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Marco Fedi. Hypothetical role of quantum space superfluid dynamics in the physics of particles and fundamental interactions.. 2015. <hal-01223102v3>

**HAL Id: hal-01223102**

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Preprint submitted on 24 Dec 2015

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# Hypothetical role of quantum space superfluid dynamics in the physics of particles and fundamental interactions.

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October 2015

## Abstract

By placing as a condition the already hypothesized and, in many respects necessary, existence of quantum space[1], its possible dynamics as a quantum superfluid is discussed to show how the phenomenon of quantum superfluid conical vortices, decreased to subatomic scale, could be able to incorporate QED, QFT, QCD, as well as to explain how the Yang-Mills theory gives rise to the physics of nuclei and nuclear constituents. It is moreover shown how this approach is compatible with special and general relativity. In light of this, the importance of mathematically scrutinizing and experimentally testing a theory of quantum space superfluidity applied to particles and fundamental interactions clearly emerges. Predictions are eventually formulated, which would besides indirectly demonstrate the existence of superfluid quantum space.

## Introduction.

As other theories, such as loop gravity[2][3], this model moves from a quantum, granular[4], view of vacuum and therefore of space itself, defined as consisting in a superfluid sea of space quanta (SQ), also indicated with the symbol small final sigma ( $\varsigma$ ). Such quanta would behave as a superfluid medium, whose perturbations – seen as topological defects of quantum space – would occur as flows and conical vortices which, along with clumping phenomena occurring among SQ, would represent particles and fundamental forces. On the other hand, the conditions of zero field energy and superfluidity would account for their continuity in time, if no events (*e.g.* scattering) interfere with them. Firstly, the hypothesis of gravity explained as absorption of SQ effected by massive particles (below described as vortices of SQ) has been tested in a set of computational fluid dynamic simulations: the result was positive in matching Newton's law of universal gravitation, also opening the way to a fluid dynamic reformulation of general relativity's time-dependent geodesic equation. This would confirm that gravity might be quantized without resorting to the graviton but simply considering a quantized space.

# 1 Fluid dynamic reformulation of gravity.

Placed the hypothesis of absorption of SQ effected by massive particles (fig. 1), the following set of simulations has been performed. Navier-Stokes equations representing mass, momentum and energy were used:

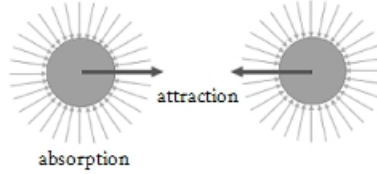


Figure 1: Two particles move the one towards the other since they're absorbing the fluid which they're immersed in (quantum space). Below, they will be actually described as superfluid vortices, which justify the absorption mechanism.

$$\frac{\partial(u_j)}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial(u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2)$$

$$\frac{\partial((\rho E + p)u_j)}{\partial x_j} = -k \frac{\partial}{\partial x_j} \left( \frac{\partial T}{\partial x_j} \right) \quad (3)$$

The condition of two stationary spheres immersed in an incompressible fluid was set and the pressure integral of the forces acting on them was calculated. The analysis took into account the response to absorption velocity and to distance between the spheres. To simplify the simulations, the system was reduced as showed in fig. 2.

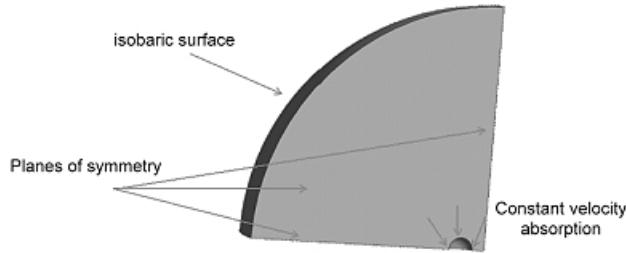


Figure 2: Simulation settings

The attractive force produced by pressure forces and momentum, where  $A$  corresponds to the surface of the inner sphere and  $\vec{d}$  is the versor for the distance between the spheres, is represented as:

$$F_a = \int_A (p + \rho(\vec{u} \cdot \vec{n})(\vec{u} \cdot \vec{d})) d\vec{A} \cdot \vec{d} \quad (4)$$

The analysis of velocity and pressure, respect to the distance from the absorbing sphere is illustrated in fig. 3 and the diagrams in fig. 5, 6, 7 and 8 show a quadratic inverse dependence on distance and a quadratic dependence on the flow intensity.

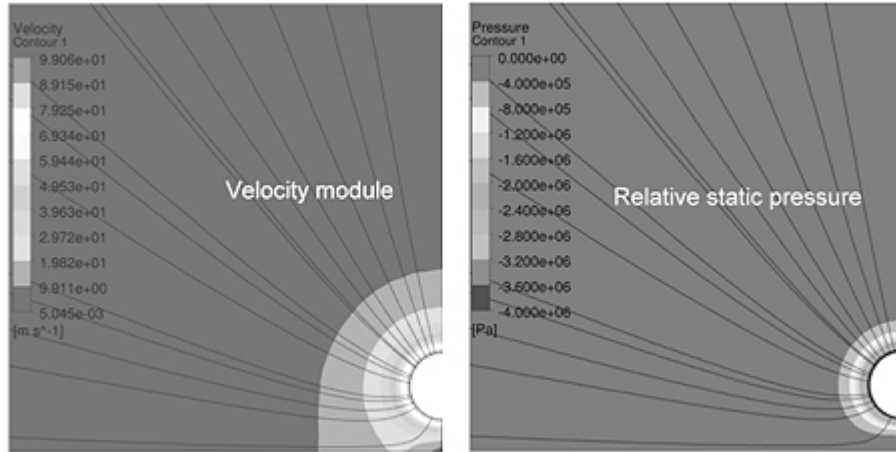


Figure 3: some results of simulations

Refinement of computational grid and domain enlargement help to reduce the curvature of the flow lines up to a virtually radial flow (fig. 4).

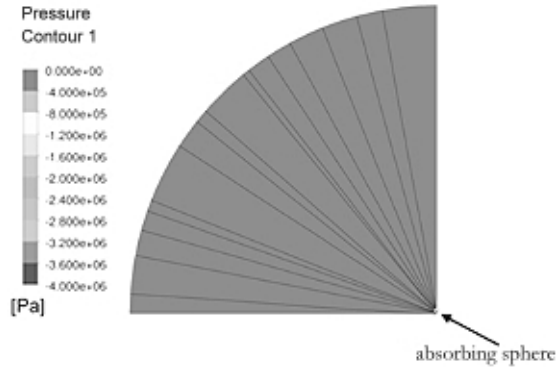


Figure 4: radial flow obtained in the simulation

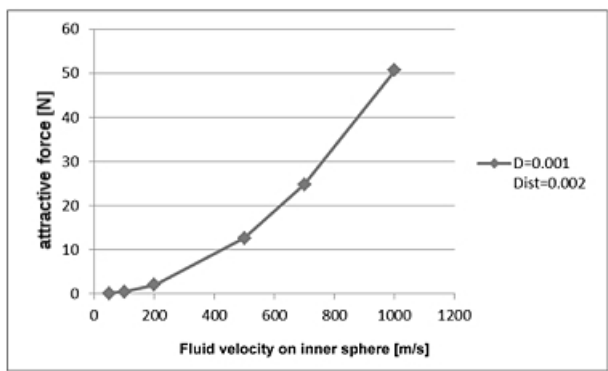


Figure 5: Test for force dependence on absorption velocity: sphere diameter 1mm, distance 2mm. Tested velocities: 50, 100, 200, 500, 700, 1000 m/s. Other tested conditions (50, 100, 200, 500 m/s) are shown in fig. 6 and 7.

Figure 6:

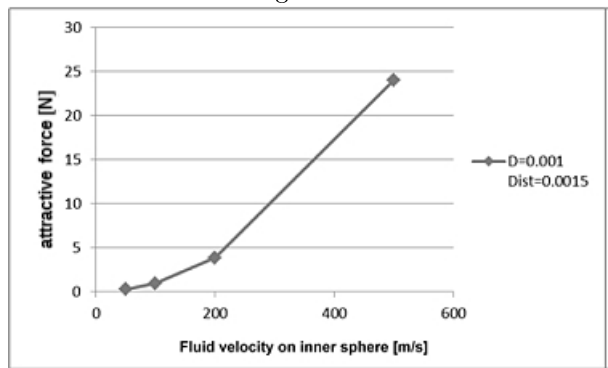


Figure 7:

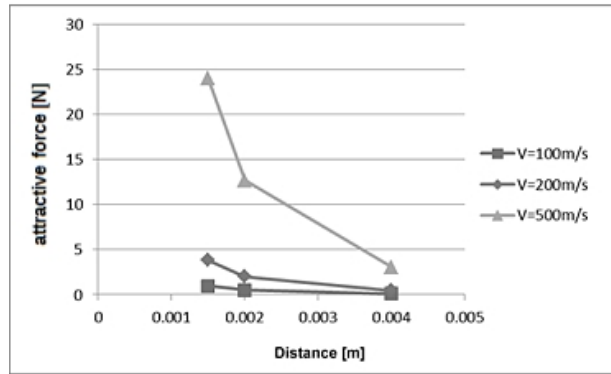
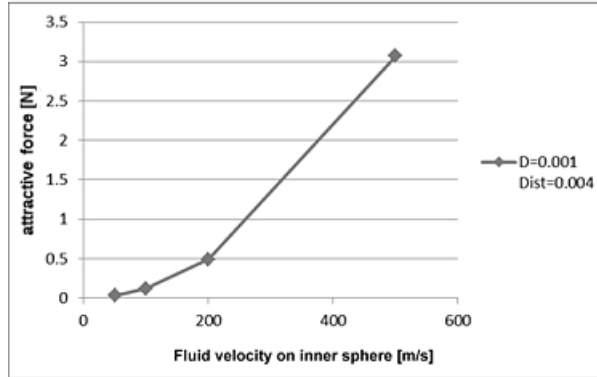


Figure 8: Test for force dependence according to distance between the spheres.

The behavior of the attractive force showed by this analysis is substantially concordant with the universal law of gravitation, since the attractive force decreases according to the inverse-square law and it quadratically grows according to the intensity of the flux (fig. 5, 6, 7). Two equal spheres have been considered, corresponding to equal masses in Newton's law.

## 2 Particles as superfluid vortices of SQ.

If gravity may be described as a flow of SQ, the physics of fluid dynamics and superfluidity gives us some examples which, suitably reduced on subatomic scale within the hypothesis of quantized space, could help us to describe particles and fundamental interactions, beyond gravity. Superfluid nanovortices have been for instance observed in superfluid helium-4. Superfluidity was originary

discovered in helium-4 by Kapitsa[5] and a recent experiment[6] has confirmed the formation of quantum vortices in superfluid helium nanodroplets, using single-shot femtosecond x-ray coherent diffractive imaging. Circulation in a superfluid vortex is quantized and similar to what described in the Bohr model, since the wavefunction must return to its same value after an integral number of turns

$$\oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{2\pi\hbar}{m} n. \quad (5)$$

where  $\hbar$  is the reduced Planck constant,  $m$  is the mass of the superfluid particle and  $2\pi n$  the phase difference around the vortex. These vortices behave as gaps in the medium where superfluidity breaks down and the presence of an axis inside them, would suggests the non-necessity of renormalization (no ultraviolet divergence) if fluid dynamic equations describing conical vortices were used to describe the force fields of subatomic particles (gravity and electrostatic interaction). The reasons for considering fermions and other particles as superfluid, conical vortices of SQ are several. We could, for instance, explain the appearance of particle-antiparticle pairs from quantum vacuum<sup>1</sup> as a perturbative phenomenon such as that described in Kármán vortex street (fig. 9)

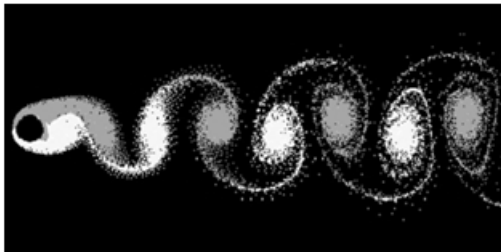


Figure 9: a computer simulation of a Kármán vortex street.

where pairs of right- and left-handed vortices occur due to a perturbation of the flow. The trigger to the formation of vortex-antivortex pairs in quantum space, could besides be a phase transition similar to the Kosterlitz-Thouless transition, where bound vortex-antivortex pairs get unpaired at some critical temperature. In the case described by Kosterlitz and Thouless, the vortices can exist above a critical temperature of

$$T_C = \frac{\kappa}{2k_B} \quad (6)$$

where  $\kappa$  is a parameter depending upon the system which the vortex is in and  $k_B$  is Boltzmann's constant. Also the mathematics of Lamb-Chaplygin dipoles is interesting for describing the dynamics of symmetric vortices. The stream function[7] of the flow is:

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<sup>1</sup>Referring, for instance, to the Casimir effect

$$\psi = \begin{cases} -\frac{2U}{kJ_0(ka)}J_1(kr)\sin\theta & r < a \\ U\left(r - \frac{a^2}{r}\right)\sin\theta & r > a \end{cases} \quad (7)$$

where  $J_0$  and  $J_1$  are the zero- and first-order Bessel functions. When the first root of  $J_1$  is  $ka$  the solution corresponds to a dipole. Single isolated point particles (here seen as point vortices[8]) could be therefore considered as vortex singlets or monopole vortices, whose origin would be however linked to another, symmetric, particle. During my investigation, I realized that vortices with conical shape are the most interesting to explain a large number of subatomic physical phenomena, especially as far as the link between gravity, electrostatic interaction and  $\beta$ -decay is concerned, as well as to explain quantum symmetries. The basic equations to describe them can be written, for greater ease, by starting from spherical coordinates  $(R, \theta, \phi)$ , where  $\phi$  is the azimuthal angle about the vortex axis  $\theta = 0$ . Setting  $x = \cos(\theta)$  and assuming

$$u_R = \frac{F'}{R}, \quad u_\theta = \frac{F(x)}{R\sin(\theta)}, \quad u_\phi = \frac{\Omega(x)}{R\sin(\theta)}$$

after some calculations[9] and by normalizing the Reynolds number to unity, we obtain two coupled nonlinear integral-differential equations in the form

$$f'^2 + f^2 = \frac{G(x)}{(1-x^2)^2} \quad (8)$$

$$\Omega'' + 2f\Omega' = 0 \quad (9)$$

where

$$G(x) \equiv \int_0^x dx \int_0^x dx \int_0^x \frac{4\Omega\Omega'}{1-x^2} dx + Px^2 + Qx + R \quad (10)$$

with  $P$ ,  $Q$  and  $R$  being integration constants linked to boundary conditions. For  $\Omega = 0$ , we have a conical similarity momentum jet.

### 3 QED in the quantum space superfluid model (QSSM): electrostatic fields/interactions and $\beta$ -decay as consequences of gravity.

By explaining gravity as absorption of SQ and assuming that they own a smallest mass-energy (infrared divergence would be moreover avoided) we'd immediately face the fact that particles' energy would diverge. An energy balance mechanism must therefore occur in the hypothesized vortices, to maintain a particle's mass-energy stable in time despite the absorption of SQ. The physics of vortices actually tells us that a vortex is able to absorb and to re-expel a fluid. If we equated this emission to the electrostatic field of charged particles, we would

immediately observe a physical link between gravity and electromagnetism, respectively seen as the input and the output of SQ into and from the superfluid vortex-particle. The output would consist in a periodic emission of clustered SQ (virtual photons) which gives rise to the static electric field of charged particles and would explain the great discrepancy in magnitude between gravitational and electrostatic interactions, since in a virtual photon the force would be concentrated into a single point and for a shorter time, respect to the absorption of many SQ during a longer time. The current Feynman diagram showing the self-energy of a charged particle (fig. 10.a) should be in this case replaced by that in fig. 10.b.

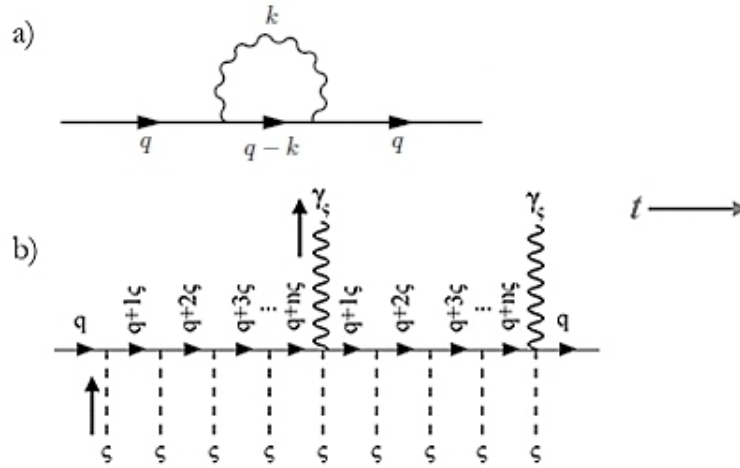


Figure 10: current (a) and suggested (b) Feynman diagram for describing a charged particle's self-energy.

Where virtual photons constituting the electric field are respectively referred to as  $k$  (10.a) and  $\gamma_\zeta$  (10.b). The reabsorption of virtual photons in (10.a) accounts for the principle of energy conservation, while in (10.b) the principle is observed thanks to the energy balance between absorbed SQ and periodically emitted virtual photons. Assuming that virtual photons are emitted from the vertex of the superfluid conical vortices, the electrostatic interaction between two charged particles may be described as in fig. 11:

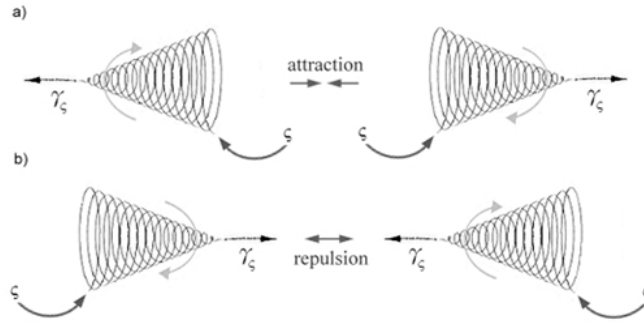


Figure 11: Representation of superfluid conical vortices of SQ corresponding to unlike (a) and like (b) charge particles. The combined action of SQ absorption and recoil due to virtual photons emission (static electric field) determines attraction or repulsion. No exchange or reabsorption of virtual photons is predicted in this theory.

We can note an interesting parallelism with the force of Van der Waals, which can be attractive or repulsive, depending on the mutual orientation of the molecules. In this case too, the electrostatic interaction between two vortex-particles would be anisotropic.

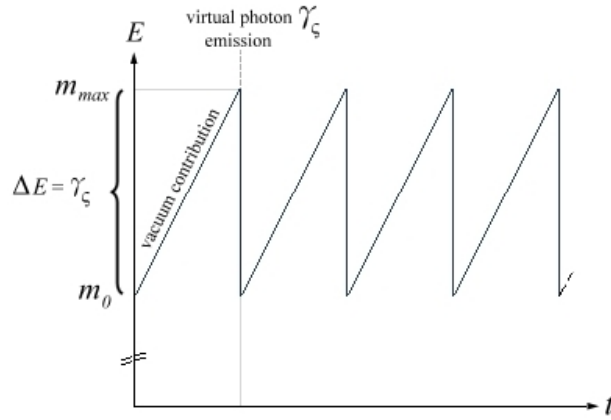


Figure 12: sawtooth electro-gravitational oscillator for a charged particle expressing its energy variation while producing gravitational pull and a static electric field.

The time-dependent equation for the chart in fig. 12 is

$$m_{eff}(t) = (t - [t])k + m_0$$

where  $m_{eff}$  is the effective mass of the particle, which would therefore rapidly oscillate between two values ( $m_0, m_{max}$ ) as described in fig. 12. It may be inferred that in absence of virtual photons emission the particle would

be forced to decay, as indeed observed in unbound neutral particles, such as neutrons (fig. 13).

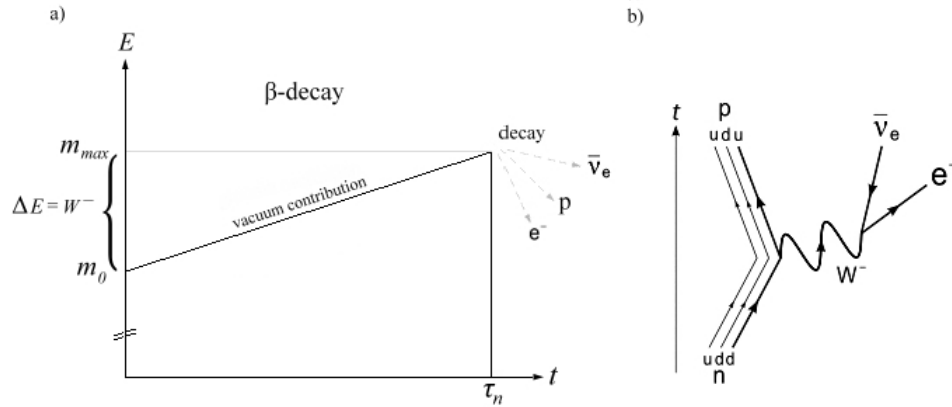


Figure 13:  $\beta$ -decay in unbound neutrons, (a) caused by vacuum energy absorption (SQ) equal to the mass of a  $W^-$  gauge boson ( $80.4 \text{ GeV}/c^2$ ), being  $\tau_n$  the neutron's mean lifetime. (b) Feynman diagram for neutron's decay.

A theory's prediction would therefore be that of a greater mass of the neutron before it decays. Electroweak interaction may consequently be explained within a QSSM and by describing particles as conical vortices of SQ. Let's now see how these assumptions may also account for quantum symmetries, by introducing the concept of mutual or conflicting torques. Fig. 14 shows what mutual torque is: a condition which two adjacent vortices don't destroy themselves in. This condition would justify Pauli exclusion principle, while that of conflicting torques accounts for the matter-antimatter annihilation, as shown in fig. 15.a.

Figure 14:



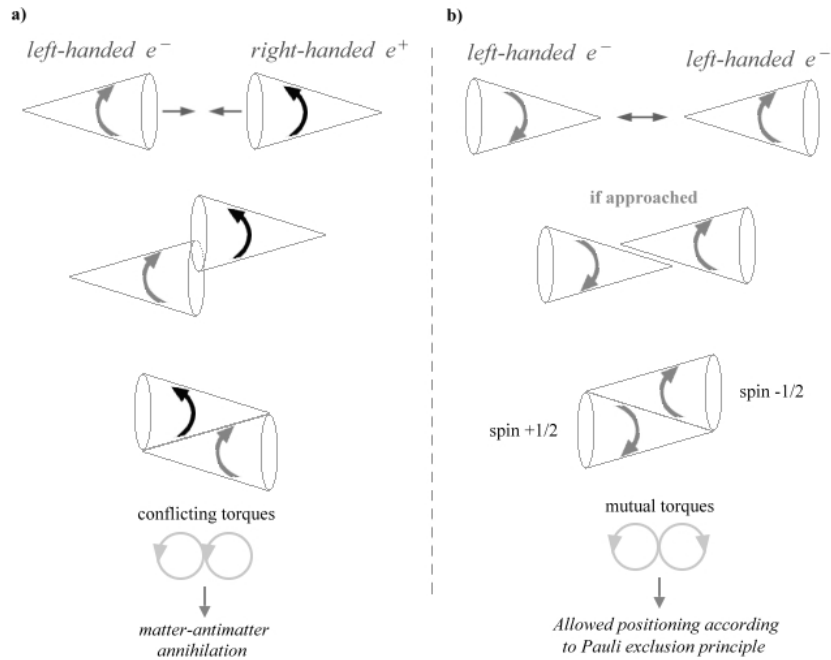


Figure 15: on the left, scattering matter-antimatter (electron-positron) showing annihilation due their conflicting torques, compared (on the right) with the spin interaction of two antisymmetric electrons (representing Pauli exclusion principle).

In fig. 16, a system with more interacting particles is shown, according to the mechanism illustrated in fig. 11 and to Coulomb's law.

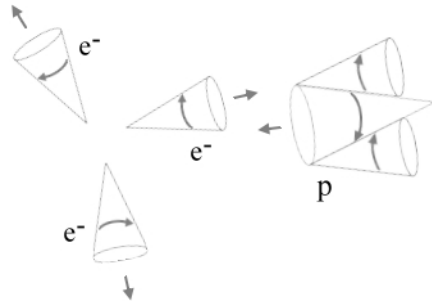


Figure 16: Representation of the interactions in a system with more particles, according to Coulomb's law.

The characteristic of gravity and electrostatic interaction of being radial forces, would be produced by the repositioning of a particle respect to another (fig. 17), while for massive, macroscopic bodies (such as Earth), the gravitational interaction would act radially because of all different positions occupied

by the large number of particles constituting them (that exerted by macroscopic bodies could be referred to as *white gravity*, in analogy to white as the sum of visible spectrum's frequencies), whose attraction - due to SQ absorption - statistically becomes radial. In this regard, it could be useful to repeat the Cavendish experiment using various crystals (differently positioned) as masses, looking for slight gravitational aberrations.

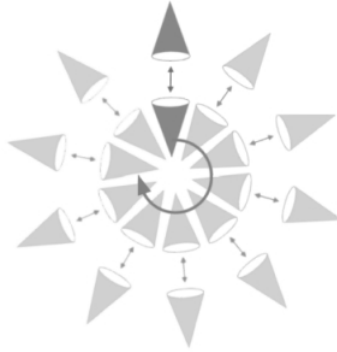


Figure 17: radially generated by the repositioning of a particle caused by the presence of a second particle (2D example).

This would also explain why atoms are neutral: their protons and electrons still emit virtual photons from the vertex of the cones but since they're already positioned in full interaction the ones respect to the others (due to proximity), no repositioning occurs respect to a third charge, unless ionization occurs. Thus, as far as the electrostatic interaction is concerned, the system is neutral. Let's now return to the neutron. Fig.18 describes it according to the QSSM, where each vortex corresponds to a quark:

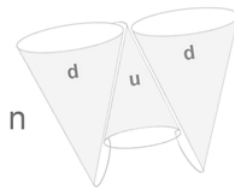


Figure 18: Structure of a neutron in the QSSM.

Three left-handed quarks are positioned with mutual torques. Each vertex is linked to another quark by a flow of gluons: this impedes the emission of virtual photons and prevents the neutron to produce a static electric field. Moreover, we see that the down quark on the right is not connected with the vertex of the central up quark. Since it possesses less connections than the other two, the ratio between absorbed energy (SQ) and shared gluonic energy will be more

disadvantageous for it and it will therefore be the first to reach energy instability. This may be the reason why one down quark in unbound neutrons decays after a mean life of about 15 minutes. The accumulation of SQ inside the down quark would correspond to what occurs during that time. A further prediction of this theory is that the quarks of a neutron should be able to be more distanced than those of a proton (before the gluonic tube breaks giving rise to a new pair quark-antiquark), because of a greater availability of SQ flowing into the gluonic tube and able to elongate it.

## 4 Quantum chromodynamics and nuclear force according to the QSSM.

The presence of less connections in a down quark may also help us to better understand what occurs when neutrons are bound in the nucleus and why, in that circumstance, they don't decay. The mechanism of the nuclear force (residual strong interaction) would indeed exactly target that *weaker* quark, by tearing it out from the neutron and preventing it to decay once it is repositioned into the proton (which emits virtual photons), as below illustrated in fig. 19.

When a neutron and a proton are at a distance of about 1 fm, a strong attractive force acts between them. The down quark in the neutron, which has a weaker bond with the central up quark, will establish a gluonic connection with the proton, because of its proximity to it. This will attract the down quark towards the proton, until the gluonic tube will break, generating a pair of down and antidown quarks. The new down quark takes the place of the other in the neutron (preventing the latter from decaying, as it was reaching its critical mass limit, as shown in fig. 13). At the same time the down antiquark binds to the old down quark, forming a meson ( $\pi^0$ ) and the reason why they do not undergo annihilation is that the antiquark is positioned upside-down (fig. 19.c). Thus their torques don't conflict as it normally occurs when matter encounters antimatter (fig. 15.a). The existence of neutral pions could therefore be an evidence for describing subatomic particles as conical superfluid vortices. Later, the antiquark annihilates the proton's down quark (this time their torques are indeed conflicting) and the inserted down quark can finally begin to emit virtual photons, getting rid of its excess mass and avoiding decay.

The concept of mutual torques also conduces to a quantum space superfluid reinterpretation of quantum chromodynamics, where the octet of gluons may be described as a flow of SQ, clustered into gluons by particles' vorticity and coursing through the vortices (quarks), a clustering process similar to virtual photons' formation (spin 1 gauge bosons in both cases), where gluons have however different energy and role. Indeed, the exchange of gluons between two vortices with mutual torques would occur along all their points of contact. For simplification, fig. 20 shows the octet on a single plane.

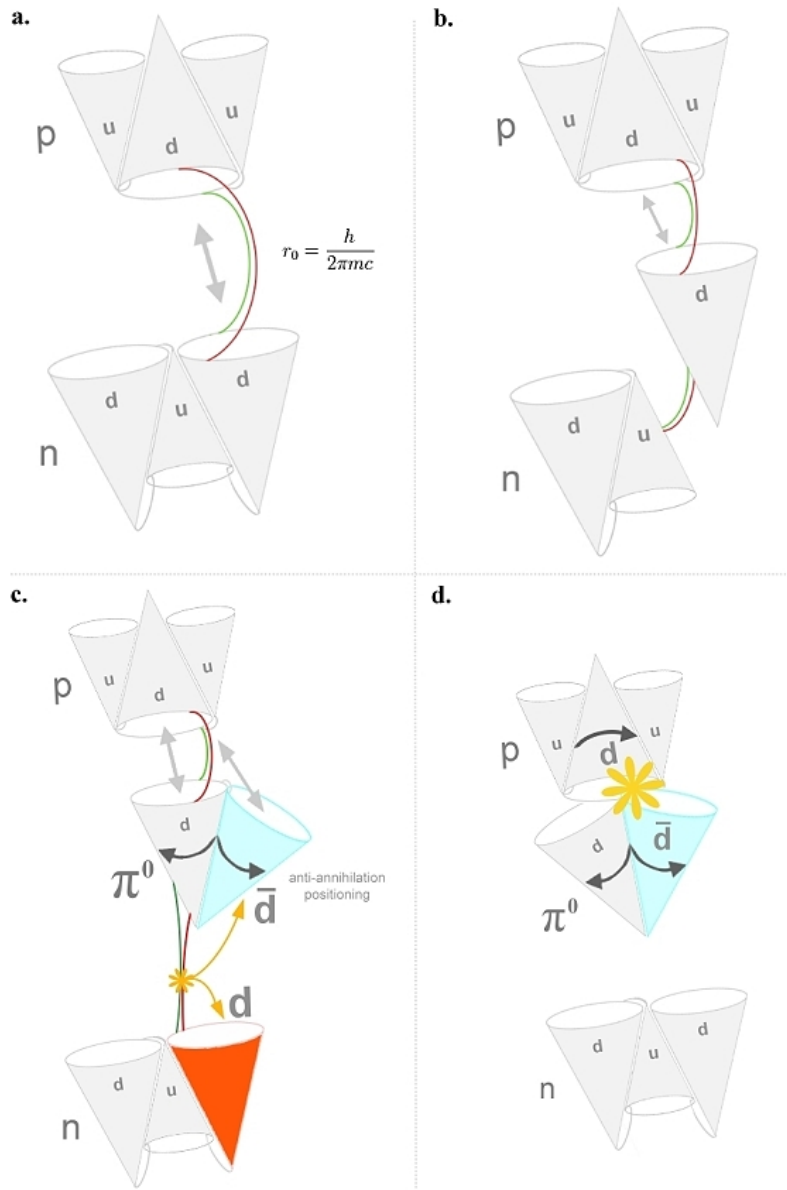


Figure 19: QSSM's reinterpretation of the nuclear force. The special upside-down positioning of the antiquark avoids annihilation with the down quark in the meson, thanks to their mutual torques.

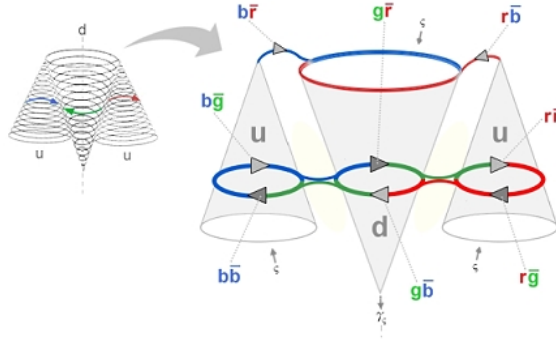


Figure 20: A proton according to the QSSM. On the left a three-quarks (uud) system possessing mutual torques. On the right the gluonic field (octet) is shown as a continuous flux among the quarks.

In agreement with the concept of mutual torques, the correct quarks geometry for a baryon would be then represented in fig. 21.a:

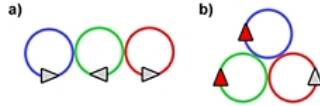


Figure 21: Suggested quarks' geometry in a baryon. The case (a) is the only one allowed, while in (b) two quarks always possess conflicting torques, causing a forbidden state according to quantum space's superfluid chromodynamics.

## 5 Merger with special and general relativity: toward superfluid relativity.

The above discussed quantum fluid dynamic reinterpretation of particles and interactions is fully compatible with special and general relativity. With the latter, since it can mathematically describe both Newton's law of universal gravitation and the time-dependent geodesic of general relativity, which expresses the attractive force.

Beyond this overview, a comprehensive analysis of special and general relativity according to superfluid quantum space can be found in another work of mine [11].

For instance, a ray of light passing near a massive body, such a star, will be deflected since moving in the strong flow of SQ directed toward the center of the mass. The mathematics of general relativity is therefore still valid according to this theory but a quantum fluid dynamic reformulation of it would be appropriate. Here *curved spacetime* is indeed a consequence of SQ absorption, not the free-standing cause of gravity.

Compatible with special relativity, since it explains *inter alia* why both the increase of gravity and that of speed conduce to the same relativistic effects of inertial mass increase and time dilation. If indeed gravity is here described as a flow of SQ, classical fluid and aerodynamics may be enough to explain the reason: both a (gravitational) flow of space quanta acting against a stationary body and a body moving through quantum space lead to the same result, just like in aerodynamics the wind exerting a force on a stationary body can be equalled to the contrary situation of the so-called aerodynamic drag, where the object moves through the fluid. In short, this is the way the QSSM explains and confirms the weak equivalence principle ( $m_i = m_g$ ), then Einstein's equivalence principle between gravitation and acceleration. Thus, a body moving at relativistic speeds will undergo gravitational mass increase like when located in a greater gravitational field. In the former case, asymptoticity in acceleration is probably due to reasons below outlined in this section.

The reason why gravity (and therefore speed) slows down time is connected to mass increase due to a greater gravitational field. Each clock (where clock broadly means any object doing actions, thus also our body and all biological processes, atoms, molecules *etc.*) indeed performs a movement thanks to a given amount of energy, which doesn't increase if its components become heavier because of a stronger gravitational field: in this case time would consequently be dilated.

While length contraction - referred to measurements, not to bodies' physical structure - is a direct consequence of time dilation and doesn't need any further justification.

The reason why photons are the fastest objects in existence and why, then, Lorentz factor mathematically uses  $c$  as a key parameter to measure the relativistic effects, may be sought among the fluid dynamic properties of quantum space<sup>2</sup>: at one point, despite the greater energy provided to the accelerated object, space's structure cannot undergo any further deformation. We could assert that, since a photon is the lightest particle (zero mass) in existence, it is understandable that any other body possessing greater mass, reaches in advance the speed which space (*i.e.* the medium which it moves through) can't undergo further deformation at. It's anyway plausible that photon, since moving through quantum space, could even be a pulse itself of SQ, just like a phonon occurring in superfluid quantum space: also this hypothesis is discussed in more detail in the document concerning superfluid relativity [11].

To conclude, the gravitational redshift is also predicted within this quantum fluid dynamic approach, since it depends on time dilation. An observer's clock located in a weaker gravitational field will tick faster, thus the observer will measure a longer period (hence a redshift) for light directed off the field, with respect to an observer located where gravity is stronger. The same equations used in GR, where  $\omega_1$  is frequency as measured by the observer can therefore still be used in the superfluid approach:

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<sup>2</sup>for instance its density, viscosity (even if near-zero), compressibility.

$$\omega_1 = \omega_0 \cdot \frac{\sqrt{1 - \frac{R_S}{r}}}{\sqrt{1 - \frac{R_S}{r'}}}$$

## 6 The double face of entropy.

According to this theory, reality may be considered as a gigantic sum of topological defects in quantum space. And if particles and fundamental forces exist thanks to a structuring of space (vortices, flows, clusterings of SQ), it follows that the value of entropy is indirectly proportional - and a zero sum parameter - when measured from our point of view respect to that of quantum space. The existence of a vortex represents, for instance, a greater entropy in quantum space respect to the initial situation of a tranquil sea of SQ. On the contrary, a particle represents for us a lower entropy state, respect to the bare sea of SQ, whose energy is not measurable or usable and therefore coincides with the maximum state of entropy. While entropy therefore increases for the universe, it decreases for quantum space and vice versa.

$$\Delta S_{su} + \Delta S_{qst} = 0 \tag{11}$$

where  $S_{su}$  is the entropy of the structured universe (corresponding to everything unless quantum space) and  $S_{qst}$  is the entropy of quantum space.

## Conclusions

By starting from the assumption that quantum space exists and by describing fundamental particles as superfluid conical vortices of space quanta, we can build up a model which seems to appropriately describe particles and interactions of the standard model, quantum gravity, as well as to explain and incorporate special and general relativity. This also suggests that the existence of quantum space could be indirectly demonstrated by confirming some predictions linked to the above described model, such as the mass-energy increase in unbound neutrons before they decay. Many other predictions of this theory already coincide with what we observe: relativistic effects, Pauli exclusion principle, matter-antimatter annihilation, quantum chromodynamics *etc.* Moreover, if particles are seen as topological defects of quantum space, entropy should be reformulated as a composed, zero sum, parameter. In light of this and after this overview, a superfluid dynamic approach to quantum physics (QFT, QED, QCD) and to special and general relativity, where quantum space is the superfluid, should be mathematically scrutinized and experimentally tested.

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