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Abstract: In this report, the fundamental limits of simultaneous information and energy transmission in the two-user Gaussian multiple access channel (G-MAC) with and without feedback are fully characterized. All the achievable information and energy transmission rates (in bits per channel use and Joules per channel use respectively) are identified. Thus, the capacity-energy region is defined in both cases. In the case without feedback, an achievability scheme based on power-splitting and successive interference cancelation is shown to be optimal. Alternatively, in the case with feedback (G-MAC-F), a simple yet optimal achievability scheme based on power-splitting and Ozarow's capacity achieving scheme is presented. Three of the most important observations in this work are: (a) The capacity-energy region of the G-MAC without feedback is a proper subset of the capacity-energy region of the G-MAC-F; (b) Feedback can at most double the energy rate for a fixed information rate; and (c) Time-sharing with power control is strictly suboptimal in terms of sum-rate in both the G-MAC with and without feedback.

Key-words: Feedback, Gaussian multiple access channel, simultaneous information and energy transmission, RF harvesting, capacity-energy region.

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L'utilisation de la voie de retour améliore la transmission simultanée d'information et d'énergie dans les canaux à accès multiple

Résumé : Dans le présent-rapport, les limites fondamentales de la transmission simultanée d'information et d'énergie dans le canal Gaussien à accès multiple (G-MAC) avec et sans voie de retour sont déterminées. L'ensemble des débits atteignables de transmission d'information et d'énergie (en bits par utilisation canal et en Joules par utilisation canal respectivement) est identifié. Ainsi, les régions de capacité-énergie sont définies dans les deux cas. Dans le cas sans voie de retour, on démontre qu'un schéma d'atteignabilité, basé sur la division de puissance et sur l'annulation successive de l'interférence, est optimal. Alternativement, dans le cas avec voie de retour (G-MAC-F), un schéma d'atteignabilité, simple mais optimal, basé sur la division de puissance et sur le schéma d'Ozarow qui atteint la capacité, est présenté. Trois parmi les observations les plus importantes dans ce travail sont: (a) la région de capacité-énergie du G-MAC sans voie de retour est un sous-ensemble propre de la région de capacité-énergie du G-MAC-F; (b) l'utilisation de la voie de retour peut au plus multiplier par deux le débit d'énergie pour un débit d'information fixé; et (c) le partage du temps avec contrôle de puissance est strictement sous-optimal en termes de débit-somme pour le G-MAC avec et sans voie de retour.

Mots-clés : Voie de retour, canal Gaussien à accès multiple (G-MAC), transmission simultanée d'information et d'énergie, collecte d'énergie RF, région de capacité-énergie.

Contents

1 Introduction

Efficient energy utilization is among the main challenges of future communication networks in order to extend their lifetime and to reduce operating costs. Towards this end, the design of energy-aware architectures and energy-saving transmission techniques has recently attracted considerable attention. For decades, the traditional engineering perspective was to use RF signals exclusively for information transmission. However, this approach has been shown to be suboptimal [2]. Indeed, an RF signal carries both energy and information. From this standpoint, a variety of modern wireless systems and proposals question this conventional assumption and suggest that RF signals can be simultaneously used for information and energy transmission [3]. Typical examples of communications technologies already exploiting this principle are RFID devices and power line communications. Beyond the existing applications, simultaneous transmission of both information and energy appears as a promising technology for a variety of emerging applications including low-power short-range communication systems, sensor networks, machine-to-machine networks and body-area networks, among others. Nevertheless, information and energy transmission are often conflicting tasks and thus subject to a trade-off between the information transmission rate (bits per channel use) and the energy transmission rate (Joules per channel use). Despite the vast existing literature on this subject, the fundamental limits of simultaneous energy and information transmission (SEIT) are still unknown in most multi-user channels. The pioneering works by Varshney in [4] and [2], as well as, Grover and Sahai in [5] provided the fundamental limits on SEIT over point-to-point channels. More specifically, in [4] the case of a the single point-to-point link was discussed and in [5] and [2] the case of parallel point-to-point links was studied.

In the context of multi-user channels, most of the existing results in this area have approached SEIT from a signal-processing or networking point of view and focused mainly on the feasibility aspects. For instance, optimization of beamforming strategies over more complex network structures was considered in [6, 7, 8], and [9] for multi-antenna broadcast channels and in [10] for multi-antenna interference channels where one receiver performs information decoding and the other performs energy harvesting. SEIT was also investigated in the general realm of cellular systems in [11] as well as in multi-hop relaying systems in [12, 13, 14, 15, 16], and [17]. This paradigm was also investigated in [18] for an interactive two-way two-node channel and in [19] for graphical unicast and multicast networks.

1.1 SEIT in Multiple Access Channels

In the particular case of the discrete memoryless multiple access channel (MAC), the trade-off between information rate and energy rate has been studied in [13]. Therein, Fouladgar *et al.* characterized the capacity-energy region of the two-user discrete memoryless MAC when a minimum energy rate is required at the input of the receiver. Such a constraint changes the dynamic of the communication system in the sense that it requires additional transmitter coordination to achieve the targeted energy rate. A straightforward extension of this work to the Gaussian MAC (G-MAC) case is far from trivial due to the fact that the capacity-energy region involves an auxiliary random-variable which cannot be eliminated as in the case without energy constraints, as independently suggested by Cover [20] and Wyner [21]. Other types of energy rate constraints for the G-MAC have been also investigated. For instance, Gastpar [22] considered the G-MAC under a maximum received energy rate constraint. Under this assumption, feedback has been shown not to increase the capacity region. However, in the case studied by Fouladgar *et al.* in [13], the effect of feedback is not yet well understood from an energy transmission perspective.

More generally, the use of feedback in the K-user G-MAC, even without energy rate constraints, has been shown to be of limited impact in terms of sum-rate improvement. This holds even in the case of perfect channel-output feedback. More specifically, the use of feedback in the G-MAC increases the sum-capacity by at most $\frac{\log_2(K)}{2}$ bits per channel use [23]. Hence, the use of feedback is difficult to justify from the point of view of information transmission.

1.2 Contributions

This report studies the fundamental limits of SEIT in the two-user G-MAC with and without feedback. It shows that when the goal is to simultaneously transmit both information and energy, feedback can significantly improve the global performance of the system in terms of both information and energy transmission rates. One of the main contributions is the identification of all the achievable information and energy transmission rates in bits per channel use and Joules per channel use, respectively. More specifically, the capacity-energy region is fully characterized in both cases. In the case without feedback, an achievability scheme based on power-splitting and successive interference cancellation is shown to be optimal. Alternatively, in the case with feedback (G-MAC-F), a simple yet optimal achievability scheme based on power-splitting and Ozarow's capacity achieving scheme is presented. Three of the most important observations in this work are: (a) The capacity-energy region of the G-MAC without feedback is a proper subset of the capacity-energy region of the G-MAC-F, that is, the former is strictly contained in the latter; (b) Feedback can at most double the energy rate for a fixed information rate; and (c) Time-sharing with power control is strictly suboptimal in both the G-MAC with and without feedback in terms of sum-rate. One of the underlying assumptions of this work is that both the receiver and the energy harvester are co-located. This implies that the channel coefficients between the transmitters and the receiver can be considered identical to those between the transmitter and the energy harvester. However, the results presented here easily extend to more general cases in which the receiver and the energy-harvester are spatially separated and thus, subject to different channel coefficients.

1.3 Organization of the Report

The remainder of the report is structured as follows. Sec. 2 formulates the problem of simultaneous energy and information transmission in the two-user G-MAC-F. Sections 3 and 4 show the main results of this report for the G-MAC without and with feedback, respectively. Namely, for both settings the following fundamental limits are derived: (a) the capacity-energy region; and (b) the maximum individual information rates and sum information rates that can be achieved given a targeted energy transmission rate. The maximum energy transmission rate improvement that can be obtained by using feedback given a targeted information transmission rate is also discussed in Sec. 4. Sec. 5 presents the proof of achievability and converse for the capacity-energy region of the G-MAC-F (Theorem 3).

2 Gaussian Multiple Access Channel with Feedback

Consider the two-user memoryless Gaussian MAC with perfect channel-output-feedback in Fig. 1. At each channel use $t \in \mathbb{N}$, $X_{1,t}$ and $X_{2,t}$ denote the real symbols sent by transmitters 1 and 2, respectively. The symbols $X_{i,1}, \ldots, X_{i,n}$ satisfy an expected average *input power constraint*

$$\frac{1}{n}\sum_{t=1}^{n} \mathsf{E}\left[X_{i,t}^2\right] \leqslant P_i, \tag{1}$$

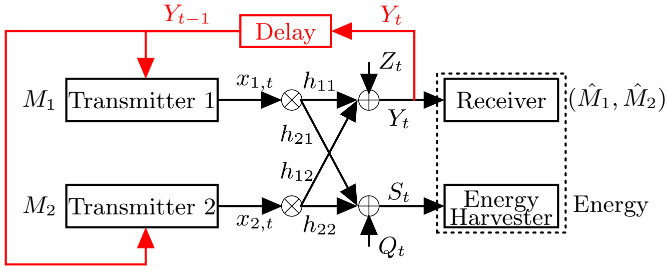

Figure 1: Two-user memoryless Gaussian MAC with feedback and energy harvester.

where P_i denotes the average transmit power of transmitter i in Joules per channel use for $i \in \{1, 2\}$. The receiver observes the real channel output

$$Y_t = h_{11}X_{1,t} + h_{12}X_{2,t} + Z_t, \tag{2}$$

while the energy harvester observes

$$S_t = h_{21}X_{1,t} + h_{22}X_{2,t} + Q_t, \tag{3}$$

where $h_{1,i}$ and $h_{2,i}$ are the corresponding constant non-negative channel coefficients from transmitter i to the receiver and energy harvester, respectively. The noise terms Z_t and Q_t are realizations of two identically distributed zero-mean unit-variance real Gaussian random variables. For ease of exposition, the receiver and the energy harvester are assumed to be co-located, which justifies that the channel coefficients are identical, that is, $h_1 = h_{11} = h_{21}$ and $h_2 = h_{12} = h_{22}$.

A perfect feedback link from the receiver to transmitter i allows at the end of each channel use t, the observation of the channel output Y_{t-d} at transmitter i, with $d \in \mathbb{N}$ the delay of the feedback channel. Without any loss of generality, the delay is assumed to be the same from the receiver to both transmitters and equivalent to one channel use, $d = 1$.

The G-MAC-F above is fully described by two parameters: the signal to noise ratios (SNRs) SNR_1 and SNR_2, which are defined as follows:

$$\text{SNR}_i \triangleq h_i^2 P_i, \quad i \in \{1, 2\}, \tag{4}$$

given the normalization over the noise power. Within this context, two main tasks are to be simultaneously accomplished: information transmission and energy transmission.

2.1 Information Transmission

The goal of the communication is to convey the independent messages M_1 and M_2 from transmitters 1 and 2 to the common receiver. The messages M_1 and M_2 are independent of the noise terms $Z_1, \ldots, Z_n$, $Q_1, \ldots, Q_n$ and uniformly distributed over the sets $\mathcal{M}_1 \triangleq \{1, \ldots, \lfloor 2^{nR_1} \rfloor\}$ and $\mathcal{M}_2 \triangleq \{1, \ldots, \lfloor 2^{nR_2} \rfloor\}$, where R_1 and R_2 denote the transmission rates and $n \in \mathbb{N}$ the block-length. The existence of feedback links allows the t-th symbol of transmitter i to be dependent on all previous channel outputs $Y_1, \ldots, Y_{t-1}$ as well as its message index M_i. More specifically,

$$X_{i,1} = f_{i,1}^{(n)}(M_i) \quad \text{and} \tag{5}$$

$$X_{i,t} = f_{i,t}^{(n)}(M_i, Y_1, \ldots, Y_{t-1}), \quad t \in \{2, \ldots, n\}, \tag{6}$$

for some encoding functions

$$f_{i,1}^{(n)} : \mathcal{M}_i \to \mathbb{R} \quad \text{and} \tag{7}$$

$$f_{i,t}^{(n)} : \mathcal{M}_i \times \mathbb{R}^{t-1} \to \mathbb{R}. \tag{8}$$

The receiver produces an estimate $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) = \Phi^{(n)}(Y^n)$ of the message-pair (M_1, M_2) via a decoding function $\Phi^{(n)} : \mathbb{R}^n \to \mathcal{M}_1 \times \mathcal{M}_2$, and the average probability of error is

$$P_{\text{error}}^{(n)} \triangleq \Pr\left\{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) \neq (M_1, M_2)\right\}. \tag{9}$$

2.2 Energy Transmission

The average received power at the energy harvester or the energy transmission rate (in Joules per channel use) is

$$B^{(n)} \triangleq \frac{1}{n} \sum_{t=1}^{n} \mathsf{E}\left[S_t^2\right] \leqslant 1 + \text{SNR}_1 + \text{SNR}_2 + 2\sqrt{\text{SNR}_1 \text{SNR}_2}. \tag{10}$$

The goal of the energy transmission is to guarantee that the energy rate $B^{(n)}$ is not less than a given constant b that must satisfy

$$0 < b \leqslant 1 + \text{SNR}_1 + \text{SNR}_2 + 2\sqrt{\text{SNR}_1 \text{SNR}_2}, \tag{11}$$

for the problem to be feasible. Hence, the probability of energy outage is defined as follows:

$$P_{\text{outage}}^{(n)}(\epsilon) = \Pr\left\{B^{(n)} < b - \epsilon\right\}, \tag{12}$$

for some $\epsilon > 0$ arbitrarily small.

2.3 Simultaneous Information and Energy Transmission

The G-MAC-F in Fig. 1 is said to operate at the information-energy rate triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ if both transmitters and the receiver use a transmit-receive configuration such that: (a) reliable communication at information rates R_1 and R_2 is ensured; and (b) the energy transmission rate during the whole block-length is not lower than B. Under these conditions, the information-energy rate triplet (R_1, R_2, B) is said to be achievable.

Definition 1 (Achievable Rates)**.** *The triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ is achievable if there exists a sequence of encoding and decoding functions $\left\{\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \Phi^{(n)}\right\}_{n=1}^{\infty}$ such that both the average error probability and the energy-outage probability tend to zero as the blocklength n tends to infinity. That is,*

$$\limsup_{n\to\infty} P_{\text{error}}^{(n)} = 0, \tag{13}$$

$$\limsup_{n\to\infty} P_{\text{outage}}^{(n)}(\epsilon) = 0 \textit{ for any } \epsilon > 0. \tag{14}$$

From Def. 1, it is clear that for any achievable triplet (R_1, R_2, B), whenever the targeted energy rate B is smaller than the minimum energy rate required to guarantee reliable communications at the information rates R_1 and R_2, the energy rate constraint is vacuous. This is basically because the energy transmission rate is always satisfied and thus, the transmitter can

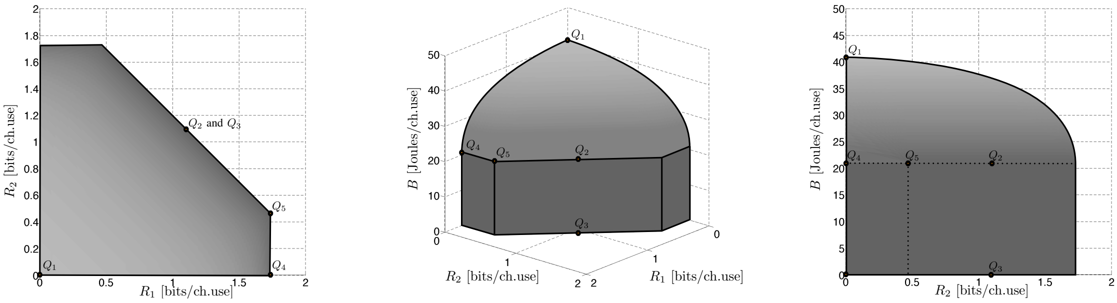

Figure 2: 3-D representation of the capacity-energy region without feedback $\mathcal{E}\,(10,10)$ in the coordinate system (R_1, R_2, B). Left and right figures represent a bi-dimensional view in the R_1-R_2 and B-R_2 planes of $\mathcal{E}\,(10,10)$, respectively. The figure in the center is a 3-D representation of $\mathcal{E}\,(10,10)$. Note that poins Q_1, Q_2 and Q_3 are coplanar and satisfy that $R_1 = R_2$. In particular, $Q_1 = (0, 0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2})$; and the points Q_2 and Q_3 are also collinear and $R_1 = R_2 = \frac{1}{4}\log_2(1 + \mathrm{SNR}_1 + \mathrm{SNR}_2)$. The points Q_2, Q_4 and Q_5 are coplanar and they satisfy that $B = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2$. In particular, $Q_4 = (\frac{1}{2}\log_2(1 + \mathrm{SNR}_1), 0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2)$ and $Q_5 = (\frac{1}{2}\log_2(1 + \mathrm{SNR}_1), \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_1}{1+\mathrm{SNR}_2}\right), 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2)$.

exclusively use the available power budget for increasing the information transmission rate. Alternatively, when the energy rate B must be higher than what is strictly necessary to guarantee reliable communication, the transmitters face a trade-off between information and energy rates. Often, increasing the energy transmission rate implies decreasing the information transmission rates and *vice versa*. This tradeoff is accurately modeled by the notion of capacity-energy region.

Definition 2 (Capacity-Energy Region)**.** *The capacity-energy region of the Gaussian MAC with perfect channel-output feedback* $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ *and without feedback* $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ *are the closure of all achievable information-energy rate triplets* (R_1, R_2, B) *in the corresponding cases.*

The main results for the G-MAC with and without feedback presented in this report are provided in terms of the capacity-energy region (Def. 2) in the following two sections.

3 Main Results: Capacity-Energy Region without Feedback

This section describes the fundamental limits of SEIT in the G-MAC for the case in which feedback is not available. These results are particular cases of the results presented in Sec. 4 in which feedback is considered. The interest of presenting these results separately stems from the need for comparing both cases.

3.1 Capacity-Energy Region without Feedback

The capacity-energy region of the G-MAC, denoted by $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, with parameters SNR_1 and SNR_2, is fully characterized by the following theorem.

Theorem 1 (Capacity-Energy Region)**.** *The capacity-energy region* $\mathcal{E}\,(\mathrm{SNR}_1, \mathrm{SNR}_2)$ *of the G-MAC without feedback is the set of all non-negative information-energy rate triplets* (R_1, R_2, B)

that satisfy

$$R_1 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\,\mathrm{SNR}_1\right), \tag{15a}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta_2\,\mathrm{SNR}_2\right), \tag{15b}$$

$$R_1+R_2 \leqslant \frac{1}{2}\log_2\big(1+\beta_1\,\mathrm{SNR}_1+\beta_2\,\mathrm{SNR}_2\big), \tag{15c}$$

$$B \leqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2}, \tag{15d}$$

with $(\beta_1,\beta_2)\in[0,1]^2$.

3.1.1 Comments on the Proof

To prove the achievability of the region presented in Theorem 1, consider that each transmitter i, with $i\in\{1,2\}$, uses a fraction $\beta_i\in[0,1]$ of its available power to transmit information and uses the remaining fraction of power $(1-\beta_i)$ to transmit energy.

Given a power-split $(\beta_1,\beta_2)\in[0,1]^2$, the achievability of information rate pairs satisfying (15a)-(15c) follows the scheme described in [20] or [21]. Additionaly, in order to satisfy the received energy constraint (15d), transmitters send common randomness that is known to both transmitters and to the receiver using all their remaining power. This common randomness does not carry any information and does not produce any interference to the information-carrying signals. More specifically, at each time t, transmitter i's channel input can be written as:

$$X_{i,t}=\sqrt{(1-\beta_i)P_i}W_t+U_{i,t},\quad i\in\{1,2\}, \tag{16}$$

for some independent zero-mean Gaussian information-carrying symbols $U_{1,t}$ and $U_{2,t}$ with variances $\beta_1 P_1$ and $\beta_2 P_2$, respectively, and independent thereof W_t is a zero-mean unit-variance Gaussian energy-carrying symbol known non-causally to all terminals. That is, the rate-pairs (R_1,R_2) can be made arbitrarily close to the upper-bounds in (15a)-(15c) by approximating transmitters 1 and 2 channel input symbols by Gaussian random variables, subject to the corresponding power constraints.

The receiver subtracts the common randomness and then performs successive decoding to recover the messages M_1 and M_2. Note that this strategy achieves the corner points of the rate-region at a given energy rate. Time-sharing between the corner points and the points on the axes is needed to achieve the remaining points.

The converse and the analysis of the average received energy rate follow the same lines as in the case with feedback described in Section 5 when the IC channel input components are assumed to be independent.

3.1.2 Example and Observations

Consider the capacity-energy region of the G-MAC without feedback $\mathcal{E}(\mathrm{SNR}_1,\mathrm{SNR}_2)$, with $\mathrm{SNR}_1=\mathrm{SNR}_2=10$ depicted in Fig. 2. Therein, left and right figures represent a bi-dimensional view in the R_1-R_2 and B-R_2 planes of $\mathcal{E}(10,10)$, respectively. The figure in the center is a 3-D representation of $\mathcal{E}(10,10)$.

For any achievable triplet (R_1,R_2,B), whenever the required minimum energy rate B is smaller than the minimum energy rate required to guarantee reliable communication at rates R_1 and R_2, the energy constraint is vacuous. Hence, transmitting information is always enough to satisfy the energy constraint. More interestingly, any intersection of the volume $\mathcal{E}(\mathrm{SNR}_1,\mathrm{SNR}_2)$ with a plane $B=b$, with $b\in[0,1+\mathrm{SNR}_1+\mathrm{SNR}_2]$, corresponds to the set of triplets (R_1,R_2,b), in which the pairs (R_1, R_2) form a set that is identical to $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, with $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ the capacity region of the G-MAC with parameters SNR_1 and SNR_2; see Fig. 2. That is, when the energy rate constraint is lower than the maximum energy rate achievable with independent symbols, all information rates of the capacity region $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ are achievable. (See for instance points Q_2, Q_3, Q_4, and Q_5 in Fig. 2.)

On the other hand, when the energy constraint consists of guaranteeing an energy rate of $B = b \in (1 + \mathrm{SNR}_1 + \mathrm{SNR}_2, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$ Joules per channel use, that is, when the energy rate B at the input of the receiver is required to be higher than what is strictly necessary to guarantee reliable communication at the sum-rate, the transmitters deal with a trade-off between information and energy transmission through power-splitting, i.e., $0 \leqslant \beta_i < 1$, for $i \in \{1, 2\}$. In this case, part of the transmitter power budget is exclusively dedicated to the transmission of energy. Thus, for all triplets (R_1, R_2, b), it follows that the corresponding information rate pairs (R_1, R_2) form a proper set of the capacity region $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2)$; see for instance Fig. 2.

Finally, requiring an energy transmission rate $B = b \in \left(1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, \infty\right)$ constrains the achievability of any positive rate (see point Q_1 in Fig. 2, which is achievable with $(\beta_1 = \beta_2 = 0)$). However, it is important to highlight that the transmitters cannot guarantee an energy rate that is higher than the energy rate achieved with full transmitter cooperation.

3.2 Individual Information Rates without Feedback and with Minimum Energy Rate

The maximum individual rate $R_i^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b)$, with $i \in \{1, 2\}$ given an energy rate constraint of b Joules per channel use at the input of the energy harvester is the solution to an optimization problem of the form

$$R_i^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \max_{(r_i, r_j, c) \in \mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2): c \geqslant b} r_i. \tag{17}$$

The following proposition characterizes the solution to the optimization problem (17).

Proposition 1 (Maximum Individual-Rate without Feedback)**.** *Let b be the minimum energy rate that must be guaranteed at the input of the energy harvester. Then, the maximum individual rates $R_i^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b)$ of the Gaussian MAC without feedback, with parameters* SNR_1 *and* SNR_2*, are given by*

$$R_i^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \frac{1}{2}\log_2\left(1 + \beta^\star(b)\mathrm{SNR}_i\right), \tag{18}$$

with $\beta^\star(b) \in [0, 1]$ defined as follows:

$$\beta^\star(b) = 1 - \left(\frac{(b - (1 + \mathrm{SNR}_1 + \mathrm{SNR}_2))^+}{2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}}\right)^2. \tag{19}$$

Proof: From the assumptions of Proposition 1 it follows that an energy transmission rate of b Joules per channel use must be guaranteed at the input of the energy harvester. Then, the set of power-splits (β_1, β_2) that satisfy this constraint must satisfy

$$1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{(1 - \beta_1)\mathrm{SNR}_1\,(1 - \beta_2)\mathrm{SNR}_2} \geqslant b. \tag{20}$$

These power-splits are referred to as *feasible power-splits.* Consider a feasible power-split in which transmitter j only transmits energy to the harvester, i.e., $\beta_j = 0$ and transmitter i transmits both information and energy, i.e., $\beta_i > 0$. The energy transmission of transmitter i is made at the minimum rate to meet the energy rate constraint. In this configuration, transmitter i is able to achieve the maximum individual rate, as it can use a power-split in which the fraction β_i is maximized. From (20), it follows that $\beta_i \leqslant \beta^\star(b)$ and the individual rates are limited by $R_i \leqslant \frac{1}{2}\log_2(1+\beta^\star(b)\mathrm{SNR}_i)$. ■

The relevance of Proposition 1 stems from the insights it adds to the understanding of the shape of the volume $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$.

Note that if $b \in [0, 1+\mathrm{SNR}_1+\mathrm{SNR}_2]$, then $\beta^\star(b) = 1$ and thus, the energy constraint does not add any additional bound on the individual rates other than (15a) and (15b). In general, any intersection of the volume $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, in the Cartesian coordinates (R_1, R_2, B), with a plane $B = b \in [0, 1+\mathrm{SNR}_1+\mathrm{SNR}_2]$ corresponds to the set of triplets (R_1, R_2, b), in which the corresponding pairs (R_1, R_2) form a set that is identical to the capacity region of the G-MAC without feedback, denoted by $\mathcal{C}(\mathrm{SNR}_1, \mathrm{SNR}_2)$. Fig. 3 shows a general example of this intersection when $\mathrm{SNR}_1 = \mathrm{SNR}_2$. Alternatively, energy transmission at a rate $b \in (1+\mathrm{SNR}_1+\mathrm{SNR}_2, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$ induces an additional constraint on the individual rates and sum-rate as shown in the following subsection.

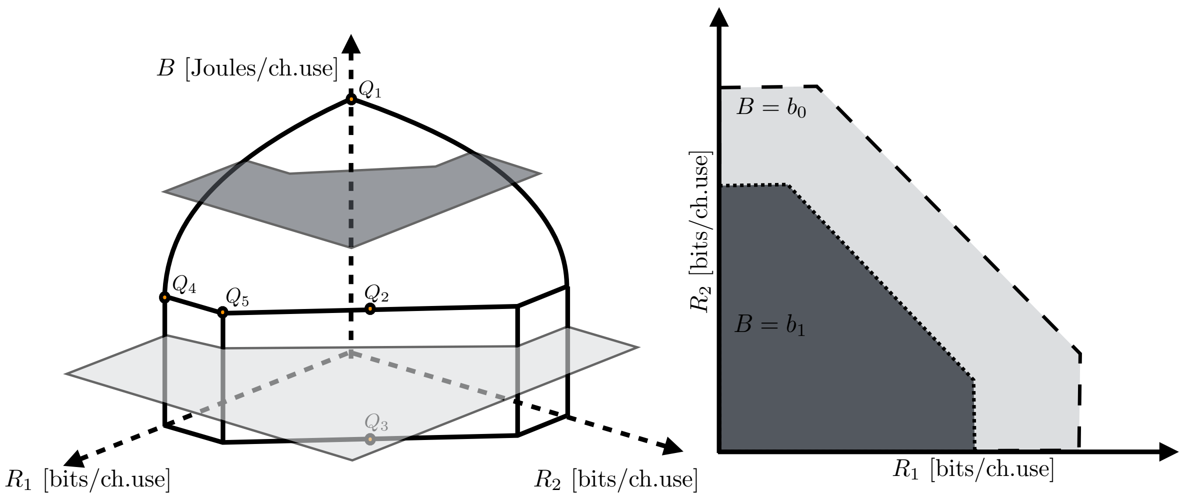

Figure 3: Intersection of the planes $B = b_0$ and $B = b_1$ with the capacity-energy region of the G-MAC without feedback $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, with $SNR_1 = SNR_2$, $b_0 \in \left[0, 1+\mathrm{SNR}_1+\mathrm{SNR}_2\right]$, $b_1 \in \left[1+\mathrm{SNR}_1+\mathrm{SNR}_2, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right]$.

3.3 Sum-Capacity with Minimum Received Energy Rate

The maximum sum-rate $R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, B)$ of the G-MAC subject to a minimum energy rate constraint B is the solution to an optimization problem of the form

$$R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, B) = \max_{(r_1, r_2, b) \in \mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2): b \geqslant B} r_1 + r_2. \tag{21}$$

The solution to this problem is given by the following theorem.

Theorem 2 (Sum-Capacity with Minimum Energy Rate)**.** *The sum-capacity of the G-MAC without feedback and with minimum energy rate of* $0 < b \leqslant 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$ *Joules per channel use is*

$$R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \frac{1}{2}\log_2\Big(1 + \beta^{\dagger}(b)(\mathrm{SNR}_1 + \mathrm{SNR}_2)\Big) \tag{22}$$

with $\beta^{\dagger}(b) = \min\left\{1, \left(\frac{1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}-b}{2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}}\right)^+\right\}$.

Proof: The sum-rate maximization problem in (21) can be written as follows:

$$R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \max_{(\beta_1,\beta_2)\in[0,1]^2} f_0(\beta_1, \beta_2) \tag{23a}$$

$$\text{subject to:} \quad g_0(\beta_1, \beta_2) \geqslant b, \tag{23b}$$

where the functions f_0 and g_0 are defined as

$$f_0(\beta_1, \beta_2) \triangleq \min\left\{\frac{1}{2}\log_2(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2), \frac{1}{2}\log_2(1+\beta_1\mathrm{SNR}_1)+\frac{1}{2}\log_2(1+\beta_2\mathrm{SNR}_2)\right\} \tag{24}$$

and

$$g_0(\beta_1, \beta_2) \triangleq 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2}. \tag{25}$$

For any nonnegative β_1 and β_2 it can be shown that

$$f_0(\beta_1, \beta_2) = \frac{1}{2}\log_2(1 + \beta_1\mathrm{SNR}_1 + \beta_2\mathrm{SNR}_2), \tag{26}$$

and thus the function f_0 is monotonically increasing in (β_1, β_2). The function g_0 is motonically decreasing in (β_1, β_2).

Note that for any $(\beta_1, \beta_2) \in [0, 1]^2$, it follows that

$$g_0(\min(\beta_1, \beta_2), \min(\beta_1, \beta_2)) \geqslant g_0(\beta_1, \beta_2) \tag{27}$$

and

$$f_0(\min(\beta_1, \beta_2), \min(\beta_1, \beta_2)) \leqslant f_0(\beta_1, \beta_2) \leqslant f_0(\max(\beta_1, \beta_2), \max(\beta_1, \beta_2)) \tag{28}$$

with equality in (27) and (28) only when $\beta_1 = \beta_2$. Consequently, the sum-rate maximization problem (23) can also be written as

$$R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \max_{\beta\in[0,1]} f_0(\beta, \beta) \tag{29a}$$

$$\text{subject to:} \quad g_0(\beta, \beta) \geqslant b. \tag{29b}$$

This implies that the optimal $\beta^{\dagger}(b)$ is the highest feasible value,

$$\beta^{\dagger}(b) = \min\left\{1, \left(\frac{1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2} - b}{2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}}\right)^+\right\},$$

which implies that $R_{\mathrm{sum}}^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \frac{1}{2}\log_2\Big(1 + \beta^{\dagger}(b)(\mathrm{SNR}_1 + \mathrm{SNR}_2)\Big)$ and completes the proof. ■

Note that, if $0 \leqslant b \leqslant 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2$, then

$$1 \leqslant \frac{1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2} - b}{2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}}. \tag{30}$$

Hence, $\beta^{\dagger}(b) = 1$ and all the available power can be dedicated to the information transmission task.

Alternatively, if $1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 < b \leqslant 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}$, then

$$0 \leqslant \frac{1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2} - b}{2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}} < 1, \tag{31}$$

and there is a strict trade-off between information and energy transmission.

Finally, if $b > 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}$, then

$$\frac{1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2} - b}{2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}} < 0, \tag{32}$$

and thus, $\beta^{\dagger}(b) = 0$. Note that any $b > 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}$ is not achievable even if all the available power is used for energy transmission. In fact, the power budgets P_1 and P_2 at transmitters 1 and 2 limit inherently the energy rate at the input of the receiver to be at maximum $1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1 \mathrm{SNR}_2}$ which is obtained when both transmitters fully cooperate and use all their available power to transmit energy.

Remark 1. *Time-sharing with power-control suggested in Fouladgar et al.'s example [13] is strictly suboptimal and does not achieve sum-capacity for a given minimum received energy constraint.*

Consider the sum-rate optimization problem proposed in [13] in which both users use time-sharing with power control. Specifically, a time-sharing parameter $\lambda \in [0, 1]$ is chosen and each transmitter i chooses powers P_i' (power dedicated to information transmission) and P_i'' (power dedicated to energy transmission). The sum-rate maximization problem proposed in [13] can be written as (using the original notation)

$$\max_{(\lambda, P_1', P_1'', P_2', P_2'') \in [0,1] \times \mathbb{R}_+^4} \frac{\lambda}{2} \log_2 \left(1 + h_1^2 P_1' + h_2^2 P_2'\right) \tag{33a}$$

$$\text{subject to :} \quad \lambda P_i' + (1-\lambda) P_i'' \leqslant P_i, \quad i \in \{1, 2\} \tag{33b}$$

$$1 + \lambda(h_1^2 P_1' + h_2^2 P_2') + (1-\lambda)(h_1\sqrt{P_1''} + h_2\sqrt{P_2''})^2 \geqslant b, \tag{33c}$$

where P_i is the total power budget of transmitter i.

For any feasible choice of $(\lambda, P_1', P_1'', P_2', P_2'')$, by the concavity of the logarithm, it follows that

$$\frac{\lambda}{2} \log_2 \left(1 + h_1^2 P_1' + h_2^2 P_2'\right) \leqslant \frac{1}{2} \log_2 \left(1 + \lambda \left(h_1^2 P_1' + h_2^2 P_2'\right)\right). \tag{34}$$

Now, when $\lambda = 1$, then (34) holds with equality and the problem (33) can be written as

$$\max_{(P_1', P_2') \in \mathbb{R}_+^2} \frac{1}{2} \log_2 \left(1 + h_1^2 P_1' + h_2^2 P_2'\right) \tag{35a}$$

$$\text{subject to :} \quad P_i' \leqslant P_i, \quad i \in \{1, 2\}, \tag{35b}$$

$$1 + h_1^2 P_1' + h_2^2 P_2' \geqslant b, \tag{35c}$$

which is infeasible for any $b > 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2$. When $\lambda \in [0, 1)$, then (34) holds with strict inequality and the rate $\frac{1}{2}\log_2\left(1 + \lambda\left(h_1^2 P_1' + h_2^2 P_2'\right)\right)$ is always achievable by a power-splitting scheme in which $\beta_i = \lambda\frac{P_i'}{P_i}$, with $i \in \{1, 2\}$, for any optimal tuple $(\lambda, P_1', P_1'', P_2', P_2'')$ in (33). This shows that the maximum-sum-rate achieved via time-sharing with power control is always bounded away from the sum-capacity (Theorem 2), except when $\lambda = 1$. That is, time-sharing with power control is optimal in terms of sum-rate only when the energy constraint is not active, i.e., $b \in [0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2]$.

4 Main Results: Capacity-Energy Region with Feedback

4.1 Capacity-Energy Region with Feedback

The capacity-energy region of the MAC-FB is fully characterized by the following theorem.

Theorem 3 (Perfect Feedback Capacity-Energy Region)**.** *The perfect feedback capacity-energy region $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ of the Gaussian MAC is the set of non-negative information-energy rate triplets (R_1, R_2, B) that satisfy*

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathrm{SNR}_1\left(1 - \rho^2\right)\right) \quad (36a)$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \beta_2\,\mathrm{SNR}_2\left(1 - \rho^2\right)\right) \quad (36b)$$

$$R_1 + R_2 \leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathrm{SNR}_1 + \beta_2\,\mathrm{SNR}_2 + 2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}\right) \quad (36c)$$

$$B \leqslant 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho\sqrt{\beta_1\mathrm{SNR}_1\,\beta_2\mathrm{SNR}_2} + 2\sqrt{(1-\beta_1)\mathrm{SNR}_1\,(1-\beta_2)\mathrm{SNR}_2}, \quad (36d)$$

with $(\rho, \beta_1, \beta_2) \in [0, 1]^3$.

Remark 2. *The capacity-energy region without feedback described by Theorem 1 is identical to the capacity-energy region described by Theorem 3 in the case in which channel inputs are chosen to be mutually independent, i.e., $\rho = 0$. This suggests that the capacity-energy region without feedback is strictly included in the capacity-energy region with feedback, and thus, for any non-zero SNR_1 and SNR_2 it holds that*

$$\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2) \subset \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2). \quad (37)$$

The proof of Theorem 3 is presented in Section 5. The remainder of this subsection highlights some important observations about Theorem 3.

4.1.1 Example

Consider the capacity-energy region $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, with $\mathrm{SNR}_1 = \mathrm{SNR}_2 = 10$ presented in Fig. 4. Therein, left and right figures represent a bi-dimensional view in the R_1-R_2 and B-R_2 planes of $\mathcal{E}_{\mathrm{FB}}(10, 10)$, respectively. The figure in the center is a 3-D representation of $\mathcal{E}_{\mathrm{FB}}(10, 10)$. The triplet with the highest energy transmission rate is denoted by $Q_1 = (0, 0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2})$. The triplets Q_2, Q_3 and Q_6 guarantee information transmission at the sum-capacity, i.e., $R_1 + R_2 = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right)$. The triplets Q_2, Q_4 and Q_5 are coplanar and they satisfy that $B = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2$. In particular, $Q_4 = \left(\frac{1}{2}\log_2(1 + \mathrm{SNR}_1), 0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2\right)$ and $Q_5 = \left(\frac{1}{2}\log_2(1 + \mathrm{SNR}_1), \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_1}{1+\mathrm{SNR}_2}\right), 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2)\right)$ are achievable with and without feedback.

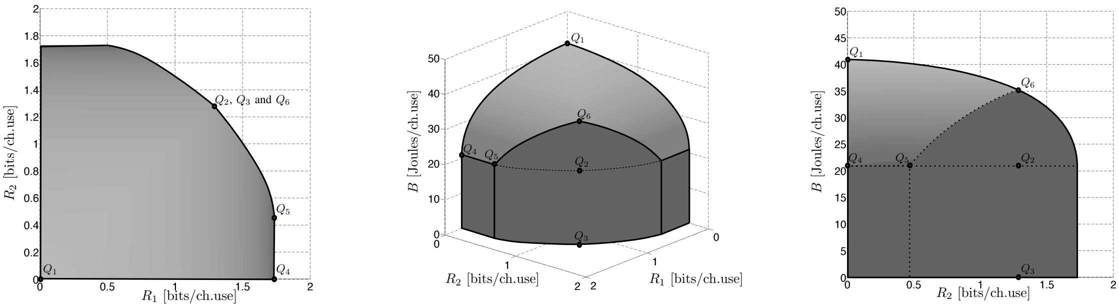

Figure 4: 3-D representation of the perfect feedback capacity-energy region of the G-MAC $\mathcal{E}_{\text{FB}}(10,10)$ in the coordinate system (R_1, R_2, B). Left and right figures represent a bi-dimensional view in the R_1-R_2 and B-R_2 planes of $\mathcal{E}_{\text{FB}}(10,10)$, respectively. The figure in the center is a 3-D representation of $\mathcal{E}_{\text{FB}}(10,10)$. Note that $Q_1 = (0, 0, 1 + \text{SNR}_1 + \text{SNR}_2 + 2\sqrt{\text{SNR}_1\text{SNR}_2})$; the points Q_2, Q_3 and Q_6 are collinear and satisfy that $R_1 + R_2 = \frac{1}{2}\log_2\left(1 + \text{SNR}_1 + \text{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\text{SNR}_1\text{SNR}_2}\right)$. The points Q_2, Q_4 and Q_5 are coplanar and they satisfy $B = 1 + \text{SNR}_1 + \text{SNR}_2$. In particular, $Q_4 = (\frac{1}{2}\log_2(1+\text{SNR}_1), 0, 1 + \text{SNR}_1 + \text{SNR}_2)$ and $Q_5 = (\frac{1}{2}\log_2(1+\text{SNR}_1), \frac{1}{2}\log_2\left(1 + \frac{\text{SNR}_1}{1+\text{SNR}_2}\right), 1 + \text{SNR}_1 + \text{SNR}_2))$.

4.1.2 Comments on the Achievability

The achievability scheme in the proof of Theorem 3 is based on power-splitting and Ozarow's capacity achieving scheme [24]. The parameters β_1 and β_2 in Theorem 3 are the fractions of average power that transmitters 1 and 2 allocate for information transmission. More specifically, transmitter i generates two signals: an information-carrying (IC) signal with average power $\beta_i P_i$ Joules per channel use; and a no-information-carrying (NIC) signal with power $(1-\beta_i)P_i$ Joules per channel use. The role of the NIC signal is exclusively energy transmission from the transmitter to the energy harvester. Conversely, the role of the IC signal is twofold: information transmission from the transmitter to the receiver and energy transmission from the transmitter to the energy harvester.

The parameter ρ is the average Pearson correlation coefficient between the IC signals sent by both transmitters. This parameter plays a fundamental role in both information transmission and energy transmission. Note for instance that the upper-bound on the energy harvested per unit time (36d) monotonically increases with ρ, whereas the upper-bounds on the individual rates (36a) and (36b) monotonically decrease with ρ. Note also that the Pearson correlation factor between the NIC signals of both transmitters does not appear in Theorem 3. This is mainly because maximum energy transmission occurs using NIC signals that are fully correlated and thus, the corresponding Pearson correlation coefficient is one. Similarly, the Pearson correlation factor between the NIC signal of transmitter i and the IC signal of transmitter j, with $j \in \{1,2\}$ and $j \neq i$, does not appear in Theorem 3 either. This stems from the fact that, without loss of optimality, NIC signals can be chosen to be independent of the messages M_1 and M_2 as well as the noise sequences, and known by both the receiver and the transmitters. Hence, NIC signals can be independent of the IC signals and more importantly, the interference they create at the receiver can be easily eliminated. Under this assumption, a power-splitting $(\beta_1, \beta_2) \in [0,1]^2$ guarantees the achievability of non-negative rate pairs (R_1, R_2) satisfying (36a)-(36c) by simply using Ozarow's capacity achieving scheme. At the energy harvester, both the IC and NIC signals contribute to the total energy harvested (10). The IC signal is able to convey at most $\beta_1\text{SNR}_1 + \beta_2\text{SNR}_2 + 2\rho\sqrt{\beta_1\text{SNR}_1\beta_2\text{SNR}_2}$ Joules per channel use, while the NIC signal is able to convey at most $(1-\beta_1)\mathrm{SNR}_1 + (1-\beta_2)\mathrm{SNR}_2 + 2\sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2}$ Joules per channel use. The sum of these two contributions as well as the contribution of the noise at the energy harvester justifies the upper-bound on the energy transmission rate in (36d).

If $\beta_1 \neq 0$ and $\beta_2 \neq 0$, let $\rho^\star(\beta_1, \beta_2)$ be the unique solution in $(0,1)$ to the following equality:

$$1 + \beta_1\mathrm{SNR}_1 + \beta_2\mathrm{SNR}_2 + 2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2} = \big(1+\beta_1\mathrm{SNR}_1(1-\rho^2)\big)\big(1+\beta_2\mathrm{SNR}_2(1-\rho^2)\big), \quad (38)$$

otherwise, let $\rho^\star(\beta_1, \beta_2) = 0$.

Note that for any power-splitting $(\beta_1, \beta_2) \in (0,1]^2$, the left hand side of (38) is monotonically increasing with ρ whereas the right hand side is monotonically decreasing with ρ. This implies that $\rho^\star(\beta_1, \beta_2)$ is a maximizer of the sum-rate. More specifically, at $\rho = \rho^\star(\beta_1, \beta_2)$, the sum of (36a) and (36b) is equal to (36c) and it corresponds to the sum-capacity of the G-MAC with feedback.

4.2 Individual Information Rates with Feedback and Minimum Energy Rate

The maximum individual rate $R_i^{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b)$, with $i \in \{1,2\}$, of the Gaussian MAC with perfect feedback, with parameters SNR_1 and SNR_2, given an energy rate constraint of b Joules per channel use at the input of the energy harvester is the solution to an optimization problem of the form

$$R_i^{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \max_{(r_i, r_j, c) \in \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2): c \geqslant b} r_i. \quad (39)$$

The solution to the optimization problem (39) is given by the following proposition.

Proposition 2 (Maximum Individual-Rate with Feedback and Minimum Energy Rate)**.** *Consider a G-MAC-F with parameters* SNR_1 *and* SNR_2*. For a given required minimum energy rate b, the maximum individual rates with feedback coincide with the maximum individual rates without feedback in Proposition 1. That is,*

$$R_i^{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = R_i^{\mathrm{NF}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b). \quad (40)$$

Proof: From the assumptions of Proposition 2 it follows for a given energy transmission rate of b Joules per channel use, a power-split (β_1, β_2) is feasible if there exists at least one $\rho \in [0,1]$ that satisfies

$$1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{(1-\beta_1)\mathrm{SNR}_1\,(1-\beta_2)\mathrm{SNR}_2} + 2\rho\sqrt{\beta_1\mathrm{SNR}_1\,\beta_2\mathrm{SNR}_2} \geqslant b. \quad (41)$$

Consider a feasible power-split in which transmitter j only transmits energy to the harvester, i.e., $\beta_j = 0$ and transmitter i transmits both information and energy, i.e., $\beta_i > 0$. Using a similar argument as in the proof of Proposition 1, it can be shown that the individual rates with feedback are limited by $R_i^{\mathrm{FB}} \leqslant \frac{1}{2}\log_2\left(1 + \beta^\star(b)\mathrm{SNR}_i\right)$ where $\beta^\star(b)$ is given by (19). ■

The relevance of Proposition 2 stems from the insights it provides to the understanding of the shape of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ in comparison to the shape of the volume $\mathcal{E}(\mathrm{SNR}_1, \mathrm{SNR}_2)$. Even if feedback does not increase the maximal individual rates that can be achieved for a given received energy rate b, it will be shown in the sequel that it increases the sum-rate that can be achieved (See Theorem 4).

Note that if $b \in [0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2]$, then $\beta^\star(b) = 1$ and thus, the energy constraint does not add any additional bound on the individual rates other than (36a) and (36b). In general, any intersection of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$, in the Cartesian coordinates (R_1, R_2, B), with

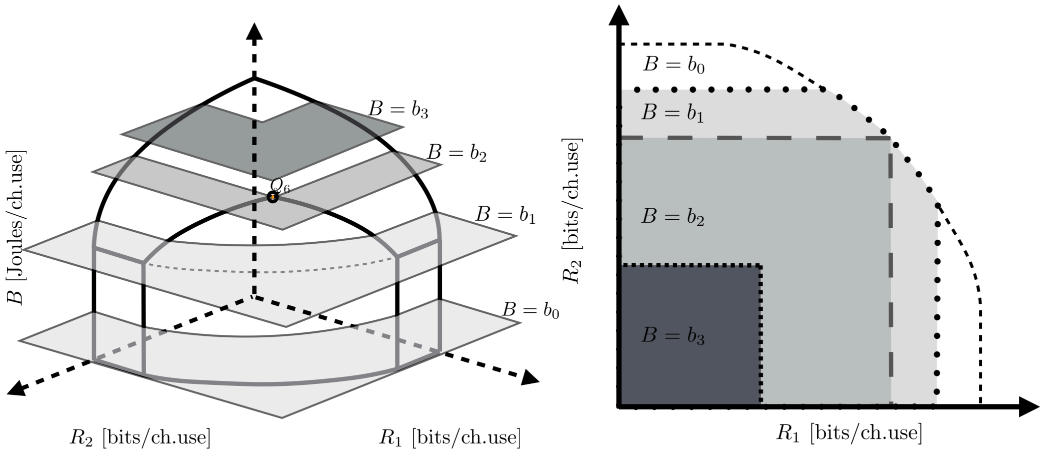

Figure 5: Intersection of the planes $B = b_1$, $B = b_2$ and $B = b_3$ with the capacity-energy region $\mathcal{E}_{\mathrm{FB}}\left(\mathrm{SNR}_1, \mathrm{SNR}_2\right)$, with $SNR_1 = SNR_2$, $b_0 \in \left[0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2\right]$, $b_1 \in \left[1 + \mathrm{SNR}_1 + \mathrm{SNR}_2, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right]$, $b_2 = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$ and $b_3 \in \left[1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2},\right]$.

a plane $B = b \in [0, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2]$ corresponds to the set of triplets (R_1, R_2, b), in which the corresponding pairs (R_1, R_2) form a set that is identical to the capacity region of the G-MAC with feedback, denoted by $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$. Fig. 5 shows a general example of this intersection when $\mathrm{SNR}_1 = \mathrm{SNR}_2$. In this case, transmitting information using all the available power budget is always enough to satisfy the energy constraint and thus, all information-transmission rate pairs of the capacity region $\mathcal{C}_{\mathrm{FB}}\left(\mathrm{SNR}_1, \mathrm{SNR}_2\right)$ are achievable. In the particular case in which $\mathrm{SNR}_1 = \mathrm{SNR}_2 = 10$ (Fig. 4), the sum-capacity points Q_2 and Q_3, as well as the corner points Q_4 and Q_5 are achievable by using Ozarow's scheme without any power-splitting, i.e., $\beta_1 = \beta_2 = 1$.

In the case in which $b \in (1 + \mathrm{SNR}_1 + \mathrm{SNR}_2, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$, if follows that $1 - \left(\rho^{\star}(1,1)\right)^2 \leqslant \beta^{\star}(b) < 1$ and thus, the energy constraint limits the individual rates. That is, the individual rate is bounded away from $\frac{1}{2}\log_2\left(1 + \mathrm{SNR}_i\right)$. The effect of these bounds can be seen in Fig. 4. Let $\mathcal{B}(b) \subset \mathbb{R}_+^2$ be a box of the form

$$\mathcal{B}(b) = \left\{(R_1, R_2) \in \mathbb{R}_+^2 : R_i \leqslant \frac{1}{2}\log_2\left(1 + \beta^{\star}(b)\mathrm{SNR}_i\right), i \in \{1, 2\}\right\}. \tag{42}$$

In general, any intersection of the volume $\mathcal{E}_{\mathrm{FB}}\left(\mathrm{SNR}_1, \mathrm{SNR}_2\right)$ with a plane $B = b \in (1 + \mathrm{SNR}_1 + \mathrm{SNR}_2, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$ is a set of triplets (R_1, R_2, b) for which the corresponding pairs (R_1, R_2) satisfy

$$(R_1, R_2) \in \mathcal{B}(b) \cap \mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2), \tag{43}$$

which form a proper subset of $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$. It is important to highlight that in this case, the sum-capacity of $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$ can always be achievable. That is, the power-split $\beta_1 = \beta_2 = 1$ is always feasible. Consider for instance the triplet Q_6 in Fig. 4. The triplet Q_6 is achievable by using Ozarow's perfect feedback capacity-achieving scheme without any power-splitting, since only information transmission satisfies the energy constraint. Fig. 5 shows the intersection of $\mathcal{E}_{\mathrm{FB}}\left(\mathrm{SNR}_1, \mathrm{SNR}_2\right)$ and a plane $B = b$, with $b \in (1 + \mathrm{SNR}_1 + \mathrm{SNR}_2, 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$. Note that this intersection always includes the triplet (R_1, R_2, b), with $R_1 + R_2 = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right)$, i.e., the sum-capacity.

Finally, in the case in which $b \in (1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$, it follows that $0 \leqslant \beta^\star(b) < 1-(\rho^\star(1,1))^2$, and thus, the individual rates are limited by $R_i < \frac{1}{2}\log_2\left(1+\left(1-(\rho^\star(1,1))^2\right)\mathrm{SNR}_i\right)$. This immediately implies that any intersection of the volume $\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2)$ with a plane $B = b \in (1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$ is a set of triplets (R_1,R_2,b) for which the corresponding pairs (R_1,R_2) satisfy

$$(R_1,R_2) \in \mathcal{B}(b) = \mathcal{B}(b) \cap \mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2). \tag{44}$$

In this case, the set $\mathcal{B}(b)$ is a proper subset of $\mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2)$ and it does not contain the sum-rate pair $(R_1,R_2) \in \mathcal{C}_{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2)$. This is clearly shown by Fig. 5. Indeed, for any $b > 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$, the set $\mathcal{B}(b)$ monotonically shrinks with b.

4.3 Feedback Sum-Capacity with Minimum Energy Constraints

The perfect feedback sum-capacity $R_{\mathrm{sum}}(\mathrm{SNR}_1,\mathrm{SNR}_2,b)$ of the Gaussian MAC, with parameters SNR_1 and SNR_2, given an energy rate constraint of b Joules per channel use at the input of the energy harvester is the solution to

$$R_{\mathrm{sum}}^{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2,b) = \max_{(r_1,r_2,c)\in\mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2):c\geqslant b} r_1+r_2. \tag{45}$$

The solution to this problem is given by the following theorem.

Theorem 4 (Perfect Feedback Sum-Capacity)**.** *Let b be the minimum energy rate that must be guaranteed at the input of the energy harvester. Then, the perfect feedback sum-capacity of the Gaussian MAC, with parameters* SNR_1 *and* SNR_2 *is*

1. for all $b \in \left[0, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right]$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2,b) = \frac{1}{2}\log_2(1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}); \tag{46}$$

2. for all $b \in \left(1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right)$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2,b) = \frac{1}{2}\log_2(1+\beta^\star(b)\mathrm{SNR}_1)+\frac{1}{2}\log_2(1+\beta^\star(b)\mathrm{SNR}_2); \tag{47}$$

3. for all $b \in \left[1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, \infty\right]$,

$$R_{\mathrm{sum}}^{\mathrm{FB}}(\mathrm{SNR}_1,\mathrm{SNR}_2,b) = 0, \tag{48}$$

with the function $\beta^\star$ defined in (19)*.*

One of the key observations from Theorem 4 is that if the minimum energy rate b required at the input of the energy harvester is less than what is needed for transmitting information using all the available power budget at the maximum sum-rate, i.e., $b \leqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$, the minimum energy constraint does not have any impact on the sum-rate (see Fig. 6). That is, in this case the requirement of a minimum energy transmission rate is automatically met by exclusively transmitting information at the sum-capacity.

Nonetheless, when the energy rate required at the input of the receiver is $b \geqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$, then there exists a loss of sum-rate induced by the fact that at least one of the fractions β_1 and β_2 is smaller than one. More specifically, for these values of b , $R_i \leqslant \frac{1}{2}\log_2\left(1+(1-(\rho(1,1)^\star)^2)\mathrm{SNR}_i\right)$ for at least one $i \in \{1,2\}$ and thus, the sum-rate is strictly smaller than the sum capacity.

Clearly, the maximum energy rate is achieved when $\beta_1 = \beta_2 = 0$, which implies that no information is conveyed from the transmitters to the receiver (see Fig. 6).

4.4 Energy Transmission Enhancement with Feedback

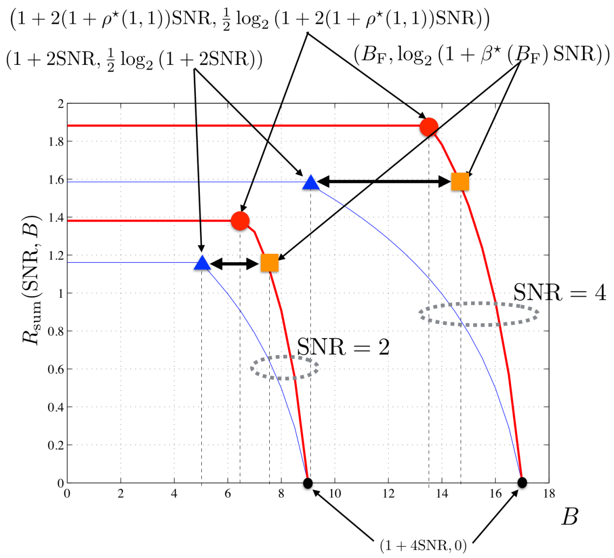

Figure 6: Sum-capacity of the symmetric two-user memoryless G-MAC with $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}$ with feedback (thick red line) and without feedback (thin blue line) as a function of B. Red (big) circles represent the pairs $(R_{\mathrm{sum}}^{\mathrm{FB}}, B_1)$ in which $R_{\mathrm{sum}}^{\mathrm{FB}}$ is the sum capacity with feedback when only information transmission is performed and $B_1 \triangleq 1 + 2(1+\rho^\star(1,1))\mathrm{SNR}$ represents the corresponding maximum energy rate that can be guaranteed at the receiver. Blue triangles represent the pairs $(R_{\mathrm{sum}}^{\mathrm{NF}}, B_{\mathrm{NF}})$ in which $R_{\mathrm{sum}}^{\mathrm{NF}}$ is the sum-capacity without feedback and B_{NF} is the corresponding maximum energy rate that can be guaranteed at the receiver without feedback. Orange squares represent the pairs $(R_{\mathrm{sum}}^{\mathrm{NF}}, B_{\mathrm{F}})$ in which B_{F} is the corresponding maximum energy rate that can be guaranteed at the receiver with feedback. Black (small) circles represent the pairs $(0, B_{\max})$ in which $B_{\max} \triangleq 1 + 4\mathrm{SNR}$ is the maximum energy rate that can be guaranteed at the receiver.

In this subsection, the enhancement on the energy transmission rate due to the use of feedback is quantized when the sum-rate is the sum-capacity without feedback (see the blue triangles and orange squares in Fig. 6). Denote by $B_{\mathrm{NF}} = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2$ the maximum energy rate that can be guaranteed at the receiver in the G-MAC without feedback when the sum-rate is the corresponding sum-capacity without feedback. Denote also by B_{F} the maximum energy rate that can be guaranteed at the receiver in the G-MAC with feedback when the sum-rate is the sum-capacity without feedback. The exact value of B_{F} is given by the following lemma.

Lemma 1. *The maximum energy rate B_{F} that can be guaranteed at the energy harvester in the G-MAC with feedback when the sum-rate is the sum-capacity without feedback is*

$$B_{\mathrm{F}} = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{(1-\gamma)\mathrm{SNR}_1\mathrm{SNR}_2}, \tag{49}$$

with $\gamma \in (0,1)$ defined as follows:

$$\gamma = -\frac{\mathrm{SNR}_1 + \mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2} + \sqrt{\left(\frac{\mathrm{SNR}_1 + \mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2}\right)^2 + 2\left(\frac{\mathrm{SNR}_1 + \mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2}\right)}. \tag{50}$$

Proof: The maximum energy transmission rate B_{F} when the information sum-rate is the sum-capacity of the G-MAC without feedback satisfies

$$
\begin{aligned}
&B_{\mathrm{F}} = \max_{(\beta_1,\beta_2,\rho)} 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\Big(\rho\sqrt{\beta_1\beta_2}+\sqrt{(1-\beta_1)(1-\beta_2)}\Big)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\\
&\text{subject to :}\\
&\log_2\left(1+\mathrm{SNR}_1+\mathrm{SNR}_2\right) = \min\Big(\log_2\left(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2+2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}\right),\\
&\qquad\qquad \log_2\left(1+\beta_1\mathrm{SNR}_1\left(1-\rho^2\right)\right)+\log_2\left(1+\beta_2\mathrm{SNR}_1\left(1-\rho^2\right)\right)\Big). \quad (51)
\end{aligned}
$$

The solution B_{F} to the optimization problem in (51) satisfies that $B_{\mathrm{F}} \in (1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}, 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}]$. Thus, from Theorem 2, it follows that the sum-rate is exclusively upper bounded by $R_1+R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta^{\star}(B_{\mathrm{F}})\mathrm{SNR}_1\right)+\frac{1}{2}\log_2\left(1+\beta^{\star}(B_{\mathrm{F}})\mathrm{SNR}_2\right)$, with the function $\beta^{\star}:\mathbb{R}_+\to[0,1]$ defined in (19). Note that $\beta^{\star}$ is monotonically decreasing with its argument. Thus, the optimization problem in (51) can be rewritten as follows:

$$
\begin{aligned}
&B_{\mathrm{F}} = \max_{b\in(b_1,b_2]} b\\
&\text{subject to :}\\
&\frac{1}{2}\log_2\left(1+\mathrm{SNR}_1+\mathrm{SNR}_2\right) = \frac{1}{2}\log_2\left(1+\beta^{\star}(b)\mathrm{SNR}_1\right)+\frac{1}{2}\log_2\left(1+\beta^{\star}(b)\mathrm{SNR}_2\right). \quad (52)
\end{aligned}
$$

where $b_1 = 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho^{\star}(1,1)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$ and $b_2 = 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}$. The constraint of the problem (52) induces a unique value for $\beta^{\star}(b)$ within $[0,1]$ for each b, and thus, the optimization is vacuous. This implies that the unique solution B_{F} satisfies

$$
\beta^{\star}(B_{\mathrm{F}}) = -\frac{\mathrm{SNR}_1+\mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2}+\sqrt{\left(\frac{\mathrm{SNR}_1+\mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2}\right)^2+2\left(\frac{\mathrm{SNR}_1+\mathrm{SNR}_2}{2\mathrm{SNR}_1\mathrm{SNR}_2}\right)}. \tag{53}
$$

Following the definition of $\beta^{\star}$ in (19) and solving for B_{F} in (53) yields (49). This completes the proof of Lemma 1. ■

The following theorem provides an upper bound on the ratio $\frac{B_{\mathrm{NF}}}{B_{\mathrm{F}}}$.

Theorem 5 (Maximum Energy Rate Improvement with Feedback)**.** *Feedback can at most double the energy transmission rate. That is,*

$$
1 \leqslant \frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}} < 2. \tag{54}
$$

Proof: The proof of Theorem 5 follows immediately from Lemma 1. The lower-bound follows from the fact that $\rho > 0$ with strict inequality (Lemma 1). Moreover, note that the ratio $\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}}$ satisfies

$$
\begin{aligned}
\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}} &= \frac{1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{(1-\gamma)\mathrm{SNR}_1\mathrm{SNR}_2}}{1+\mathrm{SNR}_1+\mathrm{SNR}_2} && (55)\\
&\overset{(a)}{<} 1+\frac{2\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}}{1+\mathrm{SNR}_1+\mathrm{SNR}_2} && (56)\\
&\overset{(b)}{\leqslant} 1+\frac{\mathrm{SNR}_1+\mathrm{SNR}_2}{1+\mathrm{SNR}_1+\mathrm{SNR}_2} && (57)\\
&< 2, && (58)
\end{aligned}
$$

where (a) follows from the fact that $\gamma \in (0,1)$ (Lemma 1); and (b) follows from the inequality $\left(\sqrt{\mathrm{SNR}_1} - \sqrt{\mathrm{SNR}_2}\right)^2 \geqslant 0$. This completes the proof of Theorem 5. ■

Fig. 7 compares the exact value of the ratio $\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}}$ in (55) to the upper-bound in (58) as a function of the SNRs in the symmetric case, i.e., $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}$. Note that the upper-bound in (58) is tight as the ratio $\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}}$ becomes arbitrarily close to two as SNR tends to infinity. In the non-symmetric cases $\mathrm{SNR}_1 \neq \mathrm{SNR}_2$, this bound is loose.

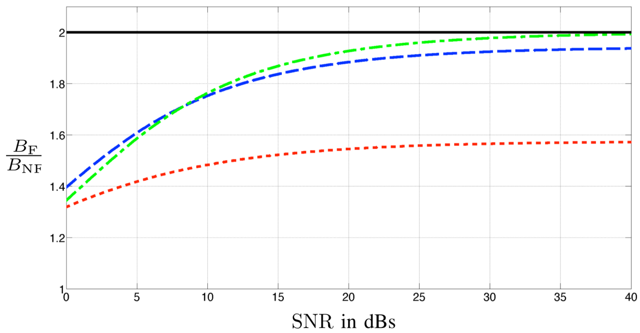

Figure 7: The ratio $\frac{B_{\mathrm{NF}}}{B_{\mathrm{N}}}$ as a function of SNR. The thickest line is the upper bound in (58); the dash-dotted line, the dashed line and the dotted line are the exact values of the ratio $\frac{B_{\mathrm{NF}}}{B_{\mathrm{N}}}$ in (55) when $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}$; $\frac{\mathrm{SNR}_1}{2} = \mathrm{SNR}_2 = \mathrm{SNR}$; and $\frac{\mathrm{SNR}_1}{10} = \mathrm{SNR}_2 = \mathrm{SNR}$, respectively.

5 Proof of Theorem 3

The proof is divided into two parts: achievability and converse parts.

5.1 Proof of Achievability

The proof of achievability uses a very simple power-splitting technique in which a fraction $\beta_i \in [0,1]$ of the power is used for information transmission and the remaining fraction $(1-\beta_i)$ for energy transmission. The information transmission is made following Ozarow's perfect feedback capacity-achieving scheme in [24]. The energy transmission is accomplished by random symbols that are known at both transmitters and the receiver. Despite its simplicity and a great deal of similarity with the scheme in [24], the complete proof is fully described hereunder for the sake of completeness.

Codebook generation: Before transmission starts, each message M_i, is mapped into the real-valued message point

$$\Theta_i(M_i) \triangleq -(M_i - 1)\Delta_i + \sqrt{P_i}, \tag{59}$$

where $\Delta_i \triangleq \frac{2\sqrt{P_i}}{\lfloor 2^{nR_i} \rfloor}$.

Encoding: The first three channel uses are part of an initialization procedure during which there is no energy transmission and the channel inputs are

$$
\begin{aligned}
t = -2 &: X_{1,-2} = 0 && \text{and} && X_{2,-2} = \Theta_2(M_2), && \text{(60a)}\\
t = -1 &: X_{1,-1} = \Theta_1(M_1) && \text{and} && X_{2,-1} = 0, && \text{(60b)}\\
t = 0 &: X_{1,0} = 0 && \text{and} && X_{2,0} = 0. && \text{(60c)}
\end{aligned}
$$

Through the feedback links, transmitter 1 observes (Z_{-1}, Z_0) and transmitter 2 observes (Z_{-2}, Z_0). After the initialization phase, each transmitter $i \in \{1, 2\}$ can thus compute

$$\Xi_i \triangleq \sqrt{1-\rho^\star(\beta_1,\beta_2)} \cdot Z_{-i} + \sqrt{\rho^\star(\beta_1,\beta_2)} \cdot Z_0, \tag{61}$$

where $\rho^\star(\beta_1, \beta_2)$ is the unique solution in $(0, 1)$ to (38).

During the remaining channel uses $1, \ldots, n$, for $i \in \{1, 2\}$, instead of repeating the message-point $\Theta_i(M_i)$, transmitter i simultaneously describes Ξ_i to the receiver and transmits energy to the energy harvester. Let β_i, with $i \in \{1, 2\}$ be the power-splitting coefficient of transmitter i. More specifically, at each time $t \in \{1, \ldots, n\}$, transmitter i sends

$$X_{i,t} = U_{i,t} + \sqrt{(1-\beta_i)P_i} W_t, \quad i \in \{1, 2\}. \tag{62}$$

Here W_t is a Gaussian zero-mean unit-variance random variable that is known non-causally to the transmitters and to the receiver and is is independent of the messages and the noise sequences. The symbol $U_{i,t}$ is a zero-mean Gaussian random variable with variance $\beta_i P_i$ and is chosen as follows:

$$U_{i,1} = \sqrt{\beta_i P_i}\, \Xi_i, \tag{63a}$$

$$U_{i,t} = \gamma_{i,t}\left(\Xi_i - \hat{\Xi}_i^{(t-1)}\right), \quad t \in \{2, \ldots, n\}, \tag{63b}$$

where the parameter $\gamma_{i,t}$ is chosen to satisfy $\mathsf{E}\left[U_{i,t}^2\right] = \beta_i P_i$ and $\hat{\Xi}_i^{(t-1)}$ is explained below.

For each $t \in \{1, \ldots, n\}$, upon receiving the channel output Y_t, the receiver subtracts the signal induced by the commom randomness to form the observation Y_t' as follows:

$$Y_t' \triangleq Y_t - (\sqrt{(1-\beta_1)P_1} + \sqrt{(1-\beta_2)P_2})W_t. \tag{64}$$

The receiver then calculates the minimum mean square error (MMSE) estimate $\hat{\Xi}_i^{(t-1)} = \mathsf{E}\left[\Xi_i | Y_1', \ldots, Y_{t-1}'\right]$ of Ξ_i given the prior observations $Y_1', \ldots, Y_{t-1}'$.

By the orthogonality principle of MMSE estimation, $(U_{1,t}, U_{2,t}, Z_t)$ are independent of the observations $Y_1', \ldots, Y_{t-1}'$ and thus of $Y_1, \ldots, Y_{t-1}$. Let ρ_t denote the correlation coefficient between $U_{1,t}$ and $U_{2,t}$, i.e., $\rho_t \triangleq \frac{\mathsf{E}[U_{1,t}U_{2,t}]}{\sqrt{\mathsf{E}[U_{1,t}^2]}\sqrt{\mathsf{E}[U_{2,t}^2]}}$. It can be shown [25] that for all $t \in \{1, \ldots, n\}$, $\rho_t = \rho^\star(\beta_1, \beta_2)$.

After reception of the output symbols $Y_{-2}, \ldots, Y_n$, the receiver forms $\hat{\Xi}_i^{(n)} \triangleq \mathsf{E}\left[\Xi_i | Y_1', \ldots, Y_n'\right]$, for $i \in \{1, 2\}$. Then, it forms an estimate of the message point $\Theta_i(M_i)$ as follows:

$$\begin{aligned}\hat{\Theta}_i &\triangleq \frac{1}{h_i}\left(Y_{-i} + \frac{\sqrt{\rho^\star(\beta_1,\beta_2)}}{\sqrt{1-\rho^\star(\beta_1,\beta_2)}} Y_0 - \frac{1}{\sqrt{1-\rho^\star(\beta_1,\beta_2)}} \hat{\Xi}_i^{(n)}\right)\\ &= \Theta_i(M_i) + \frac{1}{h_i\sqrt{1-\rho^\star(\beta_1,\beta_2)}}\left(\Xi_i - \hat{\Xi}_i^{(n)}\right).\end{aligned} \tag{65}$$

Finally, the message index estimate M_i is obtained using nearest-neighbor decoding based on the value $\hat{\Theta}_i$, as follows:

$$\hat{M}_i = \underset{m_i \in \{1, \ldots, \lfloor 2^{nR_i} \rfloor\}}{\operatorname{argmin}} \left|\Theta_i(m_i) - \hat{\Theta}_i\right|. \tag{66}$$

Analysis of the probability of error:

An error occurs whenever the receiver is not able to recover one of the messages, i.e., $(M_1, M_2) \neq (\hat{M}_1, \hat{M}_2)$ or if the received energy rate is below the desired minimum rate $B^{(n)} < B$.

First, consider the probability of a decoding error. Note that for $i \in \{1,2\}$, $\hat{M}_i = M_i$, if

$$|\Xi_i - \hat{\Xi}_i^{(n)}| \leqslant \frac{h_i\sqrt{1-\rho^\star(\beta_1,\beta_2)}\Delta_i}{2}. \tag{67}$$

Since the difference $\Xi_i - \hat{\Xi}_i^{(n)}$ is a centered Gaussian random variable, by the definition of Δ_i, the error probability $P_{e,i}$ while decoding message index M_i can be bounded as

$$P_{e,i} \leqslant 2\mathcal{Q}\left(\frac{h_i\sqrt{P_i}\sqrt{1-\rho^\star(\beta_1,\beta_2)}}{\lfloor 2^{nR_i}\rfloor\sqrt{(\sigma_i^{(n)})^2}}\right), \tag{68}$$

where

$$(\sigma_i^{(n)})^2 \triangleq \mathsf{E}\left[|\Xi_i - \hat{\Xi}_i^{(n)}|^2\right], \quad i \in \{1,2\}. \tag{69}$$

By the joint Gaussianity of Ξ_i and $\mathbf{Y}'$, the MMSE estimate $\hat{\Xi}_i^{(n)}$ is a linear function of $\mathbf{Y}'$ (see, e.g., [26]). Moreover, by the orthogonality principle, the MMSE $\Xi_i - \hat{\Xi}_i^{(n)}$ is independent of the observations $\mathbf{Y}'$. Hence, we have

$$\begin{aligned} I(\Xi_i;\mathbf{Y}') &= h(\Xi_i) - h(\Xi_i|\mathbf{Y}') \\ &\overset{(a)}{=} h(\Xi_i) - h(\Xi_i - \hat{\Xi}_i^{(n)}|\mathbf{Y}') \\ &\overset{(b)}{=} h(\Xi_i) - h(\Xi_i - \hat{\Xi}_i^{(n)}) \\ &= -\frac{1}{2}\log_2\big((\sigma_i^{(n)})^2\big), \end{aligned} \tag{70}$$

where (a) holds because $\hat{\Xi}_i^{(n)}$ is a function of $\mathbf{Y}'$, and (b) follows because $\Xi_i - \hat{\Xi}_i^{(n)}$ is independent $\mathbf{Y}'$.

Equation (70) can equivalently be written as

$$\sqrt{(\sigma_i^{(n)})^2} = 2^{-I(\Xi_i;\mathbf{Y}')}. \tag{71}$$

Combining (68) with (71) yields that the probability of error of message M_i tends to 0 as $n \to \infty$, if the rate R_i satisfies

$$R_i \leqslant \varliminf_{n\to\infty} \frac{1}{n} I(\Xi_i;\mathbf{Y}'), \qquad i \in \{1,2\}. \tag{72}$$

On the other hand, as proved in [25, Section 17.2.4],

$$I(\Xi_i;\mathbf{Y}') = \sum_{t=1}^{n} I(U_{i,t};Y'_t) \tag{73}$$

and irrespective of n and $t \in \{1,\dots,n\}$, it holds that

$$I(U_{i,t};Y'_t) = \frac{1}{2}\log_2\left(1 + \beta_i \mathrm{SNR}_i(1-(\rho^\star(\beta_1,\beta_2))^2)\right). \tag{74}$$

Hence, for $i \in \{1,2\}$ it holds that

$$\varliminf_{n\to\infty} \frac{1}{n} I(\Xi_i;\mathbf{Y}') = \frac{1}{2}\log_2\left(1 + \beta_i \mathrm{SNR}_i(1-(\rho^\star(\beta_1,\beta_2))^2)\right), \tag{75}$$

and the limit inferior is a proper limit.

Combining, (75) and (72) yields that when $n \to \infty$, this scheme can achieve all non-negative rate-pairs (R_1, R_2) that satisfy

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathrm{SNR}_1(1 - \rho^\star(\beta_1,\beta_2)^2)\right), \tag{76a}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \beta_2\,\mathrm{SNR}_2(1 - \rho^\star(\beta_1,\beta_2)^2)\right). \tag{76b}$$

and by (38) it automatically yields

$$R_1 + R_2 \leqslant \frac{1}{2}\log_2\big(1 + \beta_1\,\mathrm{SNR}_1 + \beta_2\,\mathrm{SNR}_2 + 2\rho^\star(\beta_1,\beta_2)\sqrt{\beta_1\,\mathrm{SNR}_1 \cdot \beta_2\,\mathrm{SNR}_2}\big). \tag{76c}$$

Furthermore, the total consumed power at transmitter i for $i \in \{1,2\}$ over the $n+3$ channel uses is upper bounded by $(n+1)P_i$, hence, this scheme satisfies the input-power constraints.

Average received power:

The average received power is given by $B^{(n)} \triangleq \frac{1}{n}\sum_{t=1}^{n} S_t^2$.

By the choice of the random variables, the sequence $S_1, \ldots, S_n$ is independently and identically distributed (i.i.d.) and each S_t follows a zero-mean Gaussian distribution with variance B given by

$$B \triangleq \mathsf{E}\left[S_t^2\right] = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}\;\rho^\star(\beta_1,\beta_2) + \sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2}. \tag{77}$$

By the weak law of large numbers, it holds that $\forall \epsilon > 0$

$$\lim_{n\to\infty} \Pr\left(|B^{(n)} - B| > \epsilon\right) = 0. \tag{78}$$

Consequently,

$$\lim_{n\to\infty} \Pr\left(B^{(n)} > B + \epsilon\right) = 0, \quad \text{and} \tag{79a}$$

$$\lim_{n\to\infty} \Pr\left(B^{(n)} < B - \epsilon\right) = 0. \tag{79b}$$

From (79b), it holds that $\forall b \in [0, B]$,

$$\lim_{n\to\infty} \Pr\left(B^{(n)} < b - \epsilon\right) = 0. \tag{80}$$

To sum up, any information-energy rate triplet (R_1, R_2, B) that satisfies

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathrm{SNR}_1(1 - (\rho^\star(\beta_1,\beta_2))^2)\right) \tag{81a}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \beta_2\,\mathrm{SNR}_2(1 - (\rho^\star(\beta_1,\beta_2))^2)\right) \tag{81b}$$

$$R_1 + R_2 \leqslant \frac{1}{2}\log_2\big(1 + \beta_1\,\mathrm{SNR}_1 + \beta_2\,\mathrm{SNR}_2 + 2\rho^\star(\beta_1,\beta_2)\sqrt{\beta_1\,\mathrm{SNR}_1 \cdot \beta_2\,\mathrm{SNR}_2}\big) \tag{81c}$$

$$B \leqslant 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\big(\sqrt{\beta_1\beta_2}\;\rho^\star(\beta_1,\beta_2) + \sqrt{(1-\beta_1)(1-\beta_2)}\big)\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2} \tag{81d}$$

is achievable.

To achieve other points in the capacity-energy region, transmitter 1 can split its message M_1 into two independent submessages $(M_{1,0}, M_{1,1}) \in \{1,\ldots,2^{nR_{1,0}}\} \times \{1,\ldots,2^{nR_{1,1}}\}$ such that $R_{1,0}, R_{1,1} \geq 0$ and $R_{1,0} + R_{1,1} = R_1$. It uses a power fraction $\alpha_1 \in [0,1]$ of its available information-dedicated power $\beta_1 P_1$ to transmit $M_{1,0}$ using a non-feedback Gaussian random code and uses the remaining power $(1-\alpha_1)\beta_1 P_1$ to send $M_{1,1}$ using the sum-capacity-achieving feedback scheme while treating $M_{1,0}$ as noise. Transmitter 2 sends its message M_2 using the sum-capacity-achieving feedback scheme.

Transmitter 1's IC-input is $U_{1,t} \triangleq U_{1,0,t} + U_{1,1,t}$ where $U_{1,1,t}$ is defined as in (63) but with reduced power $(1-\alpha_1)\beta_1 P_1$, and $U_{1,0,t}$ is an independent zero-mean Gaussian random variable with variance $\alpha_1\beta_1 P_1$. Transmitter 2's IC-input is defined as in (63).

The receiver first subtracts the common randomness and then decodes $(M_{1,1}, M_2)$ treating the signal encoding $M_{1,0}$ as noise. Successful decoding is possible if

$$R_{1,1} \leqslant \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_1)\beta_1 \mathrm{SNR}_1 (1-\rho_{\alpha_1}(\beta_1,\beta_2)^2)}{1+\alpha_1\beta_1\mathrm{SNR}_1}\right) \tag{82a}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \frac{\beta_2 \mathrm{SNR}_2 (1-\rho_{\alpha_1}(\beta_1,\beta_2)^2)}{1+\alpha_1\beta_1\mathrm{SNR}_1}\right) \tag{82b}$$

where $\rho_{\alpha_1}(\beta_1,\beta_2)$ is the unique solution in $(0,1)$ to

$$\begin{aligned} 1 + & \frac{(1-\alpha_1)\beta_1\mathrm{SNR}_1 + \beta_2\mathrm{SNR}_2 + 2x\sqrt{\beta_1\beta_2(1-\alpha_1)\mathrm{SNR}_1\mathrm{SNR}_2}}{1+\alpha_1\beta_1\mathrm{SNR}_1} \\ &= \left(1 + \frac{(1-\alpha_1)\beta_1\mathrm{SNR}_1}{1+\alpha_1\beta_1\mathrm{SNR}_1}(1-x^2)\right)\left(1+\frac{\beta_2\mathrm{SNR}_2}{1+\alpha_1\beta_1\mathrm{SNR}_1}(1-x^2)\right). \end{aligned} \tag{83}$$

Then, using successive cancellation, the receiver recovers $M_{1,0}$ successfully if

$$R_{1,0} \leqslant \frac{1}{2}\log_2(1+\alpha_1\beta_1\mathrm{SNR}_1). \tag{84}$$

By substituting $R_1 = R_{1,0} + R_{1,1}$, it can be seen that successful decoding of (M_1, M_2) is possible with arbitrarily small probability of error if the rates (R_1, R_2) satisfy

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_1)\beta_1\mathrm{SNR}_1\left(1-(\rho_{\alpha_1}(\beta_1,\beta_2))^2\right)}{1+\alpha_1\beta_1\mathrm{SNR}_1}\right) + \frac{1}{2}\log_2(1+\alpha_1\beta_1\mathrm{SNR}_1) \tag{85a}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \frac{\beta_2\mathrm{SNR}_2\left(1-(\rho_{\alpha_1}(\beta_1,\beta_2))^2\right)}{1+\alpha_1\beta_1\mathrm{SNR}_1}\right). \tag{85b}$$

Now, the average received power of this scheme is analyzed. The sequence $S_1, \ldots, S_n$ is i.i.d. and each S_t for $t \in \{1, \ldots, n\}$ follows a zero-mean Gaussian distribution with variance B given by

$$\begin{aligned} B = \mathsf{E}\left[S_t^2\right] = 1 + \mathrm{SNR}_1 + \mathrm{SNR}_2 + 2\sqrt{1-\alpha_1}\,\rho_{\alpha_1}(\beta_1,\beta_2)\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2} \\ + 2\sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2}\big). \end{aligned} \tag{86}$$

Here also weak law of large numbers implies that

$$\lim_{n\to\infty} \Pr\left(B^{(n)} < b - \epsilon\right) = 0 \tag{87}$$

for any $b \in [0, B]$.

Now if in constraints (85) and (86), ρ replaces $\sqrt{1-\alpha_1}\,\rho_{\alpha_1}(\beta_1,\beta_2)$ with $\alpha_1 \in [0,1]$, then any non-negative information-energy rate triplet (R_1, R_2, B) satisfying

$$R_1 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1(1-\rho^2)\right), \tag{88a}$$
$$R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2+2\rho\sqrt{\beta_1\beta_2}\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right)-\frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1(1-\rho^2)\right), \tag{88b}$$
$$B \leqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\left(\rho\sqrt{\beta_1\beta_2}+\sqrt{(1-\beta_1)(1-\beta_2)}\right)\sqrt{\mathrm{SNR}_1\,\mathrm{SNR}_2}, \tag{88c}$$

where $\rho \in [0, \rho^\star(\beta_1,\beta_2)]$ and $\rho^\star(\beta_1,\beta_2)$ uniquely satisfies equation (38), is achievable.

If the roles of transmitter 1 and 2 are reversed, it can be shown that any non-negative information-energy rate triplet (R_1, R_2, B) such that

$$R_1 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2+2\rho\sqrt{\beta_1\beta_2}\sqrt{\mathrm{SNR}_1\mathrm{SNR}_2}\right)-\frac{1}{2}\log_2\left(1+\beta_2\mathrm{SNR}_2(1-\rho^2)\right) \tag{89a}$$
$$R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta_2\mathrm{SNR}_2(1-\rho^2)\right) \tag{89b}$$
$$B \leqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}+2\sqrt{(1-\beta_1)\mathrm{SNR}_1\,(1-\beta_2)\mathrm{SNR}_2} \tag{89c}$$

for any $\rho \in [0, \rho^\star(\beta_1,\beta_2)]$, is achievable.

Time-sharing between all rate pairs in the union of the two (rectangular) regions described by the constraints (88) and (89) concludes the proof of achievability of the region

$$R_1 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1\left(1-\rho^2\right)\right) \tag{90a}$$
$$R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta_2\mathrm{SNR}_2\left(1-\rho^2\right)\right) \tag{90b}$$
$$R_1+R_2 \leqslant \frac{1}{2}\log_2\left(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2+2\rho\sqrt{\beta_1\mathrm{SNR}_1\,\beta_2\mathrm{SNR}_2}\right) \tag{90c}$$
$$B \leqslant 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}+2\sqrt{(1-\beta_1)\mathrm{SNR}_1\,(1-\beta_2)\mathrm{SNR}_2}, \tag{90d}$$

for any for any $\rho \in [0, \rho^\star(\beta_1,\beta_2)]$.

Note that for any $\rho > \rho^\star(\beta_1,\beta_2)$, the sum of (90a) and (90b) is strictly smaller than (90c). The resulting region is a rectangle that is strictly contained in the rectangle obtained for $\rho = \rho^\star(\beta_1,\beta_2)$. In other words, there is no gain in terms of rates. In terms of energy, for any $\rho > \rho^\star(\beta_1,\beta_2)$, there always exist (β_1', β_2') such that

$$\rho = \sqrt{\beta_1'\beta_2'}\rho^\star(\beta_1',\beta_2')+\sqrt{(1-\beta_1')(1-\beta_2')}.$$

This choice achieves any rate-pair (R_1, R_2) satisfying

$$R_i \leqslant \frac{1}{2}\log_2\left(1+\beta_i'\mathrm{SNR}_i(1-\rho^\star(\beta_1',\beta_2')^2)\right) \tag{91}$$

In particular, it achieves

$$R_i \leqslant \frac{1}{2}\log_2\left(1+\beta_i'\mathrm{SNR}_i(1-\rho^2)\right), \quad i \in \{1,2\} \tag{92}$$

since $\rho > \rho^\star(1,1) = \max_{\beta_1,\beta_2}\rho^\star(\beta_1,\beta_2)$. This completes the proof of the achievability part of Theorem 3.

5.2 Proof of Converse

Fix an information-energy rate triplet $(R_1, R_2, B) \in \mathcal{E}_{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2)$. For this information-energy rate triplet and for each blocklength n encoding and decoding functions are chosen such that

$$\limsup_{n\to\infty} \quad P_{\text{error}}^{(n)} = 0, \tag{93a}$$

$$\limsup_{n\to\infty} \quad P_{\text{outage}}^{(n)}(\epsilon) = 0 \text{ for any } \epsilon > 0, \tag{93b}$$

and such that the input power constraint (1) is satisfied.

Using assumption (93a), applying Fano's inequality and following similar steps as in [24], it can be shown that the rates (R_1, R_2) must satisfy

$$nR_1 \leqslant \sum_{t=1}^{n} I(X_{1,t}; Y_t | X_{2,t}) + \epsilon_1^{(n)}, \tag{94a}$$

$$nR_2 \leqslant \sum_{t=1}^{n} I(X_{2,t}; Y_t | X_{1,t}) + \epsilon_2^{(n)}, \tag{94b}$$

$$n(R_1 + R_2) \leqslant \sum_{t=1}^{n} I(X_{1,t} X_{2,t}; Y_t) + \epsilon_{12}^{(n)}, \tag{94c}$$

where $\frac{\epsilon_1^{(n)}}{n}$, $\frac{\epsilon_2^{(n)}}{n}$, and $\frac{\epsilon_1^{(n)}}{n}$ tend to zero as n tends to infinity.

Using assumption (93b), for a given $\epsilon_n > 0$, for any $\eta > 0$ there exists $n_0(\eta)$ such that for any $n \geq n_0(\eta)$ it holds that

$$\Pr(B^{(n)} < B - \epsilon_n) < \eta. \tag{95}$$

Equivalently,

$$\Pr(B^{(n)} \geqslant B - \epsilon_n) \geqslant 1 - \eta \tag{96}$$

From Markov's inequality [27], the following holds:

$$(B - \epsilon_n) \Pr\left(B^{(n)} \geqslant B - \epsilon_n\right) \leqslant \mathsf{E}\left[B^{(n)}\right]. \tag{97}$$

Combining (96) and (97) yields

$$(B - \epsilon_n)(1 - \eta) \leqslant \mathsf{E}\left[B^{(n)}\right] \tag{98}$$

which can be written as

$$(B - \delta_n) \leqslant \mathsf{E}\left[B^{(n)}\right] \tag{99}$$

for some $\delta_n > \epsilon_n$ (for sufficiently large n).

In the following, the bounds in (94) and (99) are evaluated for the Gaussian MAC.

Here it is assumed that at each time t the channel input $X_{i,t} = U_{i,t} + \alpha_{i,t} W_t$ where $U_{i,t}$ is a zero-mean IC component, independent thereof W_t is a zero-mean unit-variance NIC component that is known to all terminals, and the coefficients $\alpha_{i,t} \in \mathbb{R}$ are deterministic and known to all terminals.

For this purpose, for $t \in \{1, \ldots, n\}$ define

$$\sigma_{i,t}^2 \triangleq \mathsf{Var}(U_{i,t}), \quad i \in \{1, 2\}, \tag{100}$$

$$\lambda_t \triangleq \mathsf{Cov}[U_{1,t}, U_{2,t}]. \tag{101}$$

Define also

$$\alpha_i^2 \triangleq \frac{1}{n}\sum_{t=1}^{n} \alpha_{i,t}^2, \quad i \in \{1,2\}, \tag{102}$$

$$\sigma_i^2 \triangleq \frac{1}{n}\sum_{t=1}^{n} \sigma_{i,t}^2, \quad i \in \{1,2\}, \tag{103}$$

$$\rho \triangleq \left(\frac{1}{n}\sum_{t=1}^{n} \lambda_t\right)/\sigma_1\sigma_2. \tag{104}$$

The input sequence must satisfy the input power constraint (1) which can be written , for $i \in \{1,2\}$, as

$$\frac{1}{n}\sum_{t=1}^{n} \mathsf{E}\left[X_{i,t}^2\right] = \frac{1}{n}\sum_{t=1}^{n}\left(\sigma_{i,t}^2 + \alpha_{i,t}^2\right) = \sigma_i^2 + \alpha_i^2 \leqslant P_i. \tag{105}$$

For each $t \in \{1,\ldots,n\}$, regardless of the distribution of $(X_{1,t}, X_{2,t})$, since Z_t follows a zero-mean unit-variance Gaussian distribution it holds that

$$h(Y_t|X_{1,t}, X_{2,t}) = h(Z_t) = \frac{1}{2}\log_2\left(2\pi e\right). \tag{106}$$

Recall that by the properties of the differential entropy [28], it holds that for any random variable X of variance σ_X^2, the differential entropy $h(X)$ satisfies $h(X) \leqslant \frac{1}{2}\log_2\left(2\pi e\sigma_X^2\right)$, where the right-hand-side corresponds to the differential entropy of a Gaussian distribution with the same variance.

All terminals have access to the values of the NIC sequence $\{W_t\}_{t=1}^n$ as well as the coefficients $\{\alpha_{1,t}\}_{t=1}^n$ and $\{\alpha_{2,t}\}_{t=1}^n$ as side information. Let $Y_t' = h_1U_{1,t} + h_2U_{2,t} + Z_t$. Since at each time W_t is independent of $(U_{1,t}, U_{2,t}, Z_t)$, the bounds in (94) on the information transmission rates can be written as

$$nR_1 \leqslant \sum_{t=1}^{n} I(U_{1,t}; Y_t'|U_{2,t}) + \epsilon_1^{(n)}, \tag{107a}$$

$$nR_2 \leqslant \sum_{t=1}^{n} I(U_{2,t}; Y_t'|U_{1,t}) + \epsilon_2^{(n)}, \tag{107b}$$

$$n(R_1+R_2) \leqslant \sum_{t=1}^{n} I(U_{1,t}U_{2,t}; Y_t') + \epsilon_{12}^{(n)}. \tag{107c}$$

For the considered $(U_{1,t}, U_{2,t})$ it holds that

$$\begin{aligned}
I(U_{1,t}, U_{2,t}; Y_t') &= h(Y_t') - h(Z_t)\\
&\leqslant \frac{1}{2}\log_2\left(2\pi e \mathsf{Var}(Y_t')\right) - \frac{1}{2}\log_2\left(2\pi e\right)\\
&= \frac{1}{2}\log_2\left(h_1^2\sigma_{1,t}^2 + h_2^2\sigma_{2,t}^2 + 2h_1h_2\lambda_t + 1\right),\\
I(U_{1,t}; Y_t'|U_{2,t}) &= h(Y_t'|U_{2,t}) - h(Y_t'|U_{1,t}, U_{2,t})\\
&\leqslant \frac{1}{2}\log_2\left(2\pi e(\mathsf{Var}(Y_t'|U_{2,t}))\right) - \frac{1}{2}\log_2\left(2\pi e\right)\\
&= \frac{1}{2}\log_2\left(1 + h_1^2\sigma_{1,t}^2\left(1 - \frac{\lambda_t^2}{\sigma_{1,t}^2\sigma_{2,t}^2}\right)\right),\\
I(U_{1,t}; Y_t'|U_{2,t}) &= \frac{1}{2}\log_2\left(1 + h_2^2\sigma_{2,t}^2\left(1 - \frac{\lambda_t^2}{\sigma_{1,t}^2\sigma_{2,t}^2}\right)\right).
\end{aligned}$$

The bounds in (94) can be written as

$$\begin{aligned} nR_1 &\leqslant \sum_{t=1}^{n} \frac{1}{2} \log_2 \left(1 + h_1^2 \sigma_{1,t}^2 \left(1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2}\right)\right) + \epsilon_1^{(n)}, & \text{(108a)} \\ nR_2 &\leqslant \sum_{t=1}^{n} \frac{1}{2} \log_2 \left(1 + h_2^2 \sigma_{2,t}^2 \left(1 - \frac{\lambda_t^2}{\sigma_{1,t}^2 \sigma_{2,t}^2}\right)\right) + \epsilon_2^{(n)}, & \text{(108b)} \\ n(R_1 + R_2) &\leqslant \sum_{t=1}^{n} \frac{1}{2} \log_2 (1 + h_1^2 \sigma_{1,t}^2 + h_2^2 \sigma_{2,t}^2 + 2h_1 h_2 \lambda_t) + \epsilon_{12}^{(n)}, & \text{(108c)} \end{aligned}$$

By the concavity of the mutual information, applying Jensen's inequality [28] in the bounds (108) yields in the limit when $n \to \infty$,

$$\begin{aligned} R_1 &\leqslant \frac{1}{2} \log_2 (1 + h_1^2 \sigma_1^2 (1 - \rho^2)), \\ R_2 &\leqslant \frac{1}{2} \log_2 (1 + h_2^2 \sigma_2^2 (1 - \rho^2)), \\ R_1 + R_2 &\leqslant \frac{1}{2} \log_2 \big(1 + h_1^2 \sigma_1^2 + h_2^2 \sigma_2^2 + 2\sqrt{h_1^2 \sigma_1^2 h_2^2 \sigma_2^2} |\rho|\big). \end{aligned}$$

The average received energy is given by

$$\begin{aligned} \mathsf{E}\left[B^{(n)}\right] &= \mathsf{E}\left[\frac{1}{n} \sum_{t=1}^{n} S_t^2\right] \\ &= 1 + h_1^2 \big(\frac{1}{n} \sum_{t=1}^{n} (\sigma_{1,t}^2 + \alpha_{1,t}^2)\big) + h_2^2 \big(\frac{1}{n} \sum_{t=1}^{n} (\sigma_{2,t}^2 + \alpha_{2,t}^2)\big) + 2h_1 h_2 \big(\frac{1}{n} \sum_{t=1}^{n} (\lambda_t + \alpha_{1,t} \alpha_{2,t})\big) \\ &\overset{(a)}{\leq} 1 + h_1^2 \big(\frac{1}{n} \sum_{t=1}^{n} (\sigma_{1,t}^2 + \alpha_{1,t}^2)\big) + h_2^2 \big(\frac{1}{n} \sum_{t=1}^{n} (\sigma_{2,t}^2 + \alpha_{2,t}^2)\big) \\ &\quad + 2h_1 h_2 \left(\big(\frac{1}{n} \sum_{t=1}^{n} \lambda_t\big) + \big(\frac{1}{n} \sum_{t=1}^{n} \alpha_{1,t}^2\big)^{1/2} \big(\frac{1}{n} \sum_{t=1}^{n} \alpha_{2,t}^2\big)^{1/2} \right) \\ &= 1 + h_1^2 (\sigma_1^2 + \alpha_1^2) + h_2^2 (\sigma_2^2 + \alpha_2^2) + 2h_1 h_2 (\rho\, |\sigma_1||\sigma_2| + |\alpha_1||\alpha_2|) \end{aligned}$$

where (a) follows from the Cauchy-Schwarz inequality.

To sum up, in the limit when n tends to infinity, any information-energy rate triplet satisfying

$$\begin{aligned} R_1 &\leqslant \frac{1}{2} \log_2 (1 + h_1^2 \sigma_1^2 (1 - \rho^2)), & \text{(109a)} \\ R_2 &\leqslant \frac{1}{2} \log_2 (1 + h_2^2 \sigma_2^2 (1 - \rho^2)), & \text{(109b)} \\ R_1 + R_2 &\leqslant \frac{1}{2} \log_2 \left(1 + h_1^2 \sigma_1^2 + h_2^2 \sigma_2^2 + 2\sqrt{h_1^2 h_2^2 \sigma_1^2 \sigma_2^2} \rho\right), & \text{(109c)} \\ B &\leqslant 1 + h_1^2 (\sigma_1^2 + \alpha_1^2) + h_2^2 (\sigma_2^2 + \alpha_2^2) + 2h_1 h_2 (\rho\, |\sigma_1||\sigma_2| + |\alpha_1||\alpha_2|) & \text{(109d)} \end{aligned}$$

for some σ_1^2, σ_2^2, α_1^2, α_2^2 such that (105) is true and for some $\rho \in [-1, 1]$ is achievable. Let $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho)$ denote this region. The capacity-energy region is contained within the union of all $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho)$ over all $0 \leqslant \sigma_1^2 + \alpha_1^2 \leqslant P_1$ and $0 \leqslant \sigma_2^2 + \alpha_2^2 \leqslant P_2$ and $-1 \leqslant \rho \leqslant 1$.

In this union, it suffices to consider $0 \leqslant \rho \leqslant 1$ because for any $-1 \leqslant \rho \leqslant 1$, $\mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, \rho) \subseteq \mathcal{R}_0(\sigma_1^2, \sigma_2^2, \alpha_1^2, \alpha_2^2, |\rho|)$. Furthermore, for $0 \leqslant \rho \leqslant 1$, it suffices to consider $\alpha_1 \geqslant 0$, $\alpha_2 \geqslant 0$, and

σ_1^2, σ_2^2, α_1^2, and α_2^2 that saturate the input power constraint (i.e., (105) holds with equality). If $\beta_i \triangleq \frac{\sigma_i^2}{P_i} \in [0,1],\ i \in \{1,2\}$, such a region contains all information-energy rate triplets (R_1, R_2, B) satisfying constraints (36) which completes the converse of the proof.

6 Proof of Theorem 4

For fixed SNR_1 and SNR_2 and fixed minimum received energy rate $b \geqslant 0$ the sum-rate maximization problem in (45) can be written as

$$R_{\mathrm{sum}}^{\mathrm{FB}}(\mathrm{SNR}_1, \mathrm{SNR}_2, b) = \max_{(\beta_1,\beta_2,\rho)\in[0,1]^3} f(\beta_1,\beta_2,\rho) \tag{110a}$$
$$\text{subject to:} \qquad g(\beta_1,\beta_2,\rho) \geqslant b, \tag{110b}$$

where the functions f and g are defined as follows

$$f(\beta_1,\beta_2,\rho) \triangleq \min\Big\{\frac{1}{2}\log_2\big(1+\beta_1\mathrm{SNR}_1+\beta_2\mathrm{SNR}_2+2\rho\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}\big),$$
$$\frac{1}{2}\log_2\big(\big(1+\beta_1\mathrm{SNR}_1(1-\rho^2)\big)\big(1+\beta_2\mathrm{SNR}_2(1-\rho^2)\big)\big)\Big\}, \tag{111}$$

and

$$g(\beta_1,\beta_2,\rho) \triangleq 1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\big(\sqrt{\beta_1\beta_2}\rho+\sqrt{(1-\beta_1)(1-\beta_2)}\big)\sqrt{\mathrm{SNR}_1\ \mathrm{SNR}_2}. \tag{112}$$

Let also

$$\rho_{\min}(\beta_1,\beta_2) \triangleq \min\left(1, \frac{b-(1+\mathrm{SNR}_1+\mathrm{SNR}_2+2\sqrt{(1-\beta_1)\mathrm{SNR}_1(1-\beta_2)\mathrm{SNR}_2})}{2\sqrt{\beta_1\mathrm{SNR}_1\beta_2\mathrm{SNR}_2}}\right) \tag{113}$$

be the value of $\rho \in [0,1]$ for which $g(\beta_1,\beta_2,\rho) = b$. Note that $\rho^\star(\beta_1,\beta_2)$ can be alternatively defined as

$$\rho^\star(\beta_1,\beta_2) \triangleq \operatorname*{argmax}_{\rho\in[0,1]} \quad f(\beta_1,\beta_2,\rho). \tag{114}$$

The proof of Theorem 4 is based on the following two lemmas.

Lemma 2. *Let $(\beta_1,\beta_2,\rho) \in [0,1]^3$ be a solution to (110). Then, it holds that $(\beta_1,\beta_2,\max\{\rho_{\min}(\beta_1,\beta_2),\rho^\star(\beta_1,\beta_2)\})$ is also a solution to (110).*

Proof: Let $(\beta_1',\beta_2',\rho') \in [0,1]^3$ be a solution to (110). Hence, $\forall(\beta_1,\beta_2,\rho) \in [0,1]^3$, it follows that

$$f(\beta_1',\beta_2',\rho') \geqslant f(\beta_1,\beta_2,\rho), \text{ and} \tag{115}$$
$$g(\beta_1',\beta_2',\rho') \geqslant b. \tag{116}$$

Assume that $\rho_{\min}(\beta_1',\beta_2') \leqslant \rho^\star(\beta_1',\beta_2')$. Then $\rho^\star(\beta_1',\beta_2')$ is admissible, i.e.,

$$g(\beta_1',\beta_2',\rho^\star(\beta_1',\beta_2')) \geqslant b \tag{117}$$

and from (114), it follows that

$$f(\beta_1',\beta_2',\rho') \leqslant f(\beta_1',\beta_2',\rho^\star(\beta_1',\beta_2')). \tag{118}$$

From (115), (117), and (118) it follows that $\forall(\beta_1, \beta_2, \rho) \in [0,1]^3$

$$f(\beta_1', \beta_2', \rho^\star(\beta_1', \beta_2')) \geqslant f(\beta_1, \beta_2, \rho), \text{and} \tag{119a}$$
$$g(\beta_1', \beta_2', \rho^\star(\beta_1', \beta_2')) \geqslant b, \tag{119b}$$

which suggests that $(\beta_1', \beta_2', \rho^\star(\beta_1', \beta_2'))$ is also a solution to (110) under the assumption that $\rho_{\min}(\beta_1', \beta_2') \leqslant \rho^\star(\beta_1', \beta_2')$. Now, assume that $\rho_{\min}(\beta_1', \beta_2') \geqslant \rho^\star(\beta_1', \beta_2')$. Then, $\rho' \in [\rho_{\min}(\beta_1', \beta_2'), 1]$ and from (113) it follows that

$$g(\beta_1', \beta_2', \rho_{\min}(\beta_1', \beta_2')) = b, \tag{120}$$

and thus, $\rho_{\min}(\beta_1', \beta_2')$ is also admissible given β_1' and β_2'. Note that the function $f(\beta_1', \beta_2', \rho)$ is monotonically decreasing in ρ in the interval $[\rho^\star(\beta_1', \beta_2'), 1]$. Thus, the following inequality holds:

$$f(\beta_1', \beta_2', \rho') \leqslant f(\beta_1', \beta_2', \rho_{\min}(\beta_1', \beta_2')). \tag{121}$$

From (115), (120), and (121) it follows that $\forall(\beta_1, \beta_2, \rho) \in [0,1]^3$

$$f(\beta_1', \beta_2', \rho_{\min}(\beta_1', \beta_2')) \geqslant f(\beta_1, \beta_2, \rho), \quad \text{and} \tag{122a}$$
$$g(\beta_1', \beta_2', \rho_{\min}(\beta_1', \beta_2')) = b, \tag{122b}$$

which suggests that $(\beta_1', \beta_2', \rho_{\min}(\beta_1', \beta_2'))$ is also a solution to (110) under the assumption that $\rho_{\min}(\beta_1', \beta_2') \geqslant \rho^\star(\beta_1', \beta_2')$.

Finally, from (119) and (122), it follows that if $(\beta_1', \beta_2', \rho')$ is a solution to (110), then, the triplet $(\beta_1', \beta_2', \max\{\rho_{\min}(\beta_1', \beta_2'), \rho^\star(\beta_1', \beta_2')\})$ is also a solution, which completes the proof of Lemma 2. ■

Lemma 3. *The unique solution to* (110) *in* $[0,1]^3$ *is* $(1, 1, \bar{\rho})$ *with*

$$\bar{\rho} \triangleq \max\{\rho_{\min}(1,1), \rho^\star(1,1)\}. \tag{123}$$

Proof: Assume that there exists another solution $(\beta_1', \beta_2', \rho')$ to (110) different from $(1, 1, \bar{\rho})$. Thus, for any $(\beta_1, \beta_2, \rho) \in [0,1]^3$ it holds that

$$f(\beta_1, \beta_2, \rho) \leqslant f(\beta_1', \beta_2', \rho'). \tag{124}$$

Note that for a fixed $\rho' \in [0,1]$, $f(\beta_1, \beta_2, \rho')$ is strictly increasing in (β_1, β_2). Hence, for any $(\beta_1, \beta_2) \in [0,1]^2$,

$$f(\beta_1, \beta_2, \rho') < f(1, 1, \rho') \tag{125}$$
$$\leqslant f(1, 1, \bar{\rho}), \tag{126}$$

where the second inequality follows by Lemma 2. Moreover, since $\bar{\rho} \geqslant \rho_{\min}(1,1)$, the following inequality also holds:

$$g(1, 1, \bar{\rho}) \geqslant b. \tag{127}$$

In particular, if $(\beta_1, \beta_2) = (\beta_1', \beta_2')$ in (125), it follows that

$$f(\beta_1', \beta_2', \rho') < f(1, 1, \bar{\rho}), \tag{128}$$

which contradicts the initial assumption that there exists a solution other than $(1, 1, \bar{\rho})$. This establishes a proof by contradiction that the unique solution to (110) is $(1, 1, \bar{\rho})$. ■

Finally, the proof of Theorem 4 follows from the following equality:

$$R_{\text{sum}}^{\text{FB}}(\text{SNR}_1, \text{SNR}_2, b) = f(1, 1, \bar{\rho}), \tag{129}$$

and this completes the proof.

References

[1] S. Belhadj Amor, S. M. Perlaza, and I. Krikidis, "Simultaneous energy and information transmission in Gaussian multiple access channels," in *Proc. 5th International Conference on Communications and Networking (ComNet)*, Hammamet, Tunisia, Nov. 2015.

[2] L. R. Varshney, "On energy/information cross-layer architectures," in *Proc. IEEE International Symposium on Information Theory*, Jul. 2012, pp. 1356–1360.

[3] S. Bi, C. K. Ho, and R. Zhang, "Wireless powered communication: opportunities and challenges," *IEEE Communications Magazine*, vol. 53, pp. 117–125, Apr. 2015.

[4] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. IEEE International Symposium on Information Theory*, Jul. 2008, pp. 1612–1616.

[5] P. Grover and A. Sahai, "Shannon meets Tesla: Wireless information and power transfer," in *Proc. IEEE International Symposium on Information Theory*, Jun. 2010, pp. 2363–2367.

[6] K. Huang and E. Larsson, "Simultaneous information and power transfer for broadband wireless systems," *IEEE Transactions on Signal Processing*, vol. 61, no. 23, pp. 5972–5986, Dec. 2013.

[7] Z. Xiang and M. Tao, "Robust beamforming for wireless information and power transmission," *IEEE Wireless Communications Letters*, vol. 1, no. 4, pp. 372–375, Aug. 2012.

[8] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 1989–2001, May 2013.

[9] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: Architecture design and rate-energy tradeoff," in *Proc. 2012 IEEE Global Communications Conference (GLOBECOM)*, Dec. 2012, pp. 3982–3987.

[10] J. Park and B. Clerckx, "Joint wireless information and energy transfer in a two-user MIMO interference channel," *IEEE Transactions on Wireless Communications*, vol. 12, no. 8, pp. 4210–4221, Aug. 2013.

[11] K. Huang and V. K. N. Lau, "Enabling wireless power transfer in cellular networks: Architecture, modeling and deployment," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 902–912, Feb. 2014.

[12] Z. Ding, S. M. Perlaza, I. Esnaola, and H. V. Poor, "Power allocation strategies in energy harvesting wireless cooperative networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 846–860, Feb. 2014.

[13] A. M. Fouladgar; and O. Simeone, "On the transfer of information and energy in multi-user systems," *IEEE Communications Letters*, vol. 16, no. 11, pp. 1733–1736, Nov. 2012.

[14] B. Gurakan, O. Ozel, J. Yang, and S.Ulukus, "Energy cooperation in energy harvesting wireless communications," in *Proc. IEEE International Symposium on Information Theory*, Jul. 2012, pp. 965–969.

[15] K. Ishibashi, H. Ochiai, and V. Tarokh, "Energy harvesting cooperative communications," in *Proc. 2012 IEEE 23rd International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2012, pp. 1819–1823.

[16] Y. Luo, J. Zhang, and K. B. Letaief, "Optimal scheduling and power allocation for two-hop energy harvesting communication systems," *IEEE Transactions on Wireless Communications*, vol. 12, no. 9, pp. 4729–4741, Sep. 2013.

[17] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3622–3636, Jul. 2013.

[18] P. Popovski, A. M. Fouladgar, and O. Simeone, "Interactive joint transfer of energy and information," *IEEE Transactions on Communications*, vol. 61, no. 5, pp. 2086–2097, May 2013.

[19] A. M. Fouladgar and O. Simeone, "Information and energy flows in graphical networks with energy transfer and reuse," *IEEE Wireless Communications Letters*, vol. 2, no. 4, pp. 371–374, Aug. 2013.

[20] T. M. Cover, *Some Advances in Broadcast Channels.* Academic Press, 1975, vol. 4, ch. 4.

[21] A. D. Wyner, "Recent results in the Shannon theory," *IEEE Transactions on Information Theory*, vol. 20, no. 1, pp. 2–10, 1974.

[22] M. Gastpar, "Gaussian multiple-access channels under received-power constraints," in *Proc. IEEE Information Theory Workshop*, 2004, pp. 452–457.

[23] G. Kramer, "Feedback strategies for white Gaussian interference networks," *IEEE Transactions Information Theory*, vol. 48, no. 6, pp. 1423–1438, 2002.

[24] L. H. Ozarow, "The capacity of the white Gaussian multiple access channel with feedback," *IEEE Transactions on Information Theory*, vol. 30, no. 4, pp. 623–629, Jul. 1984.

[25] A. El Gamal and Y. H. Kim, *Network Information Theory.* Cambridge University Press, 2011.

[26] A. Lapidoth, *A Foundation in Digital Communication.* New York: Cambridge University Press, 2009.

[27] R. Durret, *Probability Theory and Examples.* New York, NY: Cambridge University Press, 2010.

[28] T. M. Cover and J. A. Thomas, *Elements of Information Theory, 2nd Ed.* Wiley, 2006.

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