature on this subject, the fundamental limits of simultaneous energy and information transmission (SEIT) are still unknown in most multi-user channels. The pioneering works by Varshney in [2] and [6], as well as, Grover and Sahai in [7] provided the fundamental limits on SEIT in point-to-point channels. More specifically, in [6] the case of a the single point-to-point link was discussed and in [7] and [2] the case of parallel point-to-point links was studied. In the context of multi-user channels, most of the existing studies of SEIT follow a signal-processing or networking approach and focus mainly on the feasibility aspects. For instance, optimization of beamforming strategies over more involved network structures was considered in [8, 9], and [10] for multi-antenna broadcast channels and in [11] for multi-antenna interference channels. SEIT was also studied in the general realm of cellular systems in [12] as well as in multi-hop relaying systems in [13, 14, 15, 16, 17], and [18]. Other studies in the two-way channel are reported in [5] and in [19] in graphical unicast and multicast networks.

### 1.1 SEIT in Multiple Access Channels (MACs)

In the particular case of the discrete memoryless multiple access channel (DM-MAC), the trade-off between information rate and energy rate has been studied in [14]. Therein, Fouladgar *et al.* characterized the information-energy capacity region of the two-user discrete memoryless MAC when a minimum energy rate is required at the input of the receiver. Such a constraint changes the dynamic of the communication system in the sense that it requires additional transmitter coordination to achieve the targeted energy rate. An extension of the work in [14] to the Gaussian MAC (G-MAC) case is far from trivial due to the fact that the information-energy capacity region involves an auxiliary random-variable which cannot be eliminated as in the case without energy constraints, as independently suggested by Cover [20] and Wyner [21]. Other types of energy rate constraints for the G-MAC have been also investigated. For instance, Gastpar [22] considered the G-MAC under a maximum received energy rate constraint. Under this assumption, channel-output feedback has been shown not to increase the capacity region. In the case



studied by Fouladgar *et al.* in [14], the effect of feedback is not yet well understood from an energy transmission perspective.

More generally, the use of feedback in the  $K$ -user G-MAC, even without energy rate constraints, has been shown to be of limited impact in terms of sum-rate improvement. This holds even in the case of perfect feedback. More specifically, the use of feedback in the G-MAC increases the sum-capacity by at most  $\frac{\log_2(K)}{2}$  bits per channel use [23]. Hence, the use of feedback is difficult to justify from the point of view of exclusive information transmission.

## 1.2 Contributions

This paper studies the fundamental limits of SEIT in the two-user G-MAC with and without feedback. It shows that when the goal is to simultaneously transmit both information and energy, feedback can significantly improve the global performance of the system in terms of both information and energy transmission rates. One of the main contributions is the identification of all the achievable information and energy transmission rates in bits per channel use and energy-units per channel use, respectively. More specifically, the information-energy capacity region is fully characterized in both cases. In the case without feedback, an achievability scheme based on power-splitting and successive interference cancellation is shown to be optimal. Alternatively, in the case with feedback (G-MAC-F), a simple yet optimal achievability scheme based on power-splitting and Ozarow's capacity achieving scheme is presented. Three of the most important observations in this work are: (a) The information-energy capacity region of the G-MAC without feedback is a proper subset of the information-energy capacity region of the G-MAC-F, that is, the former is strictly contained in the latter; (b) Feedback can at most double the energy rate for a fixed information rate; and (c) Time-sharing with power control is strictly suboptimal in the G-MAC without feedback in terms of sum-rate.

## 1.3 Organization of the Report

The remainder of the report is structured as follows. Sec. 2 formulates the problem of simultaneous energy and information transmission in the two-user G-MAC-F. Sec. 3 and Sec. 4 show the main results of this paper for the G-MAC without and with feedback, respectively. Namely, for both settings the following fundamental limits are derived: (a) the information-energy capacity region ; and (b) the maximum individual information rates and sum information rates that can be achieved given a targeted energy rate. In Sec. 4, the maximum energy rate improvement that can be obtained by using feedback given a targeted information rate is also discussed. Sec. 6 presents the proof of achievability and converse for the information-energy capacity region of the G-MAC-F (Theorem 2). Sec. 7 presents the proof of the perfect feedback information sum-capacity for a given energy rate constraint (Proposition 4).

## 2 Gaussian Multiple Access Channel with Feedback

Consider the two-user memoryless G-MAC with perfect channel-output-feedback (G-MAC-F) in Fig. 1. At each channel use  $t \in \mathbb{N}$ ,  $X_{1,t}$  and  $X_{2,t}$  denote the real symbols sent by transmitters 1 and 2, respectively. Let  $n \in \mathbb{N}$  denote the blocklength. The symbols  $X_{i,1}, \dots, X_{i,n}$  satisfy an expected average *input power constraint*

$$\frac{1}{n} \sum_{t=1}^n \mathbb{E}[X_{i,t}^2] \leq P_i, \quad (1)$$

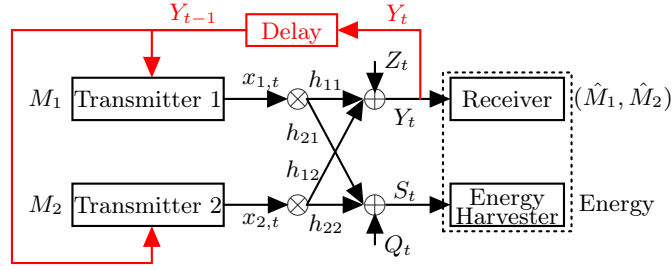


Figure 1: Two-user memoryless Gaussian MAC with feedback and energy harvester.

where  $P_i$  denotes the average transmit power of transmitter  $i$  in energy-units per channel use for  $i \in \{1, 2\}$ . The receiver observes the real channel output

$$Y_{1,t} = h_{11}X_{1,t} + h_{12}X_{2,t} + Z_t, \quad (2)$$

and the energy harvester (EH) observes

$$Y_{2,t} = h_{21}X_{1,t} + h_{22}X_{2,t} + Q_t, \quad (3)$$

where  $h_{1i}$  and  $h_{2i}$  are the corresponding constant non-negative channel coefficients from transmitter  $i$  to the receiver and EH, respectively. The channel coefficients must satisfy the following  $\mathcal{L}_2$ -norm condition:

$$\forall j \in \{1, 2\}, \quad \|\mathbf{h}_j\|^2 \leq 1, \quad (4)$$

with  $\mathbf{h}_j \triangleq (h_{j1}, h_{j2})^\top$  to ensure the principle of conservation of energy. .

The noise terms  $Z_t$  and  $Q_t$  are realizations of two identically distributed zero-mean unit-variance real Gaussian random variables. In the following, there is no particular assumption on the joint distribution of  $Q_t$  and  $Z_t$ .

A perfect feedback link from the receiver to transmitter  $i$  allows at the end of each channel use  $t$ , the observation of the channel output  $Y_{t-d}$  at transmitter  $i$ , with  $d \in \mathbb{N}$  the delay of the feedback channel. Without any loss of generality, the delay is assumed to be the same from the receiver to both transmitters and equivalent to one channel use, i.e.,  $d = 1$ .

The G-MAC-F above is fully described by the signal to noise ratios (SNRs):  $\text{SNR}_{ji}$ , with  $\forall (i, j) \in \{1, 2\}^2$  are defined as follows

$$\text{SNR}_{ji} \triangleq |h_{ji}|^2 P_i, \quad (5)$$

given the normalization over the noise powers.

Within this context, two main tasks are to be simultaneously accomplished: information transmission and energy transmission.

## 2.1 Information Transmission

The goal of the communication is to convey the independent messages  $M_1$  and  $M_2$  from transmitters 1 and 2 to the common receiver. The messages  $M_1$  and  $M_2$  are independent of the noise terms  $Z_1, \dots, Z_n, Q_1, \dots, Q_n$  and uniformly distributed over the sets  $\mathcal{M}_1 \triangleq \{1, \dots, \lfloor 2^{nR_1} \rfloor\}$  and  $\mathcal{M}_2 \triangleq \{1, \dots, \lfloor 2^{nR_2} \rfloor\}$ , where  $R_1$  and  $R_2$  denote the transmission rates and  $n \in \mathbb{N}$  the blocklength.

At each time  $t$ , the existence of feedback links allows the  $t$ -th symbol of transmitter  $i$  to be dependent on all previous channel outputs  $Y_1, \dots, Y_{t-1}$  as well as its message index  $M_i$  and a randomly generated index  $\Omega \in \{1, \dots, \lfloor 2^{nR_r} \rfloor\}$ , with  $R_r \geq 0$ , that is independent of both  $M_1$  and  $M_2$  and assumed to be known by all transmitters and the receiver. More specifically,

$$X_{i,1} = f_{i,1}^{(n)}(M_i, \Omega) \quad \text{and} \quad (6)$$

$$X_{i,t} = f_{i,t}^{(n)}(M_i, \Omega, Y_1, \dots, Y_{t-1}), \quad t \in \{2, \dots, n\}, \quad (7)$$

for some encoding functions

$$f_{i,1}^{(n)}: \mathcal{M}_i \times \mathbb{N} \rightarrow \mathbb{R} \quad \text{and} \quad (8)$$

$$f_{i,t}^{(n)}: \mathcal{M}_i \times \mathbb{N} \times \mathbb{R}^{t-1} \rightarrow \mathbb{R}. \quad (9)$$

The receiver produces an estimate  $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) = \Phi^{(n)}(Y^n)$  of the message-pair  $(M_1, M_2)$  via a decoding function  $\Phi^{(n)}: \mathbb{R}^n \rightarrow \mathcal{M}_1 \times \mathcal{M}_2$ , and the average probability of error is

$$P_{\text{error}}^{(n)}(R_1, R_2) \triangleq \Pr \{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) \neq (M_1, M_2)\}. \quad (10)$$

## 2.2 Energy Transmission

The expected energy transmission rate (in energy-units per channel use) at the EH is

$$B^{(n)} \triangleq \frac{1}{n} \sum_{t=1}^n \mathbb{E}[Y_{2,t}^2]. \quad (11)$$

The goal of the energy transmission is to guarantee that the energy rate  $B^{(n)}$  is not less than a given constant  $B$  that must satisfy

$$0 < B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, \quad (12)$$

for the problem to be feasible. Hence, the probability of energy outage is defined as follows:

$$P_{\text{outage}}^{(n)}(B) \triangleq \Pr \{B^{(n)} < B - \epsilon\}, \quad (13)$$

for some  $\epsilon > 0$  arbitrarily small.

## 2.3 Simultaneous Information and Energy Transmission

The G-MAC-F in Fig. 1 is said to operate at the information-energy rate triplet  $(R_1, R_2, B) \in \mathbb{R}_+^3$  considered if both transmitters and the receiver use a transmit-receive configuration such that: (a) reliable communication at information rates  $R_1$  and  $R_2$  is ensured; and (b) the energy transmission rate during the whole block-length is not lower than  $B$ . Under these conditions, the information-energy rate triplet  $(R_1, R_2, B)$  is said to be achievable.

**Definition 1** (Achievable Rates). *The triplet  $(R_1, R_2, B) \in \mathbb{R}_+^3$  is achievable if there exists a sequence of encoding and decoding functions  $\{\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \Phi^{(n)}\}_{n=1}^\infty$  such that both the average error probability and the energy-outage probability tend to zero as the blocklength  $n$  tends to infinity. That is,*

$$\limsup_{n \rightarrow \infty} P_{\text{error}}^{(n)}(R_1, R_2) = 0, \quad (14)$$

$$\limsup_{n \rightarrow \infty} P_{\text{outage}}^{(n)}(B) = 0 \quad \text{for any } \epsilon > 0. \quad (15)$$

From Def. 1, it is clear that for any achievable triplet  $(R_1, R_2, B)$ , whenever the targeted energy rate  $B$  is smaller than the minimum energy rate required to guarantee reliable communications at the information rates  $R_1$  and  $R_2$ , the energy rate constraint is vacuous. This is mainly because the energy transmission rate is always satisfied and thus, the transmitter can exclusively use the available power budget for increasing the information transmission rate. Alternatively, when the energy rate  $B$  must be higher than what is strictly necessary to guarantee reliable communication, the transmitters face a trade-off between information and energy rates. Often, increasing the energy transmission rate implies decreasing the information transmission rates and *vice-versa*. This trade-off is accurately modeled by the notion of *information-energy capacity region*.

**Definition 2** (Information-Energy Capacity Region). *The information-energy capacity region of the G-MAC with perfect channel-output feedback  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  and without feedback  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  are the closure of all achievable information-energy rate triplets  $(R_1, R_2, B)$  in the corresponding cases.*

The main results for the G-MAC with and without feedback presented in this paper are provided in terms of the information-energy capacity region (Def. 2) in the following two sections.

### 3 Main Results: Information-Energy Capacity Region without Feedback

This section describes the fundamental limits of SEIT in the G-MAC for the case in which feedback is not available. These results are particular cases of the results presented in Sec. 4 in which feedback is considered. The interest of presenting these results separately stems from the need for comparing both cases.

#### 3.1 Information-Energy Capacity Region without Feedback

The information-energy capacity region of the G-MAC without feedback, denoted by  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$  is fully characterized by the following theorem.

**Theorem 1** (Information-Energy Capacity Region). *The information-energy capacity region  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  of the G-MAC without feedback is the set of all non-negative information-energy rate triplets  $(R_1, R_2, B)$  that satisfy*

$$R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11}), \quad (16a)$$

$$R_2 \leq \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12}), \quad (16b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12}), \quad (16c)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (16d)$$

with  $(\beta_1, \beta_2) \in [0, 1]^2$ .

##### 3.1.1 Comments on the Proof

To prove the achievability of the region presented in Theorem 1, consider that each transmitter  $i$ , with  $i \in \{1, 2\}$ , uses a fraction  $\beta_i \in [0, 1]$  of its available power to transmit information and uses the remaining fraction of power  $(1 - \beta_i)$  to transmit energy.

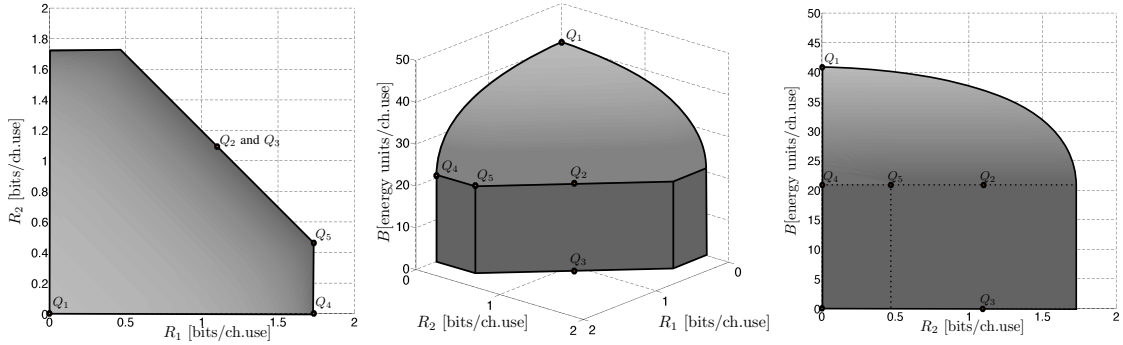


Figure 2: 3-D representation of the information-energy capacity region without feedback  $\mathcal{E}(10, 10, 10, 10)$  in the coordinate system  $(R_1, R_2, B)$ . Left and right figures represent a bi-dimensional view in the  $R_1$ - $R_2$  and  $B$ - $R_2$  planes of  $\mathcal{E}(10, 10, 10, 10)$ , respectively. The figure in the center is a 3-D representation of  $\mathcal{E}(10, 10, 10, 10)$ . Note that points  $Q_1$ ,  $Q_2$  and  $Q_3$  are coplanar and satisfy that  $R_1 = R_2$ . In particular,  $Q_1 = (0, 0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}})$ ; and the points  $Q_2$  and  $Q_3$  are also collinear and  $R_1 = R_2 = \frac{1}{4} \log_2(1 + \text{SNR}_{11} + \text{SNR}_{12})$ . The points  $Q_2$ ,  $Q_4$  and  $Q_5$  are coplanar and they satisfy that  $B = 1 + \text{SNR}_{21} + \text{SNR}_{22}$ . In particular,  $Q_4 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), 0, 1 + \text{SNR}_{21} + \text{SNR}_{22})$  and  $Q_5 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), \frac{1}{2} \log_2(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}}), 1 + \text{SNR}_{21} + \text{SNR}_{22})$ .

Given a power-split  $(\beta_1, \beta_2) \in [0, 1]^2$ , the achievability of information rate pairs satisfying (16a)-(16c) follows the scheme described in [20] or [21]. Additionally, in order to satisfy the received energy constraint (16d), transmitters send common randomness that is known to both transmitters and the receiver using all their remaining power. This common randomness does not carry any information and does not produce any interference to the information-carrying signals. More specifically, at each time  $t$ , transmitter  $i$ 's channel input can be written as:

$$X_{i,t} = \sqrt{(1 - \beta_i)P_i}W_t + U_{i,t}, \quad i \in \{1, 2\}, \quad (17)$$

for some independent zero-mean Gaussian information-carrying symbols  $U_{1,t}$  and  $U_{2,t}$  with variances  $\beta_1 P_1$  and  $\beta_2 P_2$ , respectively, and independent thereof  $W_t$  is a zero-mean unit-variance Gaussian energy-carrying symbol known non-causally to all terminals.

The receiver subtracts the common randomness and then performs successive decoding to recover the messages  $M_1$  and  $M_2$ . Note that this strategy achieves the corner points of the information rate-region at a given energy rate. Time-sharing between the corner points and the points on the axes is needed to achieve the remaining points.

The converse and the analysis of the average received energy rate follow the same lines as in the case with feedback described in Section 6 when the IC channel input components are assumed to be independent.

### 3.1.2 Example and Observations

Consider the information-energy capacity region of the G-MAC without feedback  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , with  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$  depicted in Fig. 2. Therein, left and right figures represent a bi-dimensional view in the  $R_1$ - $R_2$  and  $B$ - $R_2$  planes of  $\mathcal{E}(10, 10, 10, 10)$ , respectively. The figure in the center is a 3-D representation of  $\mathcal{E}(10, 10, 10, 10)$ .

Fig. 3 shows a general example of the intersection of the volume  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , in the Cartesian coordinates  $(R_1, R_2, B)$ , with a plane  $B = b$ , when  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22}$ .

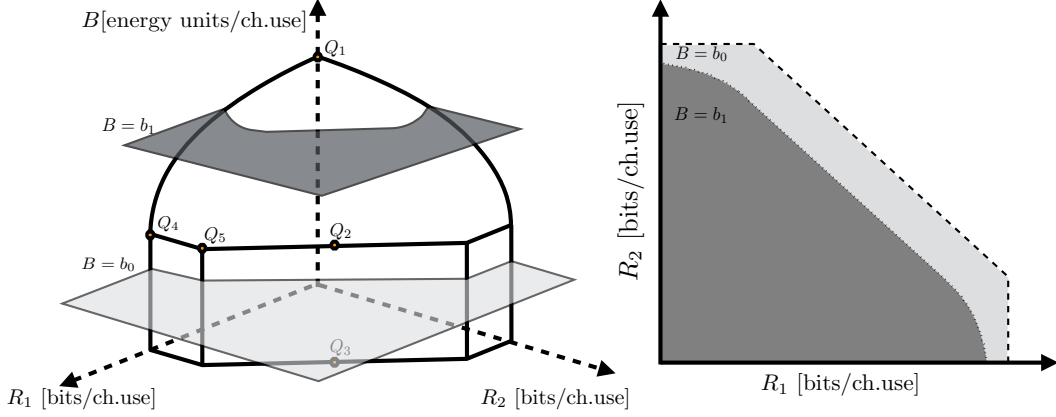


Figure 3: Intersection of the planes  $B = b_0$  and  $B = b_1$  with the information-energy capacity region of the G-MAC without feedback  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , with  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22}$ ,  $b_0 \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ ,  $b_1 \in [1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ .

For any achievable triplet  $(R_1, R_2, B)$ , whenever the required energy rate  $B$  is smaller than the minimum energy rate required to guarantee reliable communication at rates  $R_1$  and  $R_2$ , the energy constraint is vacuous. Hence, transmitting information is always enough to satisfy the energy constraint. See for instance the intersection the volume  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  with the plane  $B = b_0 \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$  in Fig. 3. This intersection corresponds to the set of triplets  $(R_1, R_2, B)$ , in which the corresponding pairs  $(R_1, R_2)$  form a set that is identical to the capacity region of the G-MAC without feedback, denoted by  $\mathcal{C}(\text{SNR}_{11}, \text{SNR}_{12})$ . That is, when the energy rate constraint is lower than the maximum energy rate achievable with independent symbols, all information rates of the information capacity region  $\mathcal{C}(\text{SNR}_{11}, \text{SNR}_{12})$  are achievable. (See for instance points  $Q_2, Q_3, Q_4$ , and  $Q_5$  in Fig. 3.)

On the other hand, when the energy constraint consists of guaranteeing an energy rate of  $B \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$  energy-units per channel use, that is, when the energy rate  $B$  at the input of the energy harvester is required to be higher than what is strictly necessary to guarantee reliable communication at the information sum-rate, the transmitters deal with a trade-off between information and energy transmission. This can be interpreted in terms of power-splits, i.e.,  $0 \leq \beta_i < 1$ , for  $i \in \{1, 2\}$ . In this case, part of the transmitter power budget is exclusively dedicated to the transmission of energy. Thus, for all triplets  $(R_1, R_2, B)$ , it follows that the corresponding information rate pairs  $(R_1, R_2)$  form a proper set of the capacity region  $\mathcal{C}(\text{SNR}_{11}, \text{SNR}_{12})$ ; see for instance the intersection of the volume  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  with the plane  $B = b_1 \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$  in Fig. 3.

Finally, requiring an energy transmission rate  $B \in (1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \infty)$  constrains the achievability of any positive rate (see point  $Q_1$  in Fig. 3, which is achievable with  $(\beta_1 = \beta_2 = 0)$ ). However, it is important to highlight that the transmitters cannot guarantee an energy rate that is higher than the energy rate achieved with full transmitter cooperation.

### 3.2 Information-Energy Transmission without Feedback and with Minimum Energy Rate Constraint $b$

Let  $0 \leq b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  denote the minimum energy rate that must be guaranteed at the input of the EH in the G-MAC without feedback and with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ .

In the following, for a fixed minimum energy rate constraint  $b$ , the maximum individual information rates as well as the information sum-capacity are identified.

#### 3.2.1 Maximum Individual Information Rates without Feedback and with Minimum Energy Rate Constraint $b$

The maximum individual rate  $R_i^{\text{NF}}(b)$ , with  $i \in \{1, 2\}$  given an energy rate constraint of  $b$  energy-units per channel use at the input of the EH is the solution to an optimization problem of the form

$$R_i^{\text{NF}}(b) = \max_{(r_i, r_j, c) \in \mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}): c \geq b} r_i. \quad (18)$$

The following proposition characterizes the solution to the optimization problem (18).

**Proposition 1** (Maximum Individual Information Rates without Feedback with Minimum Energy Rate Constraint  $b$ ). *For a fixed minimum energy rate  $b$  required at the input of the EH, the maximum individual rates  $R_i^{\text{NF}}(b)$  of the G-MAC without feedback, with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ , are given by*

$$R_i^{\text{NF}}(b) = \frac{1}{2} \log_2 (1 + \beta^*(b) \text{SNR}_{1i}), \quad (19)$$

with  $\beta^*(b) \in [0, 1]$  defined as follows:

$$\beta^*(b) = 1 - \left( \frac{(b - (1 + \text{SNR}_{21} + \text{SNR}_{22}))^+}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} \right)^2. \quad (20)$$

*Proof:*

From the assumptions of Proposition 1 it follows that an energy transmission rate of  $b$  energy-units per channel use must be guaranteed at the input of the energy harvester. Then, the set of power-splits  $(\beta_i, \beta_j)$  that satisfy this constraint must satisfy

$$g_0(\beta_i, \beta_j) \geq b, \quad (21)$$

with

$$g_0(\beta_i, \beta_j) \triangleq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_i)\text{SNR}_{21}(1 - \beta_j)\text{SNR}_{22}}. \quad (22)$$

These power-splits are referred to as *feasible power-splits*.

Using a Fourier-Motzkin elimination in the constraints (16a)-(16c) to eliminate  $R_j$ , it can be shown that transmitter  $i$ 's individual rate maximization problem in (18) can be written as

$$R_i^{\text{NF}}(b) = \max_{(\beta_i, \beta_j) \in [0, 1]^2} f_i(\beta_i, \beta_j), \quad (23a)$$

$$\text{subject to: } g_0(\beta_i, \beta_j) \geq b, \quad (23b)$$

with

$$f_i(\beta_i, \beta_j) \triangleq \min \left\{ \frac{1}{2} \log_2 (1 + \beta_i \text{SNR}_{1i}), \frac{1}{2} \log_2 (1 + \beta_i \text{SNR}_{1i} + \beta_j \text{SNR}_{1j}) \right\} \quad (24)$$

and  $g_0(\beta_1, \beta_2)$  is defined in (22).

For any feasible power-split  $(\beta_i, \beta_j)$ , it holds that

$$f_i(\beta_i, \beta_j) = \frac{1}{2} \log_2 (1 + \beta_i \text{SNR}_{1i}). \quad (25)$$

The target function  $f_i(\beta_i, \beta_j)$  is increasing in  $\beta_i$  and is independent of  $\beta_j$ . Since the constraint function is monotonically decreasing in  $(\beta_i, \beta_j)$ , in order to maximize transmitter  $i$ 's individual rate, the optimal power-split should be a feasible power-split in which  $\beta_i$  is maximized while  $\beta_j$  can be forced to 0. Thus, the maximization problem in (18) can be written as follows:

$$R_i^{\text{NF}}(b) = \max_{\beta_i \in [0,1]} \frac{1}{2} \log_2 (1 + \beta_i \text{SNR}_{1i}), \quad (26a)$$

$$\text{subject to: } g_0(\beta_i, 0) \geq b. \quad (26b)$$

Transmitter  $i$ 's achievable information rate is increasing in  $\beta_i$  and the energy rate constraint is decreasing in  $\beta_i$ . Hence, transmitter  $i$  is able to achieve the maximum individual rate if the energy transmission of transmitter  $i$  is made at the minimum rate to meet the energy rate constraint, i.e., if there is equality in (21). In this configuration, transmitter  $i$  can use a power-split in which the  $\beta_i = \beta^*(b)$  with  $\beta^*(b)$  defined in (20) which yields the maximum individual rate  $R_i(b) = \frac{1}{2} \log_2 (1 + \beta^*(b) \text{SNR}_{1i})$ . ■

### 3.2.2 Information Sum-Capacity with Minimum Received Energy Rate Constraint $b$

The information sum-capacity  $R_{\text{sum}}^{\text{NF}}(b)$  (i.e., the maximum information sum-rate) of the G-MAC subject to a minimum energy rate constraint  $b$  is the solution to an optimization problem of the form

$$R_{\text{sum}}^{\text{NF}}(b) = \max_{(r_1, r_2, c) \in \mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}): c \geq b} r_1 + r_2. \quad (27)$$

The solution to this problem is given by the following theorem.

**Proposition 2** (Information Sum-Capacity with Minimum Energy Rate Constraint  $b$ ). *For a fixed minimum energy rate  $b$  required at the input of the EH, the information sum-capacity of the G-MAC without feedback and with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$  is*

$$R_{\text{sum}}^{\text{NF}}(b) = \frac{1}{2} \log_2 \left( 1 + \beta^\dagger(b) (\text{SNR}_{11} + \text{SNR}_{12}) \right) \quad (28)$$

with

$$\beta^\dagger(b) = \min \left\{ 1, \frac{(1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}} - b)^+}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} \right\}. \quad (29)$$

*Proof:* The sum-rate maximization problem in (27) can be written as follows:

$$R_{\text{sum}}^{\text{NF}}(b) = \max_{(\beta_1, \beta_2) \in [0,1]^2} f_0(\beta_1, \beta_2) \quad (30a)$$

$$\text{subject to: } g_0(\beta_1, \beta_2) \geq b, \quad (30b)$$



where the functions  $f_0$  and  $g_0$  are defined as

$$f_0(\beta_1, \beta_2) \triangleq \min \left\{ \frac{1}{2} \log_2(1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12}), \frac{1}{2} \log_2(1 + \beta_1 \text{SNR}_{11}) + \frac{1}{2} \log_2(1 + \beta_2 \text{SNR}_{12}) \right\} \quad (31)$$

and  $g_0(\beta_1, \beta_2)$  defined as in (22).

For any nonnegative  $\beta_1$  and  $\beta_2$  it can be shown that

$$f_0(\beta_1, \beta_2) = \frac{1}{2} \log_2(1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12}), \quad (32)$$

and thus the function  $f_0$  is monotonically increasing in  $(\beta_1, \beta_2)$ . The function  $g_0$  is monotonically decreasing in  $(\beta_1, \beta_2)$ .

Note that for any  $(\beta_1, \beta_2) \in [0, 1]^2$ , it follows that

$$g_0(\min(\beta_1, \beta_2), \min(\beta_1, \beta_2)) \geq g_0(\beta_1, \beta_2), \quad (33)$$

and

$$f_0(\min(\beta_1, \beta_2), \min(\beta_1, \beta_2)) \leq f_0(\beta_1, \beta_2) \leq f_0(\max(\beta_1, \beta_2), \max(\beta_1, \beta_2)) \quad (34)$$

with equality in (33) and (34) only when  $\beta_1 = \beta_2$ . Consequently, the sum-rate maximization problem (30) can also be written as

$$R_{\text{sum}}^{\text{NF}}(b) = \max_{\beta \in [0, 1]} f_0(\beta, \beta) \quad (35a)$$

$$\text{subject to: } g_0(\beta, \beta) \geq b. \quad (35b)$$

This implies that the optimal  $\beta^\dagger(b)$  is the highest feasible value,

$$\beta^\dagger(b) = \min \left\{ 1, \left( \frac{1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}} - b}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} \right)^+ \right\},$$

which implies (28) and completes the proof.  $\blacksquare$

Note that, if  $0 \leq b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$ , then

$$1 \leq \frac{1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}} - b}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}}. \quad (36)$$

Hence,  $\beta^\dagger(b) = 1$  and all the available power can be dedicated to the information transmission task.

Alternatively, if  $1 + \text{SNR}_{21} + \text{SNR}_{22} < b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , then

$$0 \leq \frac{1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}} - b}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} < 1, \quad (37)$$

and there is a strict trade-off between information and energy transmission.

Finally, if  $b > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , then

$$\frac{1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}} - b}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} < 0, \quad (38)$$

and thus,  $\beta^\dagger(b) = 0$ . Note that any  $b > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  is not achievable even if all the available power is used for energy transmission. In fact, the power budgets  $P_1$  and  $P_2$  at transmitters 1 and 2 limit inherently the energy rate at the input of the receiver to be at maximum  $1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , which is obtained when both transmitters fully cooperate and use all their available power to transmit energy.

**Remark 1.** *Time-sharing with power-control suggested in Fouladgar et al.'s example [14] is strictly suboptimal and does not achieve sum-capacity for a given minimum received energy constraint.*

Consider the sum-rate optimization problem proposed in [14] in which both users use time-sharing with power control. Specifically, a time-sharing parameter  $\lambda \in [0, 1]$  is chosen and each transmitter  $i$  chooses powers  $P'_i$  (power dedicated to information transmission) and  $P''_i$  (power dedicated to energy transmission). The sum-rate maximization problem proposed in [14] can be written as (using the original notation)

$$\max_{(\lambda, P'_1, P''_1, P'_2, P''_2) \in [0, 1] \times \mathbb{R}_+^4} \frac{\lambda}{2} \log_2 (1 + h_{11}^2 P'_1 + h_{12}^2 P'_2) \quad (39a)$$

subject to :

$$\lambda P'_i + (1 - \lambda) P''_i \leq P_i, \quad i \in \{1, 2\} \quad (39b)$$

$$1 + \lambda(h_{21}^2 P'_1 + h_{22}^2 P'_2) + (1 - \lambda)(h_{21} \sqrt{P''_1} + h_{22} \sqrt{P''_2})^2 \geq b, \quad (39c)$$

where  $P_i$  is the total power budget of transmitter  $i$ .

For any feasible choice of  $(\lambda, P'_1, P''_1, P'_2, P''_2)$ , by the concavity of the logarithm, it follows that

$$\frac{\lambda}{2} \log_2 (1 + h_{11}^2 P'_1 + h_{12}^2 P'_2) \leq \frac{1}{2} \log_2 (1 + \lambda (h_{11}^2 P'_1 + h_{12}^2 P'_2)). \quad (40)$$

Now, when  $\lambda = 1$ , then (40) holds with equality and the problem (39) can be written as

$$\max_{(P'_1, P'_2) \in \mathbb{R}_+^2} \frac{1}{2} \log_2 (1 + h_{11}^2 P'_1 + h_{12}^2 P'_2) \quad (41a)$$

$$\text{subject to : } P'_i \leq P_i, \quad i \in \{1, 2\}, \quad (41b)$$

$$1 + h_{21}^2 P'_1 + h_{22}^2 P'_2 \geq b, \quad (41c)$$

which is infeasible for any  $b > 1 + \text{SNR}_{21} + \text{SNR}_{22}$ . When  $\lambda \in [0, 1)$ , then (40) holds with strict inequality and the rate  $\frac{1}{2} \log_2 (1 + \lambda (h_{11}^2 P'_1 + h_{12}^2 P'_2))$  is always achievable by a power-splitting scheme in which  $\beta_i = \lambda \frac{P'_i}{P_i}$ , with  $i \in \{1, 2\}$ , for any optimal tuple  $(\lambda, P'_1, P''_1, P'_2, P''_2)$  in (39). This shows that the maximum-sum-rate achieved via time-sharing with power control is always bounded away from the sum-capacity (Proposition 2), except when  $\lambda = 1$ . That is, time-sharing with power control is optimal in terms of sum-rate only when the energy constraint is not active, i.e.,  $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ .

## 4 Main Results: Information-Energy Capacity Region with Feedback

The information-energy capacity region of the G-MAC-F is fully characterized by the following theorem.

**Theorem 2** (Perfect Feedback Information-Energy Capacity Region). *The perfect feedback information-energy capacity region  $\mathcal{E}_{\text{FB}}$  ( $\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}$ ) of the G-MAC-F is the*

set of non-negative information-energy rate triplets  $(R_1, R_2, B)$  that satisfy

$$R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} (1 - \rho^2)) \quad (42a)$$

$$R_2 \leq \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12} (1 - \rho^2)) \quad (42b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho\sqrt{\beta_1 \text{SNR}_{11} \beta_2 \text{SNR}_{12}}) \quad (42c)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho\sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21} (1 - \beta_2)\text{SNR}_{22}}, \quad (42d)$$

with  $(\rho, \beta_1, \beta_2) \in [0, 1]^3$ .

**Remark 2.** The information-energy capacity region without feedback described by Theorem 1 is identical to the information-energy capacity region described by Theorem 2 in the case in which channel inputs are chosen to be mutually independent, i.e.,  $\rho = 0$ . This suggests that the information-energy capacity region without feedback is strictly included in the information-energy capacity region with feedback, and thus, for any non-zero  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ , it holds that

$$\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}) \subset \mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}). \quad (43)$$

The proof of Theorem 2 is presented in Section 6. The remainder of this subsection highlights some important observations about Theorem 2.

#### 4.0.1 Comments on the Achievability

The achievability scheme in the proof of Theorem 2 is based on power-splitting and Ozarow's capacity achieving scheme [24]. From an achievability standpoint, the parameters  $\beta_1$  and  $\beta_2$  in Theorem 2 might be interpreted as the fractions of average power that transmitters 1 and 2 allocate for information transmission. More specifically, transmitter  $i$  generates two signals: an information-carrying (IC) signal with average power  $\beta_i P_i$  energy-units per channel use; and a no-information-carrying (NIC) signal with power  $(1 - \beta_i) P_i$  energy-units per channel use. The role of the NIC signal is exclusively energy transmission from the transmitter to the energy harvester. Conversely, the role of the IC signal is twofold: information transmission from the transmitter to the receiver and energy transmission from the transmitter to the energy harvester.

The parameter  $\rho$  is the average Pearson correlation coefficient between the IC signals sent by both transmitters. This parameter plays a fundamental role in both information transmission and energy transmission. Note for instance that the upper-bound on the energy harvested per unit time (42d) monotonically increases with  $\rho$ , whereas the upper-bounds on the individual rates (42a) and (42b) monotonically decrease with  $\rho$ . Note also that the Pearson correlation factor between the NIC signals of both transmitters does not appear in Theorem 2. This is mainly because maximum energy transmission occurs using NIC signals that are fully correlated and thus, the corresponding Pearson correlation coefficient is one. Similarly, the Pearson correlation factor between the NIC signal of transmitter  $i$  and the IC signal of transmitter  $j$ , with  $j \in \{1, 2\}$  and  $j \neq i$ , does not appear in Theorem 2 either. This observation stems from the fact that, without loss of optimality, NIC signals can be chosen to be independent of the messages  $M_1$  and  $M_2$  as well as the noise sequences, and known by both the receiver and the transmitters. Hence, NIC signals can be independent of the IC signals and more importantly, the interference they create at the receiver can be easily eliminated. Under this assumption, a power-splitting  $(\beta_1, \beta_2) \in [0, 1]^2$  guarantees the achievability of non-negative rate pairs  $(R_1, R_2)$  satisfying (42a)-(42c) by simply using Ozarow's capacity achieving scheme. At the EH, both the IC and NIC

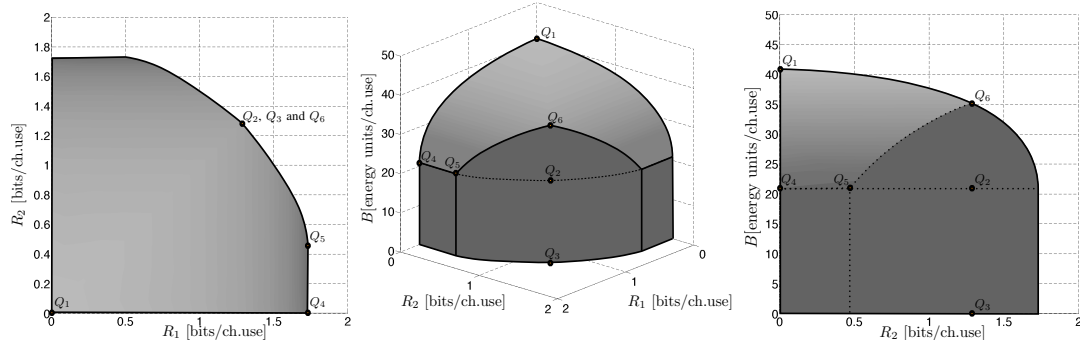


Figure 4: 3-D representation of the perfect feedback information-energy capacity region of the G-MAC  $\mathcal{E}_{\text{FB}}(10, 10, 10, 10)$  in the coordinate system  $(R_1, R_2, B)$ . Left and right figures represent a bi-dimensional view in the  $R_1$ - $R_2$  and  $B$ - $R_2$  planes of  $\mathcal{E}_{\text{FB}}(10, 10, 10, 10)$ , respectively. The figure in the center is a 3-D representation of  $\mathcal{E}_{\text{FB}}(10, 10, 10, 10)$ . Note that  $Q_1 = (0, 0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}})$ ; the points  $Q_2, Q_3$  and  $Q_6$  are collinear and satisfy that  $R_1 + R_2 = \frac{1}{2} \log_2(1 + \text{SNR}_{11} + \text{SNR}_{12} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{11}\text{SNR}_{12}})$ . The points  $Q_2, Q_4$  and  $Q_5$  are coplanar and they satisfy  $B = 1 + \text{SNR}_{21} + \text{SNR}_{22}$ . In particular,  $Q_4 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), 0, 1 + \text{SNR}_{21} + \text{SNR}_{22})$  and  $Q_5 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), \frac{1}{2} \log_2(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}}), 1 + \text{SNR}_{21} + \text{SNR}_{22})$ .

signals contribute to the total harvested energy (11). The IC signal is able to convey at most  $\beta_1 \text{SNR}_{21} + \beta_2 \text{SNR}_{22} + 2\rho\sqrt{\beta_1 \text{SNR}_{21}\beta_2 \text{SNR}_{22}}$  energy-units per channel use, while the NIC signal is able to convey at most  $(1 - \beta_1)\text{SNR}_{21} + (1 - \beta_2)\text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}$  energy-units per channel use. The sum of these two contributions as well as the contribution of the noise at the EH justifies the upper-bound on the energy transmission rate in (42d).

If  $\beta_1 \neq 0$  and  $\beta_2 \neq 0$ , let  $\rho^*(\beta_1, \beta_2)$  be the unique solution in  $(0, 1)$  to the following equality:

$$1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho\sqrt{\beta_1 \text{SNR}_{11}\beta_2 \text{SNR}_{12}} = (1 + \beta_1 \text{SNR}_{11}(1 - \rho^2))(1 + \beta_2 \text{SNR}_{12}(1 - \rho^2)), \quad (44)$$

otherwise, let  $\rho^*(\beta_1, \beta_2) = 0$ .

Note that for any power-splitting  $(\beta_1, \beta_2) \in (0, 1]^2$ , the left hand side of (44) is monotonically increasing with  $\rho$  whereas the right hand side is monotonically decreasing with  $\rho$ . This implies that  $\rho^*(\beta_1, \beta_2)$  is a maximizer of the sum-rate. More specifically, at  $\rho = \rho^*(\beta_1, \beta_2)$ , the sum of (42a) and (42b) is equal to (42c) and it corresponds to the sum-capacity of the G-MAC-F.

#### 4.0.2 Example

Consider the information-energy capacity region  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , with  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$  presented in Fig. 4. Therein, left and right figures represent a bi-dimensional view in the  $R_1$ - $R_2$  and  $B$ - $R_2$  planes of  $\mathcal{E}_{\text{FB}}(10, 10, 10, 10)$ , respectively. The figure in the center is a 3-D representation of  $\mathcal{E}_{\text{FB}}(10, 10, 10, 10)$ . The triplet with the highest energy transmission rate is denoted by  $Q_1 = (0, 0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}})$ . The triplets  $Q_2, Q_3$  and  $Q_6$  guarantee information transmission at the sum-capacity, i.e.,  $R_1 + R_2 = \frac{1}{2} \log_2(1 + \text{SNR}_{11} + \text{SNR}_{12} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{11}\text{SNR}_{12}})$ . The triplets  $Q_2, Q_4$  and  $Q_5$  are coplanar and they satisfy that  $B = 1 + \text{SNR}_{21} + \text{SNR}_{22}$ . In particular,  $Q_4 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), 0, 1 + \text{SNR}_{21} + \text{SNR}_{22})$ .

$\text{SNR}_{21} + \text{SNR}_{22}$ ) and  $Q_5 = (\frac{1}{2} \log_2(1 + \text{SNR}_{11}), \frac{1}{2} \log_2(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}}), 1 + \text{SNR}_{21} + \text{SNR}_{22})$  are achievable with and without feedback.

#### 4.1 Information-Energy Transmission with Feedback and with Minimum Energy Rate Constraint $b$

Let  $0 \leq b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  denote the minimum energy rate that must be guaranteed at the input of the EH in the G-MAC-F with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ .

In the following, for a fixed minimum energy rate constraint  $b$ , the maximum individual information rates as well as the information sum-capacity are identified. Hence, the set of all information-energy rate triplets that are achievable given an energy rate constraint of  $b$  energy-units per channel use at the input of the EH is fully characterized.

##### 4.1.1 Individual Information Rates with Feedback and Minimum Energy Rate Constraint $b$

The maximum individual information rate  $R_i^{\text{FB}}(b)$ , with  $i \in \{1, 2\}$ , of the G-MAC-F, with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ , given an energy rate constraint of  $b$  energy-units per channel use at the input of the EH is the solution to an optimization problem of the form

$$R_i^{\text{FB}}(b) = \max_{(r_i, r_j, c) \in \mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}) : c \geq b} r_i. \quad (45)$$

The solution to the optimization problem (45) is given by the following proposition.

**Proposition 3** (Maximum Individual Information Rates with Feedback and Minimum Energy Rate Constraint  $b$ ). *Consider a G-MAC-F with parameters  $\text{SNR}_{11}$ ,  $\text{SNR}_{12}$ ,  $\text{SNR}_{21}$ , and  $\text{SNR}_{22}$ . For a given required minimum energy rate  $b$ , the maximum individual information rates with feedback coincide with the maximum individual information rates without feedback in Proposition 1. That is,*

$$R_i^{\text{FB}}(b) = R_i^{\text{NF}}(b). \quad (46)$$

*Proof:* From the assumptions of Proposition 3 it follows that for a given energy transmission rate of  $b$  energy-units per channel use, a power-split  $(\beta_i, \beta_j)$  is feasible if there exists at least one  $\rho \in [0, 1]$  that satisfies

$$g^{\text{FB}}(\beta_i, \beta_j, \rho) \geq b, \quad (47)$$

with

$$g^{\text{FB}}(\beta_i, \beta_j, \rho) \triangleq 1 + \text{SNR}_{2i} + \text{SNR}_{2j} + 2\sqrt{(1 - \beta_i)\text{SNR}_{2i}(1 - \beta_j)\text{SNR}_{2j}} + 2\rho\sqrt{\beta_i\text{SNR}_{2i}\beta_j\text{SNR}_{2j}}. \quad (48)$$

Using a Fourier-Motzkin elimination in the constraints (42a)-(42c) to eliminate  $R_j$ , it can be shown that transmitter  $i$ 's individual rate maximization problem (45) is equivalent to

$$R_i^{\text{FB}}(b) = \max_{(\beta_i, \beta_j, \rho) \in [0, 1]^3} f_i^{\text{FB}}(\beta_i, \beta_j, \rho), \quad (49a)$$

$$\text{subject to: } g^{\text{FB}}(\beta_i, \beta_j, \rho) \geq b, \quad (49b)$$

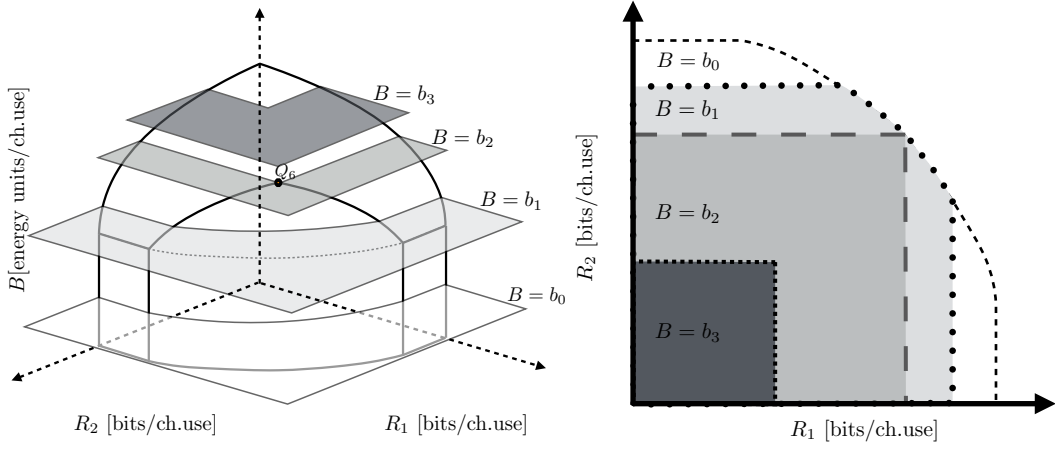


Figure 5: Intersection of the planes  $B = b_1$ ,  $B = b_2$  and  $B = b_3$  with the information-energy capacity region  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , with  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22}$ ,  $b_0 \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ ,  $b_1 \in [1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ ,  $b_2 = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  and  $b_3 \in [1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ .

with

$$f_i^{\text{FB}}(\beta_i, \beta_j, \rho) \triangleq \min\left\{\frac{1}{2}\log_2(1 + \beta_i \text{SNR}_{1i}(1 - \rho^2)), \frac{1}{2}\log_2(1 + \beta_i \text{SNR}_{1i} + \beta_j \text{SNR}_{1j} + 2\rho\sqrt{\beta_i \text{SNR}_{1i}\beta_j \text{SNR}_{1j}})\right\}. \quad (50)$$

For a given triplet  $(\beta_i, \beta_j, \rho)$ , there are two cases: either it satisfies

$$-\rho^2\beta_i \text{SNR}_{1i} > \beta_j \text{SNR}_{1j} + 2\rho\sqrt{\beta_i \text{SNR}_{1i}\beta_j \text{SNR}_{1j}}, \quad (51)$$

which implies that

$$f_i^{\text{FB}}(\beta_i, \beta_j, \rho) = \frac{1}{2}\log_2(1 + \beta_i \text{SNR}_{1i} + \beta_j \text{SNR}_{1j} + 2\rho\sqrt{\beta_i \text{SNR}_{1i}\beta_j \text{SNR}_{1j}}); \quad (52)$$

or it satisfies

$$-\rho^2\beta_i \text{SNR}_{1i} \leq \beta_j \text{SNR}_{1j} + 2\rho\sqrt{\beta_i \text{SNR}_{1i}\beta_j \text{SNR}_{1j}}, \quad (53)$$

and in this case

$$f_i^{\text{FB}}(\beta_i, \beta_j, \rho) = \frac{1}{2}\log_2(1 + \beta_i \text{SNR}_{1i}(1 - \rho^2)). \quad (54)$$

In the first case, condition (51) does not hold true for all triplets  $(\beta_i, \beta_j, \rho) \in [0, 1]^3$  and thus, this case should be excluded.

In the second case, the function  $f_i^{\text{FB}}(\beta_i, \beta_j, \rho)$  is decreasing in  $\rho$ . Thus, it holds that

$$f_i^{\text{FB}}(\beta_i, \beta_j, \rho) \leq f_i^{\text{FB}}(\beta_i, \beta_j, 0), \quad (55)$$

and the triplet  $(\beta_i, \beta_j, 0)$  is feasible if and only if  $g^{\text{FB}}(\beta_i, \beta_j, 0) \geq b$ . Note that  $f_i^{\text{FB}}(\beta_i, \beta_j, 0)$  in (54) is monotonically increasing in  $\beta_i$  and does not depend on  $\beta_j$ . Alternatively,  $g^{\text{FB}}(\beta_i, \beta_j, 0)$

is monotonically decreasing in  $(\beta_i, \beta_j)$ . These observations lead to the choice  $\beta_j = 0$ , given that it maximizes the energy transmission rate and lets  $\beta_i$  be chosen to maximize the individual information rate of transmitter  $i$ . Following this intuition, the optimization problem in (49) reduces to the optimization problem in (26) in the proof of Proposition 1. This shows that the individual rates with feedback are limited by  $R_i \leq \frac{1}{2} \log_2(1 + \beta^*(b) \text{SNR}_{1i})$  where  $\beta^*(b)$  is given by (20). ■

The relevance of Proposition 3 stems from the insights it provides to the understanding of the shape of the volume  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  in comparison to the shape of the volume  $\mathcal{E}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ . Even if feedback does not increase the maximal individual rates that can be achieved for a given received energy rate  $b$ , it will be shown in the sequel that it increases the sum-rate that can be achieved (See Proposition 4).

Note that if  $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ , then  $\beta^*(b) = 1$  and thus, the energy constraint does not add any additional bound on the individual rates other than (42a) and (42b). In general, any intersection of the volume  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ , in the Cartesian coordinates  $(R_1, R_2, B)$ , with a plane  $B = b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$  corresponds to the set of triplets  $(R_1, R_2, b)$ , in which the corresponding pairs  $(R_1, R_2)$  form a set that is identical to the capacity region of the G-MAC with feedback and without EH, denoted by  $\mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$ . Fig. 5 shows a general example of this intersection when  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22}$ . In this case, transmitting information using all the available power budget is always enough to satisfy the energy constraint and thus, all information-transmission rate pairs of the capacity region  $\mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$  are achievable. In the particular case in which  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$  (Fig. 4), the sum-capacity points  $Q_2$  and  $Q_3$ , as well as the corner points  $Q_4$  and  $Q_5$  are achievable by using Ozarow's scheme without any power-splitting, i.e.,  $\beta_1 = \beta_2 = 1$ .

In the case in which  $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ , it follows that  $1 - (\rho^*(1, 1))^2 \leq \beta^*(b) < 1$  and thus, the energy constraint limits the individual rates. That is, the individual rate is bounded away from  $\frac{1}{2} \log_2(1 + \text{SNR}_{1i})$ . The effect of these bounds can be seen in Fig. 4. Let  $\mathcal{B}(b) \subset \mathbb{R}_+^2$  be a box of the form

$$\mathcal{B}(b) = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : R_i \leq \frac{1}{2} \log_2(1 + \beta^*(b) \text{SNR}_{1i}), i \in \{1, 2\} \right\}. \quad (56)$$

In general, any intersection of the volume  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  with a plane  $B = b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$  is a set of triplets  $(R_1, R_2, B)$  for which the corresponding pairs  $(R_1, R_2)$  satisfy

$$(R_1, R_2) \in \mathcal{B}(b) \cap \mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}), \quad (57)$$

which form a proper subset of  $\mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$ . It is important to highlight that in this case, the sum-capacity of  $\mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$  can always be achievable. That is, the power-split  $\beta_1 = \beta_2 = 1$  is always feasible. Consider for instance the triplet  $Q_6$  in Fig. 4. The triplet  $Q_6$  is achievable by using Ozarow's perfect feedback capacity-achieving scheme without any power-splitting, since only information transmission satisfies the energy constraint. Fig. 5 shows the intersection of  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  and a plane  $B = b$ , with  $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ . Note that this intersection always includes the triplet  $(R_1, R_2, B)$ , with  $R_1 + R_2 = \frac{1}{2} \log_2(1 + \text{SNR}_{11} + \text{SNR}_{12} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{11}\text{SNR}_{12}})$ , i.e., the information sum-capacity.

Finally, in the case in which  $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ , it follows that  $0 \leq \beta^*(b) < 1 - (\rho^*(1, 1))^2$ , and thus, the individual rates are limited by  $R_i < \frac{1}{2} \log_2\left(1 + \left(1 - (\rho^*(1, 1))^2\right) \text{SNR}_i\right)$ . This immediately implies that any intersection of the volume  $\mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$  with a plane  $B = b \in (1 +$

$\text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  is a set of triplets  $(R_1, R_2, B)$  for which the corresponding pairs  $(R_1, R_2)$  satisfy

$$(R_1, R_2) \in \mathcal{B}(b) = \mathcal{B}(b) \cap \mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}). \quad (58)$$

In this case, the set  $\mathcal{B}(b)$  is a proper subset of  $\mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$  and it does not contain the sum-rate pair  $(R_1, R_2) \in \mathcal{C}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12})$ . This is clearly shown by Fig. 5. Indeed, for any  $b > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , the set  $\mathcal{B}(b)$  monotonically shrinks with  $b$ .

#### 4.1.2 Information Sum-Capacity with Feedback and with Minimum Energy Constraint $b$

The perfect feedback information sum-capacity  $R_{\text{sum}}^{\text{FB}}(b)$  of the G-MAC-F, with parameters  $\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}$ , and  $\text{SNR}_{22}$ , given an energy rate constraint of  $b$  energy-units per channel use at the input of the EH is the solution to

$$R_{\text{sum}}^{\text{FB}}(b) = \max_{(r_1, r_2, c) \in \mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22}): c \geq b} r_1 + r_2. \quad (59)$$

The solution to this problem is given by the following theorem.

**Proposition 4** (Perfect Feedback Information Sum-Capacity with Minimum Energy Rate Constraint  $b$ ). *Let  $b$  be the minimum energy rate that must be guaranteed at the input of the EH. Then, the perfect feedback information sum-capacity of the G-MAC-F, with parameters  $\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}$ , and  $\text{SNR}_{22}$  is*

$$1. \quad \forall b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}],$$

$$R_{\text{sum}}^{\text{FB}}(b) = \frac{1}{2} \log_2(1 + \text{SNR}_{11} + \text{SNR}_{12} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{11}\text{SNR}_{12}}); \quad (60)$$

$$2. \quad \forall b \in (1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}),$$

$$R_{\text{sum}}^{\text{FB}}(b) = \frac{1}{2} \log_2(1 + \beta^*(b)\text{SNR}_{11}) + \frac{1}{2} \log_2(1 + \beta^*(b)\text{SNR}_{12}); \quad (61)$$

$$3. \quad \forall b \in [1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, \infty],$$

$$R_{\text{sum}}^{\text{FB}}(b) = 0, \quad (62)$$

with the function  $\beta^*(b)$  defined in (20).

The proof of Proposition 4 is provided in Section 7. One of the key observations from Proposition 4 is that if the minimum energy rate  $b$  required at the input of the EH is less than what is needed for transmitting information using all the available power budget at the maximum sum-rate, i.e.,  $b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , the minimum energy constraint does not have any impact on the sum-rate (see Fig. 6). That is, in this case the requirement of a minimum energy transmission rate is automatically met by exclusively transmitting information at the sum-capacity.

Nonetheless, when the energy rate required at the input of the receiver is  $b \geq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ , then there exists a loss of sum-rate induced by the fact that at least one of the fractions  $\beta_1$  and  $\beta_2$  is smaller than one. More specifically, for these values of  $b$ ,  $R_i \leq \frac{1}{2} \log_2(1 + (1 - (\rho(1, 1)^*)^2)\text{SNR}_{1i})$  for at least one  $i \in \{1, 2\}$  and thus, the sum-rate is strictly smaller than the sum capacity.

Clearly, the maximum energy rate is achieved when  $\beta_1 = \beta_2 = 0$ , which implies that no information is conveyed from the transmitters to the receiver (see Fig. 6).



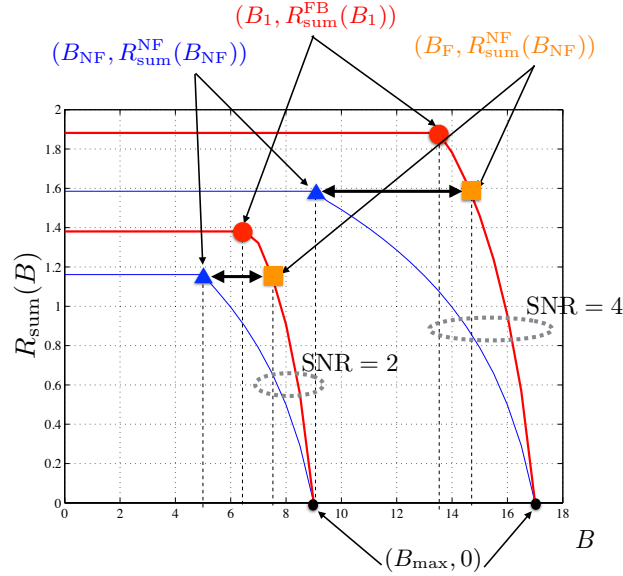


Figure 6: Information sum-capacity of the symmetric two-user memoryless G-MAC with co-located receiver and EH, with  $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = \text{SNR}$ , with feedback (thick red line) and without feedback (thin blue line) as a function of  $B$ . Red (big) circles represent the pairs  $(B_1, R_{\text{sum}}^{\text{FB}}(B_1))$  in which  $R_{\text{sum}}^{\text{FB}}(B_1)$  is the information sum-capacity with feedback when only information transmission is performed and  $B_1 \triangleq 1 + 2(1 + \rho^*(1, 1))\text{SNR}$  represents the corresponding maximum energy rate that can be guaranteed at the EH. Blue triangles represent the pairs  $(B_{\text{NF}}, R_{\text{sum}}^{\text{NF}}(B_{\text{NF}}))$  in which  $R_{\text{sum}}^{\text{NF}}(B_{\text{NF}})$  is the information sum-capacity without feedback and  $B_{\text{NF}} \triangleq 1 + 2\text{SNR}$  is the corresponding maximum energy rate that can be guaranteed at the EH without feedback. Orange squares represent the pairs  $(B_{\text{F}}, R_{\text{sum}}^{\text{NF}}(B_{\text{F}}))$  in which  $B_{\text{F}}$  is the corresponding maximum energy rate that can be guaranteed at the EH with feedback. Black (small) circles represent the pairs  $(B_{\text{max}}, 0)$  in which  $B_{\text{max}} \triangleq 1 + 4\text{SNR}$  is the maximum energy rate that can be guaranteed at the EH.

## 5 Main Results: Energy Transmission Enhancement with Feedback

In this section, the enhancement on the energy transmission rate due to the use of feedback is quantized when the information sum-rate is the information sum-capacity without feedback (see the blue triangles and orange squares in Fig. 6).

Denote by  $B_{\text{NF}} = 1 + \text{SNR}_{21} + \text{SNR}_{22}$  the maximum energy rate that can be guaranteed at the EH in the G-MAC without feedback when the information sum-rate is the corresponding information sum-capacity without feedback. Denote also by  $B_{\text{F}}$  the maximum energy rate that can be guaranteed at the EH in the G-MAC with feedback when the information sum-rate is the information sum-capacity without feedback. The exact value of  $B_{\text{F}}$  is given by the following lemma.

**Lemma 1.** *The maximum energy rate  $B_{\text{F}}$  that can be guaranteed at the EH in the G-MAC with feedback when the information sum-rate is the information sum-capacity without feedback is*

$$B_{\text{F}} = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \gamma)\text{SNR}_{21}\text{SNR}_{22}}, \quad (63)$$

with  $\gamma \in (0, 1)$  defined as follows:

$$\gamma = -\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}} + \sqrt{\left(\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}}\right)^2 + 2\left(\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}}\right)}. \quad (64)$$

*Proof:* The maximum energy transmission rate  $B_F$  when the information sum-rate is the information sum-capacity of the G-MAC without feedback satisfies

$$B_F = \max_{(\beta_1, \beta_2, \rho) \in [0, 1]^3} 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\left(\rho\sqrt{\beta_1\beta_2} + \sqrt{(1-\beta_1)(1-\beta_2)}\right)\sqrt{\text{SNR}_{21}\text{SNR}_{22}} \quad (65a)$$

subject to :

$$1 + \text{SNR}_{11} + \text{SNR}_{12} = \min\left\{1 + \beta_1\text{SNR}_{11} + \beta_2\text{SNR}_{12} + 2\rho\sqrt{\beta_1\text{SNR}_{11}\beta_2\text{SNR}_{12}}, \right. \\ \left. (1 + \beta_1\text{SNR}_{11}(1 - \rho^2))(1 + \beta_2\text{SNR}_{12}(1 - \rho^2))\right\}. \quad (65b)$$

The solution  $B_F$  to the optimization problem in (65) satisfies that  $B_F \in (1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ . Thus, from Theorem 3, it follows that the information sum-rate is exclusively upper bounded by  $R_1 + R_2 \leq \frac{1}{2}\log_2(1 + \beta^*(B_F)\text{SNR}_{11}) + \frac{1}{2}\log_2(1 + \beta^*(B_F)\text{SNR}_{12})$ , with the function  $\beta^* : \mathbb{R}_+ \rightarrow [0, 1]$  defined in (20). Note that  $\beta^*$  is monotonically decreasing with its argument. Thus, the optimization problem in (65) can be rewritten as follows:

$$B_F = \max_{b \in (b_1, b_2]} b \quad (66a)$$

subject to :

$$\frac{1}{2}\log_2(1 + \text{SNR}_{11} + \text{SNR}_{12}) = \frac{1}{2}\log_2(1 + \beta^*(b)\text{SNR}_{11}) + \frac{1}{2}\log_2(1 + \beta^*(b)\text{SNR}_{12}). \quad (66b)$$

where  $b_1 = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho^*(1, 1)\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$  and  $b_2 = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ . The constraint of the problem (66b) induces a unique value for  $\beta^*(b)$  within  $[0, 1]$  for each  $b$ , and thus, the optimization is vacuous. This implies that the unique solution  $B_F$  satisfies

$$\beta^*(B_F) = -\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}} + \sqrt{\left(\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}}\right)^2 + 2\left(\frac{\text{SNR}_{11} + \text{SNR}_{12}}{2\text{SNR}_{11}\text{SNR}_{12}}\right)}. \quad (67)$$

Following the definition of  $\beta^*$  in (20) and solving for  $B_F$  in (67) yields (63). This completes the proof of Lemma 1.  $\blacksquare$

The following theorem provides an upper bound on the ratio  $\frac{B_{NF}}{B_F}$ .

**Theorem 3** (Maximum Energy Rate Improvement with Feedback). *Feedback can at most double the energy transmission rate. That is,*

$$1 < \frac{B_F}{B_{NF}} < 2. \quad (68)$$

*Proof:* The proof of Theorem 3 follows immediately from Lemma 1. The lower-bound follows from the fact that  $0 < \gamma < 1$  with strict inequality (Lemma 1). Moreover, note that the

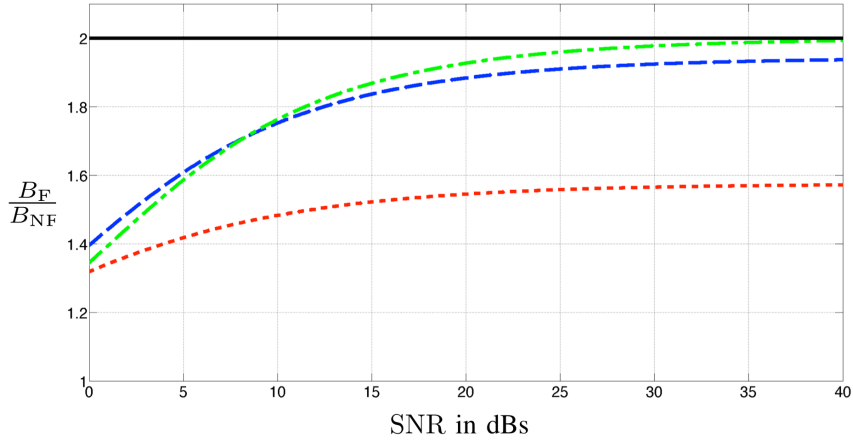


Figure 7: The ratio  $\frac{B_F}{B_{NF}}$  as a function of SNR when the receiver and the EH are co-located, i.e.,  $\text{SNR}_{11} = \text{SNR}_{21} = \text{SNR}_1$  and  $\text{SNR}_{12} = \text{SNR}_{22} = \text{SNR}_2$ . The thickest line is the upper bound in (72); the dash-dotted line, the dashed line and the dotted line are the exact values of the ratio  $\frac{B_F}{B_{NF}}$  in (69) when  $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$ ;  $\frac{\text{SNR}_1}{2} = \text{SNR}_2 = \text{SNR}$ ; and  $\frac{\text{SNR}_1}{10} = \text{SNR}_2 = \text{SNR}$ , respectively.

ratio  $\frac{B_F}{B_{NF}}$  satisfies

$$\frac{B_F}{B_{NF}} = \frac{1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1-\gamma)\text{SNR}_{21}\text{SNR}_{22}}}{1 + \text{SNR}_{21} + \text{SNR}_{22}} \quad (69)$$

$$\stackrel{(a)}{<} 1 + \frac{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}}{1 + \text{SNR}_{21} + \text{SNR}_{22}} \quad (70)$$

$$\stackrel{(b)}{\leq} 1 + \frac{\text{SNR}_{21} + \text{SNR}_{22}}{1 + \text{SNR}_{21} + \text{SNR}_{22}} \quad (71)$$

$$< 2, \quad (72)$$

where (a) follows from the fact that  $\gamma \in (0, 1)$  (Lemma 1); and (b) follows from the inequality  $(\sqrt{\text{SNR}_{21}} - \sqrt{\text{SNR}_{22}})^2 \geq 0$ . This completes the proof of Theorem 3. ■

Fig. 7 compares the exact value of the ratio  $\frac{B_F}{B_{NF}}$  in (69) to the upper-bound in (72) as a function of the SNRs in the special case in which the receiver and the EH are co-located. This implies that the channel coefficients between the transmitters and the receiver can be considered identical to those between the transmitter and the EH, i.e.,  $\text{SNR}_{11} = \text{SNR}_{21} = \text{SNR}_1$  and  $\text{SNR}_{12} = \text{SNR}_{22} = \text{SNR}_2$ . Note that in the symmetric case, i.e.,  $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$ , the upper-bound in (72) is tight as the ratio  $\frac{B_F}{B_{NF}}$  becomes arbitrarily close to two as SNR tends to infinity. In the non-symmetric cases  $\text{SNR}_1 \neq \text{SNR}_2$ , this bound is loose.

## 6 Proof of Theorem 2

The proof is divided into two parts: achievability and converse parts.

## 6.1 Proof of Achievability

The proof of achievability uses a very simple power-splitting technique in which a fraction  $\beta_i \in [0, 1]$  of the power is used for information transmission and the remaining fraction  $(1 - \beta_i)$  for energy transmission. The information transmission is made following Ozarow's perfect feedback capacity-achieving scheme in [24]. The energy transmission is accomplished by random symbols that are known at both transmitters and the receiver. Despite its simplicity and a great deal of similarity with the scheme in [24], the complete proof is fully described hereunder for the sake of completeness.

**Codebook generation:** At the beginning of transmission, each message  $M_i$  is mapped into the real-valued message point

$$\Theta_i(M_i) \triangleq -(M_i - 1)\Delta_i + \sqrt{P_i}, \quad (73)$$

where

$$\Delta_i \triangleq \frac{2\sqrt{P_i}}{\lfloor 2^{nR_i} \rfloor}. \quad (74)$$

**Encoding:** The first three channel uses are part of an initialization procedure during which there is no energy transmission and the channel inputs are

$$t = -2 : X_{1,-2} = 0 \quad \text{and} \quad X_{2,-2} = \Theta_2(M_2), \quad (75a)$$

$$t = -1 : X_{1,-1} = \Theta_1(M_1) \quad \text{and} \quad X_{2,-1} = 0, \quad (75b)$$

$$t = 0 : X_{1,0} = 0 \quad \text{and} \quad X_{2,0} = 0. \quad (75c)$$

Through the feedback links, transmitter 1 observes  $(Z_{-1}, Z_0)$  and transmitter 2 observes  $(Z_{-2}, Z_0)$ . After the initialization phase, each transmitter  $i \in \{1, 2\}$  can thus compute

$$\Xi_i \triangleq \sqrt{1 - \rho^*(\beta_1, \beta_2)} \cdot Z_{-i} + \sqrt{\rho^*(\beta_1, \beta_2)} \cdot Z_0, \quad (76)$$

where  $\rho^*(\beta_1, \beta_2)$  is the unique solution in  $(0, 1)$  to (44).

During the remaining channel uses  $1, \dots, n$ , for  $i \in \{1, 2\}$ , instead of repeating the message-point  $\Theta_i(M_i)$ , transmitter  $i$  simultaneously describes  $\Xi_i$  to the receiver and transmits energy to the energy harvester. Let  $\beta_i$ , with  $i \in \{1, 2\}$  be the power-splitting coefficient of transmitter  $i$ . More specifically, at each time  $t \in \{1, \dots, n\}$ , transmitter  $i$  sends

$$X_{i,t} = U_{i,t} + \sqrt{(1 - \beta_i)P_i}W_t, \quad i \in \{1, 2\}. \quad (77)$$

Here  $W_t$  is a Gaussian zero-mean unit-variance random variable that is known non-causally to the transmitters and to the receiver and is independent of the messages and the noise sequences. The symbol  $U_{i,t}$  is a zero-mean Gaussian random variable with variance  $\beta_i P_i$  and is chosen as follows:

$$U_{i,1} = \sqrt{\beta_i P_i} \Xi_i, \quad (78a)$$

$$U_{i,t} = \gamma_{i,t} \left( \Xi_i - \hat{\Xi}_i^{(t-1)} \right), \quad t \in \{2, \dots, n\}, \quad (78b)$$

where the parameter  $\gamma_{i,t}$  is chosen to satisfy  $\mathbf{E}[U_{i,t}^2] = \beta_i P_i$  and  $\hat{\Xi}_i^{(t-1)}$  is explained below.

For each  $t \in \{1, \dots, n\}$ , upon receiving the channel output  $Y_{1,t}$ , the receiver subtracts the signal induced by the common randomness to form the observation  $Y'_{1,t}$  as follows:

$$Y'_{1,t} \triangleq Y_{1,t} - (\sqrt{(1 - \beta_1)P_1} + \sqrt{(1 - \beta_2)P_2})W_t. \quad (79)$$

The receiver then calculates the minimum mean square error (MMSE) estimate  $\hat{\Xi}_i^{(t-1)} = \mathbb{E}[\Xi_i | Y'_{1,1}, \dots, Y'_{1,t-1}]$  of  $\Xi_i$  given the prior observations  $Y'_{1,1}, \dots, Y'_{1,t-1}$ .

By the orthogonality principle of MMSE estimation,  $(U_{1,t}, U_{2,t}, Z_t)$  are independent of the observations  $Y'_{1,1}, \dots, Y'_{1,t-1}$  and thus of  $Y_{1,1}, \dots, Y_{1,t-1}$ . Let  $\rho_t$  denote the correlation coefficient between  $U_{1,t}$  and  $U_{2,t}$ , i.e.,  $\rho_t \triangleq \frac{\mathbb{E}[U_{1,t}U_{2,t}]}{\sqrt{\mathbb{E}[U_{1,t}^2]}\sqrt{\mathbb{E}[U_{2,t}^2]}}$ . It can be shown [25] that for all  $t \in \{1, \dots, n\}$ ,  $\rho_t = \rho^*(\beta_1, \beta_2)$ .

After reception of the output symbols  $Y_{1,-2}, \dots, Y_{1,n}$ , the receiver forms  $\hat{\Xi}_i^{(n)} \triangleq \mathbb{E}[\Xi_i | Y'_{1,1}, \dots, Y'_{1,n}]$ , for  $i \in \{1, 2\}$ . Then, it forms an estimate of the message point  $\Theta_i(M_i)$  as follows:

$$\begin{aligned} \hat{\Theta}_i &\triangleq \frac{1}{h_i} \left( Y_{1,-i} + \sqrt{\frac{\rho^*(\beta_1, \beta_2)}{1 - \rho^*(\beta_1, \beta_2)}} Y_{1,0} - \frac{1}{\sqrt{1 - \rho^*(\beta_1, \beta_2)}} \hat{\Xi}_i^{(n)} \right) \\ &= \Theta_i(M_i) + \frac{1}{h_i \sqrt{1 - \rho^*(\beta_1, \beta_2)}} \left( \Xi_i - \hat{\Xi}_i^{(n)} \right). \end{aligned} \quad (80)$$

Finally, the message index estimate  $M_i$  is obtained using nearest-neighbor decoding based on the value  $\hat{\Theta}_i$ , as follows:

$$\hat{M}_i = \underset{m_i \in \{1, \dots, \lfloor 2^{nR_i} \rfloor\}}{\operatorname{argmin}} |\Theta_i(m_i) - \hat{\Theta}_i|. \quad (81)$$

#### Analysis of the probability of error:

An error occurs whenever the receiver is not able to recover one of the messages, i.e.,  $(M_1, M_2) \neq (\hat{M}_1, \hat{M}_2)$  or if the received energy rate is below the desired minimum rate  $B^{(n)} < B$ .

First, consider the probability of a decoding error. Note that for  $i \in \{1, 2\}$ ,  $\hat{M}_i = M_i$ , if

$$|\Xi_i - \hat{\Xi}_i^{(n)}| \leq \frac{h_{1i} \sqrt{1 - \rho^*(\beta_1, \beta_2)} \Delta_i}{2}. \quad (82)$$

Since the difference  $\Xi_i - \hat{\Xi}_i^{(n)}$  is a centered Gaussian random variable, by the definition of  $\Delta_i$  in (74), the error probability  $P_{e,i}$  while decoding message index  $M_i$  can be bounded as

$$P_{e,i} \leq 2\mathcal{Q} \left( \frac{\sqrt{\text{SNR}_{1i}} \sqrt{1 - \rho^*(\beta_1, \beta_2)}}{\lfloor 2^{nR_i} \rfloor \sqrt{(\sigma_i^{(n)})^2}} \right), \quad (83)$$

where

$$(\sigma_i^{(n)})^2 \triangleq \mathbb{E} \left[ |\Xi_i - \hat{\Xi}_i^{(n)}|^2 \right], \quad i \in \{1, 2\}. \quad (84)$$

By the joint Gaussianity of  $\Xi_i$  and  $\mathbf{Y}'_1$ , the MMSE estimate  $\hat{\Xi}_i^{(n)}$  is a linear function of  $\mathbf{Y}'_1$  (see, e.g., [26]). Moreover, by the orthogonality principle, the MMSE  $\Xi_i - \hat{\Xi}_i^{(n)}$  is independent of the observations  $\mathbf{Y}'_1$ . Hence, we have

$$\begin{aligned} I(\Xi_i; \mathbf{Y}'_1) &= h(\Xi_i) - h(\Xi_i | \mathbf{Y}'_1) \\ &\stackrel{(a)}{=} h(\Xi_i) - h(\Xi_i - \hat{\Xi}_i^{(n)} | \mathbf{Y}'_1) \\ &\stackrel{(b)}{=} h(\Xi_i) - h(\Xi_i - \hat{\Xi}_i^{(n)}) \\ &= -\frac{1}{2} \log_2((\sigma_i^{(n)})^2), \end{aligned} \quad (85)$$

where (a) holds because  $\hat{\Xi}_i^{(n)}$  is a function of  $\mathbf{Y}'_1$ , and (b) follows because  $\Xi_i - \hat{\Xi}_i^{(n)}$  is independent of  $\mathbf{Y}'_1$ .

Equation (85) can equivalently be written as

$$\sqrt{(\sigma_i^{(n)})^2} = 2^{-I(\Xi_i; \mathbf{Y}'_1)}. \quad (86)$$

Combining (83) with (86) yields that the probability of error of message  $M_i$  tends to 0 as  $n \rightarrow \infty$ , if the rate  $R_i$  satisfies

$$R_i \leq \liminf_{n \rightarrow \infty} \frac{1}{n} I(\Xi_i; \mathbf{Y}'_1), \quad i \in \{1, 2\}. \quad (87)$$

On the other hand, as proved in [25, Section 17.2.4],

$$I(\Xi_i; \mathbf{Y}'_1) = \sum_{t=1}^n I(U_{i,t}; Y'_{1,t}) \quad (88)$$

and irrespective of  $n$  and  $t \in \{1, \dots, n\}$ , it holds that

$$I(U_{i,t}; Y'_{1,t}) = \frac{1}{2} \log_2 \left( 1 + \beta_i \text{SNR}_{1i} (1 - (\rho^*(\beta_1, \beta_2))^2) \right). \quad (89)$$

Hence, for  $i \in \{1, 2\}$  it holds that

$$\liminf_{n \rightarrow \infty} \frac{1}{n} I(\Xi_i; \mathbf{Y}'_1) = \frac{1}{2} \log_2 \left( 1 + \beta_i \text{SNR}_{1i} (1 - (\rho^*(\beta_1, \beta_2))^2) \right), \quad (90)$$

and the limit inferior is a proper limit.

Combining, (87) and (90) yields that when  $n \rightarrow \infty$ , this scheme can achieve all non-negative rate-pairs  $(R_1, R_2)$  that satisfy

$$R_1 \leq \frac{1}{2} \log_2 \left( 1 + \beta_1 \text{SNR}_{11} (1 - \rho^*(\beta_1, \beta_2))^2 \right), \quad (91a)$$

$$R_2 \leq \frac{1}{2} \log_2 \left( 1 + \beta_2 \text{SNR}_{12} (1 - \rho^*(\beta_1, \beta_2))^2 \right). \quad (91b)$$

and by (44) it automatically yields

$$R_1 + R_2 \leq \frac{1}{2} \log_2 \left( 1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho^*(\beta_1, \beta_2) \sqrt{\beta_1 \text{SNR}_{11} \cdot \beta_2 \text{SNR}_{12}} \right). \quad (91c)$$

Furthermore, the total consumed power at transmitter  $i$  for  $i \in \{1, 2\}$  over the  $n + 3$  channel uses is upper bounded by  $(n + 1)P_i$ , hence, this scheme satisfies the input-power constraints.

**Average received energy rate:**

The average received energy rate is given by  $B^{(n)} \triangleq \frac{1}{n} \sum_{t=1}^n Y_{2,t}^2$ .

By the choice of the random variables, the sequence  $Y_{2,1}, \dots, Y_{2,n}$  is independently and identically distributed (i.i.d.) and each  $Y_{2,t}$  follows a zero-mean Gaussian distribution with variance  $\bar{B}$  given by

$$\begin{aligned} \bar{B} \triangleq \mathbb{E}[Y_{2,t}^2] &= 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}} \rho^*(\beta_1, \beta_2) \\ &\quad + \sqrt{(1 - \beta_1) \text{SNR}_{21} (1 - \beta_2) \text{SNR}_{22}}. \end{aligned} \quad (92)$$

By the weak law of large numbers, it holds that  $\forall \epsilon > 0$

$$\lim_{n \rightarrow \infty} \Pr \left( |B^{(n)} - \bar{B}| > \epsilon \right) = 0. \quad (93)$$

Consequently,

$$\lim_{n \rightarrow \infty} \Pr \left( B^{(n)} > \bar{B} + \epsilon \right) = 0, \quad \text{and} \quad (94a)$$

$$\lim_{n \rightarrow \infty} \Pr \left( B^{(n)} < \bar{B} - \epsilon \right) = 0. \quad (94b)$$

From (94b), it holds that for any energy rate  $B$  which satisfies  $0 < B \leq \bar{B}$ , it holds that

$$\lim_{n \rightarrow \infty} \Pr \left( B^{(n)} < B - \epsilon \right) = 0. \quad (95)$$

To sum up, any information-energy rate triplet  $(R_1, R_2, B)$  that satisfies

$$R_1 \leq \frac{1}{2} \log_2 \left( 1 + \beta_1 \text{SNR}_{11} (1 - (\rho^*(\beta_1, \beta_2))^2) \right) \quad (96a)$$

$$R_2 \leq \frac{1}{2} \log_2 \left( 1 + \beta_2 \text{SNR}_{12} (1 - (\rho^*(\beta_1, \beta_2))^2) \right) \quad (96b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2 \left( 1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho^*(\beta_1, \beta_2) \sqrt{\beta_1 \text{SNR}_{11} \cdot \beta_2 \text{SNR}_{12}} \right) \quad (96c)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2 \left( \sqrt{\beta_1 \beta_2} \rho^*(\beta_1, \beta_2) + \sqrt{(1 - \beta_1)(1 - \beta_2)} \right) \sqrt{\text{SNR}_{21} \text{SNR}_{22}} \quad (96d)$$

is achievable.

To achieve other points in the information-energy capacity region, transmitter 1 can split its message  $M_1$  into two independent submessages  $(M_{1,0}, M_{1,1}) \in \{1, \dots, 2^{nR_{1,0}}\} \times \{1, \dots, 2^{nR_{1,1}}\}$  such that  $R_{1,0}, R_{1,1} \geq 0$  and  $R_{1,0} + R_{1,1} = R_1$ . It uses a power fraction  $\alpha_1 \in [0, 1]$  of its available information-dedicated power  $\beta_1 P_1$  to transmit  $M_{1,0}$  using a non-feedback Gaussian random code and uses the remaining power  $(1 - \alpha_1)\beta_1 P_1$  to send  $M_{1,1}$  using the sum-capacity-achieving feedback scheme while treating  $M_{1,0}$  as noise. Transmitter 2 sends its message  $M_2$  using the sum-capacity-achieving feedback scheme.

Transmitter 1's IC-input is  $U_{1,t} \triangleq U_{1,0,t} + U_{1,1,t}$  where  $U_{1,1,t}$  is defined as in (78) but with reduced power  $(1 - \alpha_1)\beta_1 P_1$ , and  $U_{1,0,t}$  is an independent zero-mean Gaussian random variable with variance  $\alpha_1 \beta_1 P_1$ . Transmitter 2's IC-input is defined as in (78).

The receiver first subtracts the common randomness and then decodes  $(M_{1,1}, M_2)$  treating the signal encoding  $M_{1,0}$  as noise. Successful decoding is possible if

$$R_{1,1} \leq \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \alpha_1)\beta_1 \text{SNR}_{11} (1 - \rho_{\alpha_1}(\beta_1, \beta_2)^2)}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} \right) \quad (97a)$$

$$R_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\beta_2 \text{SNR}_{12} (1 - \rho_{\alpha_1}(\beta_1, \beta_2)^2)}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} \right) \quad (97b)$$

where  $\rho_{\alpha_1}(\beta_1, \beta_2)$  is the unique solution in  $(0, 1)$  to

$$\begin{aligned} & 1 + \frac{(1 - \alpha_1)\beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2x \sqrt{\beta_1 \beta_2 (1 - \alpha_1) \text{SNR}_{11} \text{SNR}_{12}}}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} \\ & = \left( 1 + \frac{(1 - \alpha_1)\beta_1 \text{SNR}_{11}}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} (1 - x^2) \right) \left( 1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} (1 - x^2) \right). \end{aligned} \quad (98)$$

Then, using successive cancellation, the receiver recovers  $M_{1,0}$  successfully if

$$R_{1,0} \leq \frac{1}{2} \log_2 (1 + \alpha_1 \beta_1 \text{SNR}_{11}). \quad (99)$$

By substituting  $R_1 = R_{1,0} + R_{1,1}$ , it can be seen that successful decoding of  $(M_1, M_2)$  is possible with arbitrarily small probability of error if the rates  $(R_1, R_2)$  satisfy

$$R_1 \leq \frac{1}{2} \log_2 \left( 1 + \frac{(1 - \alpha_1) \beta_1 \text{SNR}_{11} (1 - (\rho_{\alpha_1}(\beta_1, \beta_2))^2)}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} \right) + \frac{1}{2} \log_2 (1 + \alpha_1 \beta_1 \text{SNR}_{11}) \quad (100a)$$

$$R_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\beta_2 \text{SNR}_{12} (1 - (\rho_{\alpha_1}(\beta_1, \beta_2))^2)}{1 + \alpha_1 \beta_1 \text{SNR}_{11}} \right). \quad (100b)$$

Now, the average received energy rate of this scheme is analyzed. The sequence  $Y_{2,1}, \dots, Y_{2,n}$  is i.i.d. and each  $Y_{2,t}$  for  $t \in \{1, \dots, n\}$  follows a zero-mean Gaussian distribution with variance  $B$  given by

$$B = \mathbb{E}[Y_{2,t}^2] = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{1 - \alpha_1} \rho_{\alpha_1}(\beta_1, \beta_2) \sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}} \\ + 2\sqrt{(1 - \beta_1) \text{SNR}_{21} (1 - \beta_2) \text{SNR}_{22}}. \quad (101)$$

Here also weak law of large numbers implies that

$$\lim_{n \rightarrow \infty} \Pr(B^{(n)} < b - \epsilon) = 0 \quad (102)$$

for any  $b \in [0, B]$ .

Now if  $\rho$  replaces  $\sqrt{1 - \alpha_1} \rho_{\alpha_1}(\beta_1, \beta_2)$  with  $\alpha_1 \in [0, 1]$  in constraints (100) and (101), then any non-negative information-energy rate triplet  $(R_1, R_2, B)$  satisfying

$$R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} (1 - \rho^2)), \quad (103a)$$

$$R_2 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho \sqrt{\beta_1 \text{SNR}_{11} \beta_2 \text{SNR}_{12}}) \\ - \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} (1 - \rho^2)), \quad (103b)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2(\rho \sqrt{\beta_1 \beta_2} + \sqrt{(1 - \beta_1)(1 - \beta_2)}) \sqrt{\text{SNR}_{21} \text{SNR}_{22}}, \quad (103c)$$

where  $\rho \in [0, \rho^*(\beta_1, \beta_2)]$  and  $\rho^*(\beta_1, \beta_2)$  uniquely satisfies equation (44), is achievable.

If the roles of transmitters 1 and 2 are reversed, it can be shown that any non-negative information-energy rate triplet  $(R_1, R_2, B)$  such that

$$R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho \sqrt{\beta_1 \text{SNR}_{11} \beta_2 \text{SNR}_{12}}) - \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12} (1 - \rho^2)), \quad (104a)$$

$$R_2 \leq \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12} (1 - \rho^2)), \quad (104b)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho \sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}} + 2\sqrt{(1 - \beta_1) \text{SNR}_{21} (1 - \beta_2) \text{SNR}_{22}}, \quad (104c)$$

for any  $\rho \in [0, \rho^*(\beta_1, \beta_2)]$ , is achievable.



Time-sharing between all information-energy rate triplets in the union of the two regions described by the constraints (103) and (104) concludes the proof of achievability of the region

$$R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} (1 - \rho^2)), \quad (105a)$$

$$R_2 \leq \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12} (1 - \rho^2)), \quad (105b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho \sqrt{\beta_1 \text{SNR}_{11} \beta_2 \text{SNR}_{12}}), \quad (105c)$$

$$B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\rho \sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}} + 2\sqrt{(1 - \beta_1) \text{SNR}_{21} (1 - \beta_2) \text{SNR}_{22}}, \quad (105d)$$

for any  $\rho \in [0, \rho^*(\beta_1, \beta_2)]$ .

Note that for any  $\rho > \rho^*(\beta_1, \beta_2)$ , the sum of (105a) and (105b) is strictly smaller than (105c). The resulting information region is a rectangle that is strictly contained in the rectangle obtained for  $\rho = \rho^*(\beta_1, \beta_2)$ . In other words, there is no gain in terms of information rates. In terms of energy rates, for any  $\rho > \rho^*(\beta_1, \beta_2)$ , there always exist  $(\beta'_1, \beta'_2)$  such that

$$\rho = \sqrt{\beta'_1 \beta'_2} \rho^*(\beta'_1, \beta'_2) + \sqrt{(1 - \beta'_1)(1 - \beta'_2)}.$$

This choice achieves any information rate pair  $(R_1, R_2)$  satisfying

$$R_i \leq \frac{1}{2} \log_2 (1 + \beta'_i \text{SNR}_{1i} (1 - \rho^*(\beta'_1, \beta'_2)^2)). \quad (106)$$

In particular, it achieves

$$R_i \leq \frac{1}{2} \log_2 (1 + \beta'_i \text{SNR}_{1i} (1 - \rho^2)), \quad i \in \{1, 2\}, \quad (107)$$

since  $\rho > \rho^*(1, 1) = \max_{(\beta_1, \beta_2) \in [0, 1]^2} \rho^*(\beta_1, \beta_2)$ . This completes the proof of the achievability part of Theorem 2.

## 6.2 Proof of Converse

Fix an information-energy rate triplet  $(R_1, R_2, B) \in \mathcal{E}_{\text{FB}}(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ . For this information-energy rate triplet and for each blocklength  $n$  encoding and decoding functions are chosen such that

$$\limsup_{n \rightarrow \infty} P_{\text{error}}^{(n)} = 0, \quad (108a)$$

$$\limsup_{n \rightarrow \infty} P_{\text{outage}}^{(n)} = 0 \text{ for any } \epsilon > 0, \quad (108b)$$

and such that the input power constraint (1) is satisfied.

Using assumption (108a), applying Fano's inequality and following similar steps as in [24], it can be shown that the rates  $(R_1, R_2)$  must satisfy

$$nR_1 \leq \sum_{t=1}^n I(X_{1,t}; Y_{1,t} | X_{2,t}, \Omega) + \epsilon_1^{(n)}, \quad (109a)$$

$$nR_2 \leq \sum_{t=1}^n I(X_{2,t}; Y_{1,t} | X_{1,t}, \Omega) + \epsilon_2^{(n)}, \quad (109b)$$

$$n(R_1 + R_2) \leq \sum_{t=1}^n I(X_{1,t} X_{2,t}; Y_{1,t} | \Omega) + \epsilon_{12}^{(n)}, \quad (109c)$$

where  $\frac{\epsilon_1^{(n)}}{n}$ ,  $\frac{\epsilon_2^{(n)}}{n}$ , and  $\frac{\epsilon_1^{(n)}}{n}$  tend to zero as  $n$  tends to infinity.

Using assumption (108b), for a given  $\epsilon_n > 0$ , for any  $\eta > 0$  there exists  $n_0(\eta)$  such that for any  $n \geq n_0(\eta)$  it holds that

$$\Pr(B^{(n)} < B - \epsilon_n) < \eta. \quad (110)$$

Equivalently,

$$\Pr(B^{(n)} \geq B - \epsilon_n) \geq 1 - \eta \quad (111)$$

From Markov's inequality [27], the following holds:

$$(B - \epsilon_n) \Pr(B^{(n)} \geq B - \epsilon_n) \leq \mathbb{E}[B^{(n)}]. \quad (112)$$

Combining (111) and (112) yields

$$(B - \epsilon_n)(1 - \eta) \leq \mathbb{E}[B^{(n)}] \quad (113)$$

which can be written as

$$(B - \delta_n) \leq \mathbb{E}[B^{(n)}] \quad (114)$$

for some  $\delta_n > \epsilon_n$  (for sufficiently large  $n$ ).

In the following, the bounds in (109) and (114) are evaluated for the G-MAC-F. Assume that at each time  $t$  the channel input is  $X_{i,t} = U_{i,t} + \alpha_{i,t}W_t$ , with  $U_{i,t}$  a zero-mean IC component that depends only on the message  $M_i$ , and independent thereof  $W_t$  a zero-mean unit-variance NIC component that depends only on  $\Omega$  and that is known to all terminals. The coefficients  $\alpha_{i,t} \in \mathbb{R}$  are deterministic and known to all terminals.

For this purpose, for  $t \in \{1, \dots, n\}$  define

$$\sigma_{i,t}^2 \triangleq \text{Var}(U_{i,t}), \quad i \in \{1, 2\}, \quad (115)$$

$$\lambda_t \triangleq \text{Cov}[U_{1,t}, U_{2,t}]. \quad (116)$$

Define also

$$\alpha_i^2 \triangleq \frac{1}{n} \sum_{t=1}^n \alpha_{i,t}^2, \quad i \in \{1, 2\}, \quad (117)$$

$$\sigma_i^2 \triangleq \frac{1}{n} \sum_{t=1}^n \sigma_{i,t}^2, \quad i \in \{1, 2\}, \quad (118)$$

$$\rho \triangleq \left( \frac{1}{n} \sum_{t=1}^n \lambda_t \right) / \sigma_1 \sigma_2. \quad (119)$$

The input sequence must satisfy the input power constraint (1) which can be written, for  $i \in \{1, 2\}$ , as

$$\frac{1}{n} \sum_{t=1}^n \mathbb{E}[X_{i,t}^2] = \frac{1}{n} \sum_{t=1}^n (\sigma_{i,t}^2 + \alpha_{i,t}^2) = \sigma_i^2 + \alpha_i^2 \leq P_i. \quad (120)$$

For each  $t \in \{1, \dots, n\}$ , regardless of the distribution of  $(X_{1,t}, X_{2,t})$ , since  $Z_t$  follows a zero-mean unit-variance Gaussian distribution it holds that

$$h(Y_{1,t}|X_{1,t}, X_{2,t}, \Omega) = h(Z_t) = \frac{1}{2} \log_2(2\pi e). \quad (121)$$

Recall that by the properties of the differential entropy [28], it holds that for any random variable  $X$  of variance  $\sigma_X^2$ , the differential entropy  $h(X)$  satisfies  $h(X) \leq \frac{1}{2} \log_2 (2\pi e \sigma_X^2)$ , where the right-hand-side corresponds to the differential entropy of a Gaussian distribution with the same variance.

All terminals have access to the values of the NIC sequence  $\{W_t\}_{t=1}^n$  as well as the coefficients  $\{\alpha_{1,t}\}_{t=1}^n$  and  $\{\alpha_{2,t}\}_{t=1}^n$  as side information. Let  $Y'_{1,t} = h_1 U_{1,t} + h_2 U_{2,t} + Z_t$ . Since at each time  $W_t$  is independent of  $(U_{1,t}, U_{2,t}, Z_t)$ , the bounds in (109) on the information transmission rates can be written as

$$nR_1 \leq \sum_{t=1}^n I(U_{1,t}; Y'_{1,t} | U_{2,t}) + \epsilon_1^{(n)}, \quad (122a)$$

$$nR_2 \leq \sum_{t=1}^n I(U_{2,t}; Y'_{1,t} | U_{1,t}) + \epsilon_2^{(n)}, \quad (122b)$$

$$n(R_1 + R_2) \leq \sum_{t=1}^n I(U_{1,t} U_{2,t}; Y'_{1,t}) + \epsilon_{12}^{(n)}. \quad (122c)$$

For the considered  $(U_{1,t}, U_{2,t})$  it holds that

$$\begin{aligned} I(U_{1,t}, U_{2,t}; Y'_{1,t}) &= h(Y'_{1,t}) - h(Z_t) \\ &\leq \frac{1}{2} \log_2 (2\pi e \text{Var}(Y'_{1,t})) - \frac{1}{2} \log_2 (2\pi e) \\ &= \frac{1}{2} \log_2 (h_{11}^2 \sigma_{1,t}^2 + h_{12}^2 \sigma_{2,t}^2 + 2h_{11}h_{12}\lambda_t + 1), \\ I(U_{1,t}; Y'_{1,t} | U_{2,t}) &= h(Y'_{1,t} | U_{2,t}) - h(Y'_{1,t} | U_{1,t}, U_{2,t}) \\ &\leq \frac{1}{2} \log_2 (2\pi e (\text{Var}(Y'_{1,t} | U_{2,t}))) - \frac{1}{2} \log_2 (2\pi e) \\ &= \frac{1}{2} \log_2 \left( 1 + h_{11}^2 \sigma_{1,t}^2 \left( 1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2} \right) \right), \\ I(U_{1,t}; Y'_{1,t} | U_{2,t}) &= \frac{1}{2} \log_2 \left( 1 + h_{12}^2 \sigma_{2,t}^2 \left( 1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2} \right) \right). \end{aligned}$$

The bounds in (109) can be written as

$$nR_1 \leq \sum_{t=1}^n \frac{1}{2} \log_2 \left( 1 + h_{11}^2 \sigma_{1,t}^2 \left( 1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2} \right) \right) + \epsilon_1^{(n)}, \quad (123a)$$

$$nR_2 \leq \sum_{t=1}^n \frac{1}{2} \log_2 \left( 1 + h_{12}^2 \sigma_{2,t}^2 \left( 1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2} \right) \right) + \epsilon_2^{(n)}, \quad (123b)$$

$$n(R_1 + R_2) \leq \sum_{t=1}^n \frac{1}{2} \log_2 (1 + h_{11}^2 \sigma_{1,t}^2 + h_{12}^2 \sigma_{2,t}^2 + 2h_{11}h_{12}\lambda_t) + \epsilon_{12}^{(n)}, \quad (123c)$$

By the concavity of the mutual information, applying Jensen's inequality [28] in the bounds (123) yields in the limit when  $n \rightarrow \infty$ ,

$$\begin{aligned} R_1 &\leq \frac{1}{2} \log_2 (1 + h_{11}^2 \sigma_1^2 (1 - \rho^2)), \\ R_2 &\leq \frac{1}{2} \log_2 (1 + h_{12}^2 \sigma_2^2 (1 - \rho^2)), \\ R_1 + R_2 &\leq \frac{1}{2} \log_2 (1 + h_{11}^2 \sigma_1^2 + h_{12}^2 \sigma_2^2 + 2\sqrt{h_{11}^2 \sigma_1^2 h_{12}^2 \sigma_2^2} |\rho|). \end{aligned}$$

The average received energy is given by

$$\begin{aligned}
 \mathbb{E}[B^{(n)}] &= \mathbb{E}\left[\frac{1}{n}\sum_{t=1}^n Y_{2,t}^2\right] \\
 &= 1 + h_{21}^2\left(\frac{1}{n}\sum_{t=1}^n(\sigma_{1,t}^2 + \alpha_{1,t}^2)\right) + h_{22}^2\left(\frac{1}{n}\sum_{t=1}^n(\sigma_{2,t}^2 + \alpha_{2,t}^2)\right) \\
 &\quad + 2h_{21}h_{22}\left(\frac{1}{n}\sum_{t=1}^n(\lambda_t + \alpha_{1,t}\alpha_{2,t})\right) \\
 &\stackrel{(a)}{\leq} 1 + h_{21}^2\left(\frac{1}{n}\sum_{t=1}^n(\sigma_{1,t}^2 + \alpha_{1,t}^2)\right) + h_{22}^2\left(\frac{1}{n}\sum_{t=1}^n(\sigma_{2,t}^2 + \alpha_{2,t}^2)\right) \\
 &\quad + 2h_{21}h_{22}\left(\left(\frac{1}{n}\sum_{t=1}^n\lambda_t\right) + \left(\frac{1}{n}\sum_{t=1}^n\alpha_{1,t}^2\right)^{1/2}\left(\frac{1}{n}\sum_{t=1}^n\alpha_{2,t}^2\right)^{1/2}\right) \\
 &= 1 + h_{21}^2(\sigma_1^2 + \alpha_1^2) + h_{22}^2(\sigma_2^2 + \alpha_2^2) + 2h_{21}h_{22}(\rho|\sigma_1||\sigma_2| + |\alpha_1||\alpha_2|)
 \end{aligned}$$

where (a) follows from the Cauchy-Schwarz inequality.

To sum up, in the limit when  $n$  tends to infinity, any information-energy rate triplet satisfying

$$R_1 \leq \frac{1}{2} \log_2(1 + h_{11}^2 \sigma_1^2 (1 - \rho^2)), \quad (124a)$$

$$R_2 \leq \frac{1}{2} \log_2(1 + h_{12}^2 \sigma_2^2 (1 - \rho^2)), \quad (124b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log_2(1 + h_{11}^2 \sigma_1^2 + h_{12}^2 \sigma_2^2 + 2\sqrt{h_{11}^2 h_{12}^2 \sigma_1^2 \sigma_2^2} \rho), \quad (124c)$$

$$B \leq 1 + h_{21}^2(\sigma_1^2 + \alpha_1^2) + h_{22}^2(\sigma_2^2 + \alpha_2^2) + 2h_{21}h_{22}(\rho|\sigma_1||\sigma_2| + |\alpha_1||\alpha_2|) \quad (124d)$$

for some  $\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2$  such that (120) is true and for some  $\rho \in [-1, 1]$  is achievable. Let  $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho)$  denote this region. The information-energy capacity region is contained within the union of all  $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho)$  over all  $0 \leq \sigma_1^2 + \alpha_1^2 \leq P_1$  and  $0 \leq \sigma_2^2 + \alpha_2^2 \leq P_2$  and  $-1 \leq \rho \leq 1$ .

In this union, it suffices to consider  $0 \leq \rho \leq 1$  because for any  $-1 \leq \rho \leq 1$ ,  $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho) \subseteq \mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, |\rho|)$ . Furthermore, for  $0 \leq \rho \leq 1$ , it suffices to consider  $\alpha_1 \geq 0, \alpha_2 \geq 0$ , and  $\sigma_1^2, \sigma_2^2, \alpha_1^2$ , and  $\alpha_2^2$  that saturate the input power constraint (i.e., (120) holds with equality). If  $\beta_i \triangleq \frac{\sigma_i^2}{P_i} \in [0, 1], i \in \{1, 2\}$ , such a region contains all information-energy rate triplets  $(R_1, R_2, B)$  satisfying constraints (42) which completes the converse of the proof.

## 7 Proof of Proposition 4

For fixed  $\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}$ , and  $\text{SNR}_{22}$  and fixed minimum received energy rate  $b \geq 0$ , the information sum-rate maximization problem in (59) can be written as

$$R_{\text{sum}}^{\text{FB}}(b) = \max_{(\beta_1, \beta_2, \rho) \in [0, 1]^3} f(\beta_1, \beta_2, \rho) \quad (125a)$$

$$\text{subject to: } g(\beta_1, \beta_2, \rho) \geq b, \quad (125b)$$

where the functions  $f$  and  $g$  are defined as follows

$$f(\beta_1, \beta_2, \rho) \triangleq \min \left\{ \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12} + 2\rho \sqrt{\beta_1 \text{SNR}_{11} \beta_2 \text{SNR}_{12}}), \right. \\ \left. \frac{1}{2} \log_2 ( (1 + \beta_1 \text{SNR}_{11} (1 - \rho^2)) (1 + \beta_2 \text{SNR}_{12} (1 - \rho^2)) ) \right\}, \quad (126)$$

and

$$g(\beta_1, \beta_2, \rho) \triangleq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2(\sqrt{\beta_1 \beta_2} \rho + \sqrt{(1 - \beta_1)(1 - \beta_2)}) \sqrt{\text{SNR}_{21} \text{SNR}_{22}}. \quad (127)$$

Let also

$$\rho_{\min}(\beta_1, \beta_2) \triangleq \min \left( 1, \frac{b - (1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}})}{2\sqrt{\beta_1 \text{SNR}_{21} \beta_2 \text{SNR}_{22}}} \right) \quad (128)$$

be the value of  $\rho \in [0, 1]$  for which  $g(\beta_1, \beta_2, \rho) = b$ . Note that  $\rho^*(\beta_1, \beta_2)$  can be alternatively defined as

$$\rho^*(\beta_1, \beta_2) \triangleq \operatorname{argmax}_{\rho \in [0, 1]} f(\beta_1, \beta_2, \rho). \quad (129)$$

The proof of Proposition 4 is based on the following two lemmas.

**Lemma 2.** *Let  $(\beta_1, \beta_2, \rho) \in [0, 1]^3$  be a solution to (125). Then, it holds that  $(\beta_1, \beta_2, \max\{\rho_{\min}(\beta_1, \beta_2), \rho^*(\beta_1, \beta_2)\})$  is also a solution to (125).*

*Proof:* Let  $(\beta'_1, \beta'_2, \rho') \in [0, 1]^3$  be a solution to (125). Hence,  $\forall (\beta_1, \beta_2, \rho) \in [0, 1]^3$ , it follows that

$$f(\beta'_1, \beta'_2, \rho') \geq f(\beta_1, \beta_2, \rho), \quad \text{and} \quad (130)$$

$$g(\beta'_1, \beta'_2, \rho') \geq b. \quad (131)$$

Assume that  $\rho_{\min}(\beta'_1, \beta'_2) \leq \rho^*(\beta'_1, \beta'_2)$ . Then  $\rho^*(\beta'_1, \beta'_2)$  is admissible, i.e.,

$$g(\beta'_1, \beta'_2, \rho^*(\beta'_1, \beta'_2)) \geq b, \quad (132)$$

and from (129), it follows that

$$f(\beta'_1, \beta'_2, \rho') \leq f(\beta'_1, \beta'_2, \rho^*(\beta'_1, \beta'_2)). \quad (133)$$

From (130), (132), and (133) it follows that  $\forall (\beta_1, \beta_2, \rho) \in [0, 1]^3$

$$f(\beta'_1, \beta'_2, \rho^*(\beta'_1, \beta'_2)) \geq f(\beta_1, \beta_2, \rho), \quad \text{and} \quad (134a)$$

$$g(\beta'_1, \beta'_2, \rho^*(\beta'_1, \beta'_2)) \geq b, \quad (134b)$$

which suggests that  $(\beta'_1, \beta'_2, \rho^*(\beta'_1, \beta'_2))$  is also a solution to (125) under the assumption that  $\rho_{\min}(\beta'_1, \beta'_2) \leq \rho^*(\beta'_1, \beta'_2)$ . Now, assume that  $\rho_{\min}(\beta'_1, \beta'_2) \geq \rho^*(\beta'_1, \beta'_2)$ . Then,  $\rho' \in [\rho_{\min}(\beta'_1, \beta'_2), 1]$  and from (128) it follows that

$$g(\beta'_1, \beta'_2, \rho_{\min}(\beta'_1, \beta'_2)) = b, \quad (135)$$

and thus,  $\rho_{\min}(\beta'_1, \beta'_2)$  is also admissible given  $\beta'_1$  and  $\beta'_2$ . Note that the function  $f(\beta'_1, \beta'_2, \rho)$  is monotonically decreasing in  $\rho$  in the interval  $[\rho^*(\beta'_1, \beta'_2), 1]$ . Thus, the following inequality holds:

$$f(\beta'_1, \beta'_2, \rho') \leq f(\beta'_1, \beta'_2, \rho_{\min}(\beta'_1, \beta'_2)). \quad (136)$$

From (130), (135), and (136) it follows that  $\forall(\beta_1, \beta_2, \rho) \in [0, 1]^3$

$$f(\beta'_1, \beta'_2, \rho_{\min}(\beta'_1, \beta'_2)) \geq f(\beta_1, \beta_2, \rho), \quad \text{and} \quad (137a)$$

$$g(\beta'_1, \beta'_2, \rho_{\min}(\beta'_1, \beta'_2)) = b, \quad (137b)$$

which suggests that  $(\beta'_1, \beta'_2, \rho_{\min}(\beta'_1, \beta'_2))$  is also a solution to (125) under the assumption that  $\rho_{\min}(\beta'_1, \beta'_2) \geq \rho^*(\beta'_1, \beta'_2)$ .

Finally, from (134) and (137), it follows that if  $(\beta'_1, \beta'_2, \rho')$  is a solution to (125), then, the triplet  $(\beta'_1, \beta'_2, \max\{\rho_{\min}(\beta'_1, \beta'_2), \rho^*(\beta'_1, \beta'_2)\})$  is also a solution, which completes the proof of Lemma 2.  $\blacksquare$

**Lemma 3.** *The unique solution to (125) in  $[0, 1]^3$  is  $(1, 1, \bar{\rho})$  with*

$$\bar{\rho} \triangleq \max\{\rho_{\min}(1, 1), \rho^*(1, 1)\}. \quad (138)$$

*Proof:* Assume that there exists another solution  $(\beta'_1, \beta'_2, \rho')$  to (125) different from  $(1, 1, \bar{\rho})$ . Thus, for any  $(\beta_1, \beta_2, \rho) \in [0, 1]^3$  it holds that

$$f(\beta_1, \beta_2, \rho) \leq f(\beta'_1, \beta'_2, \rho'). \quad (139)$$

Note that for a fixed  $\rho' \in [0, 1]$ ,  $f(\beta_1, \beta_2, \rho')$  is strictly increasing in  $(\beta_1, \beta_2)$ . Hence, for any  $(\beta_1, \beta_2) \in [0, 1]^2$ ,

$$f(\beta_1, \beta_2, \rho') < f(1, 1, \rho') \quad (140)$$

$$\leq f(1, 1, \bar{\rho}), \quad (141)$$

where the second inequality follows by Lemma 2. Moreover, since  $\bar{\rho} \geq \rho_{\min}(1, 1)$ , the following inequality also holds:

$$g(1, 1, \bar{\rho}) \geq b. \quad (142)$$

In particular, if  $(\beta_1, \beta_2) = (\beta'_1, \beta'_2)$  in (140), it follows that

$$f(\beta'_1, \beta'_2, \rho') < f(1, 1, \bar{\rho}), \quad (143)$$

which contradicts the initial assumption that there exists a solution other than  $(1, 1, \bar{\rho})$ . This establishes a proof by contradiction that the unique solution to (125) is  $(1, 1, \bar{\rho})$ .  $\blacksquare$

Finally, the proof of Proposition 4 follows from the following equality:

$$R_{\text{sum}}^{\text{FB}}(b) = f(1, 1, \bar{\rho}), \quad (144)$$

and this completes the proof.

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Inria  
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ISSN 0249-6399