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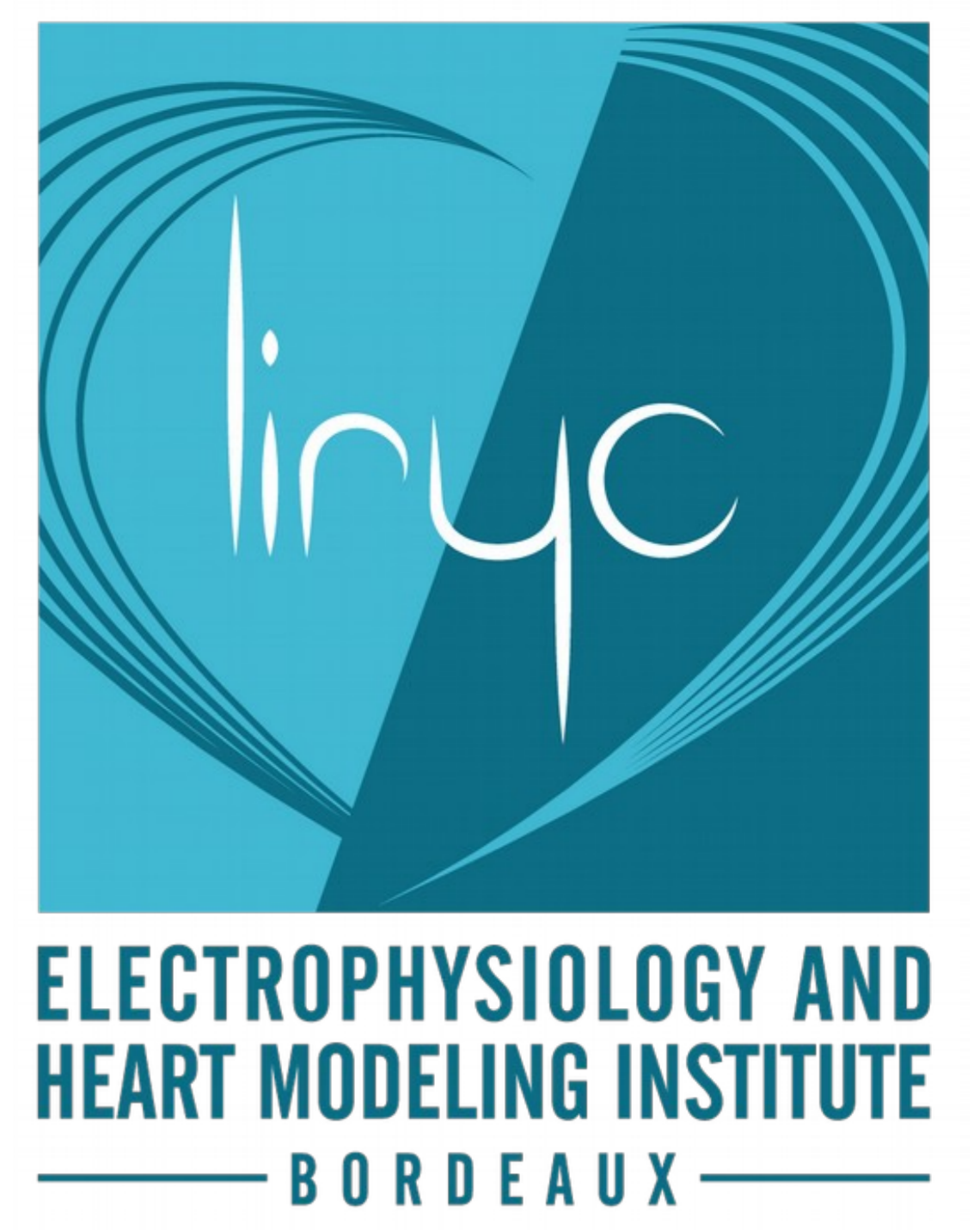
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# Inria New Mathematical approaches in Electrocardiography Imaging inverse problem

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## Context and objectives

### Major objectives

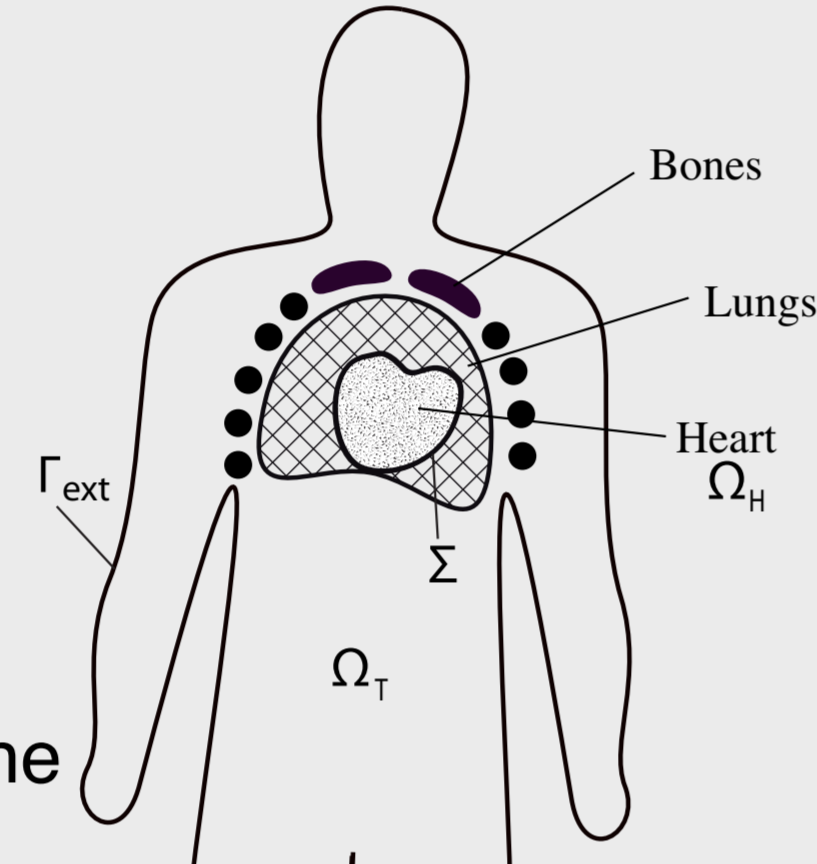
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

## Mathematical model

### Forward model

- If we know the heart potential we can compute the electrical potential

$$\begin{cases} \operatorname{div}(\sigma_T \nabla u_T) = 0, & \text{in } \Omega_T, \\ \sigma_T \nabla u_T \cdot \mathbf{n} = 0, & \text{on } \Gamma_{\text{ext}}, \\ u_T = u_e, & \text{on } \Sigma. \end{cases}$$



### Inverse problem

- If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

$$\begin{cases} \operatorname{div}(\sigma_T \nabla u_T) = 0, & \text{in } \Omega_T, \\ \sigma_T \nabla u_T \cdot \mathbf{n} = 0, \text{ and } u_T = T, & \text{on } \Gamma_{\text{ext}}, \\ u_T = ?, & \text{on } \Sigma. \end{cases}$$

## MFS approach

- Solve the linear system

$$\hat{A} \vec{a} = \vec{b}$$

$$\hat{A} = \begin{pmatrix} 1 & f(\|x_1 - y_1\|) & \dots & f(\|x_1 - y_M\|) \\ \vdots & \vdots & \dots & \vdots \\ 1 & f(\|x_N - y_1\|) & \dots & f(\|x_N - y_M\|) \\ 0 & \frac{\partial f(\|x_1 - y_1\|)}{\partial n} & \dots & \frac{\partial f(\|x_1 - y_M\|)}{\partial n} \\ \vdots & \vdots & \dots & \vdots \\ 0 & \frac{\partial f(\|x_N - y_1\|)}{\partial n} & \dots & \frac{\partial f(\|x_N - y_M\|)}{\partial n} \end{pmatrix} \quad \vec{a} = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_M \end{pmatrix} \quad \vec{b} = \begin{pmatrix} u_T(x_1) \\ \vdots \\ u_T(x_N) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$f(r) = \frac{1}{4\pi r} \quad \begin{matrix} x_1, \dots, x_N : \text{Torso points} \\ y_1, \dots, y_M : \text{Heart points} \end{matrix}$$

- Regularization with CRESO

## Optimal control approach

- Poincaré–Steklov variational formulation of the inverse problem.

- Minimize the following energy functional

$$J(\lambda) = \frac{1}{2} \int_{\Omega_T} (\nabla u_D(\lambda) - \nabla u_N(\lambda))^2.$$

Subject to

$$\begin{cases} \operatorname{div}(\sigma_T \nabla u_D(\lambda)) = 0, & \text{in } \Omega_T, \\ u_D(\lambda) = T, & \text{on } \Gamma_{\text{ext}}, \\ u_D(\lambda) = \lambda, & \text{on } \Sigma. \end{cases} \quad \begin{cases} \operatorname{div}(\sigma_T \nabla u_N(\lambda)) = 0, & \text{in } \Omega_T, \\ \sigma_T \nabla u_N(\lambda) \cdot \mathbf{n} = 0, & \text{on } \Gamma_{\text{ext}}, \\ u_N(\lambda) = \lambda, & \text{on } \Sigma. \end{cases}$$

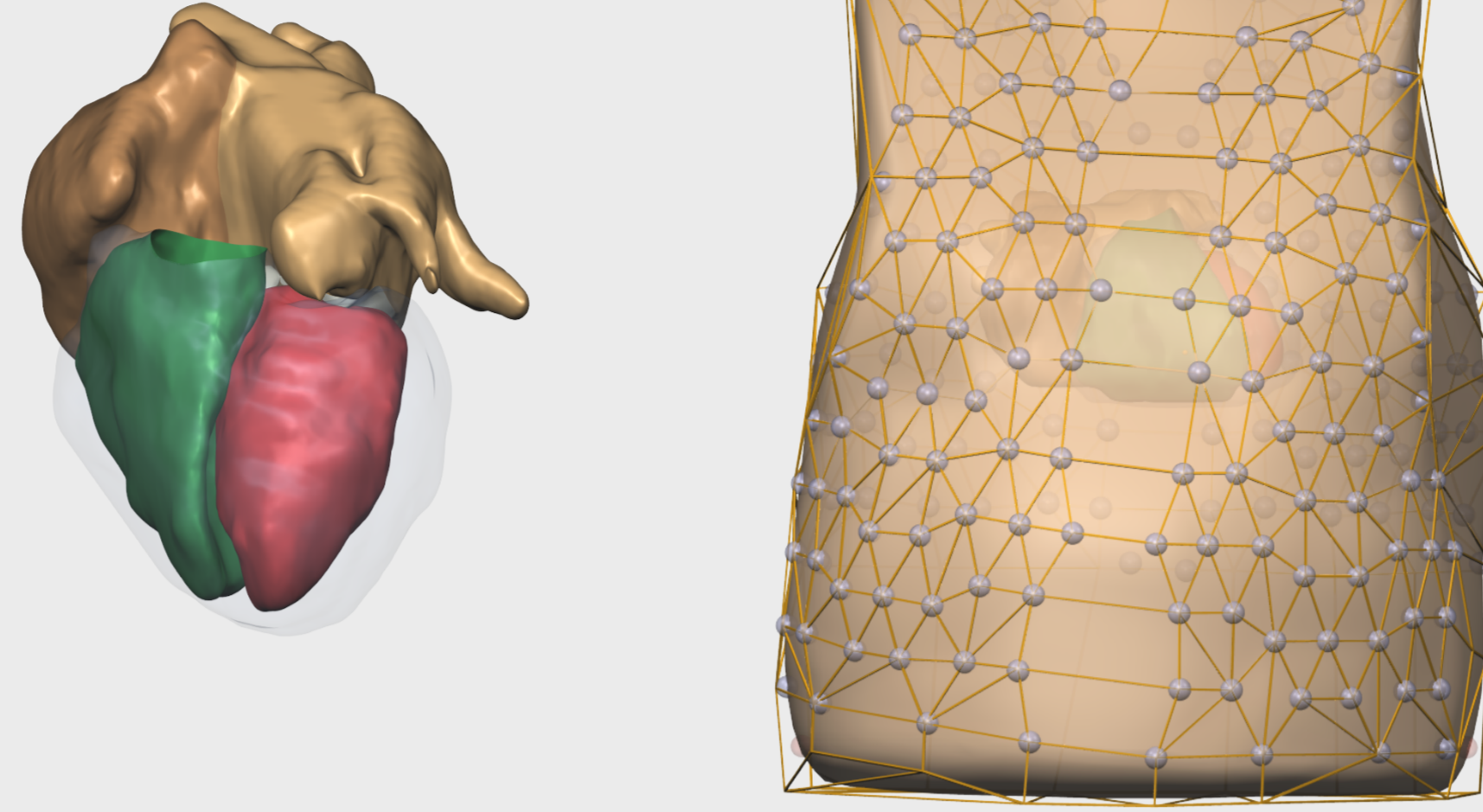
- Descent gradient methods

$$\nabla_{\lambda} J(\lambda) = \sigma_T (\nabla u_D(\lambda) - \nabla u_N(\lambda)) \cdot \mathbf{n}_{\Sigma}$$

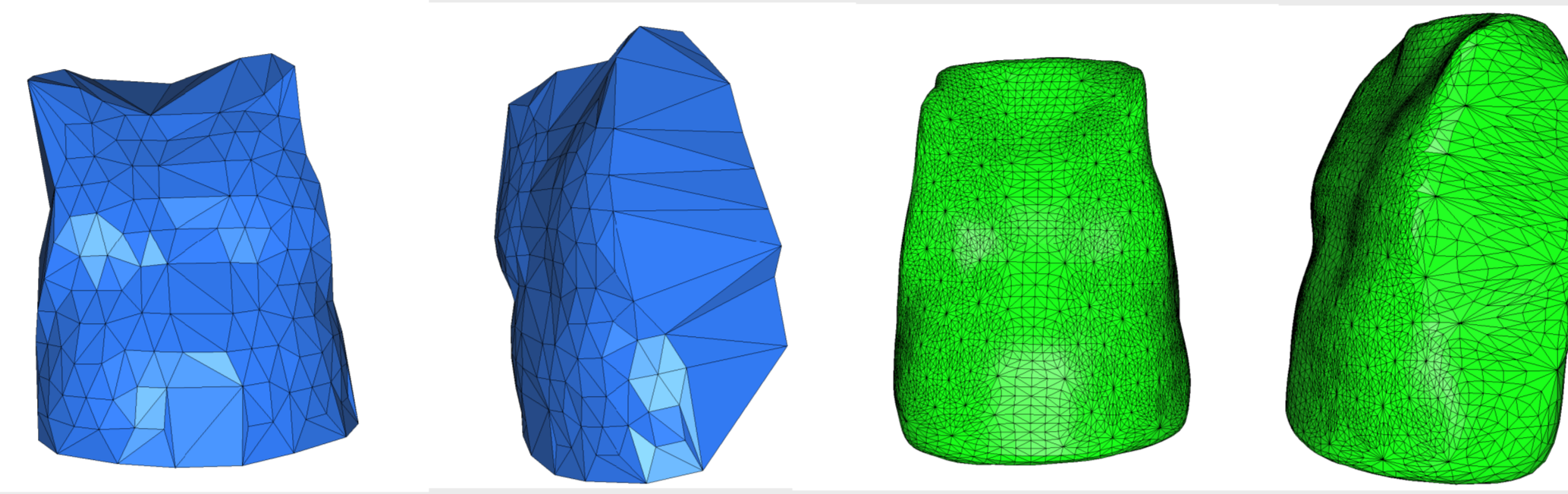
- Discretization with Finite elements method.

## In silico gold standard

### Anatomical data



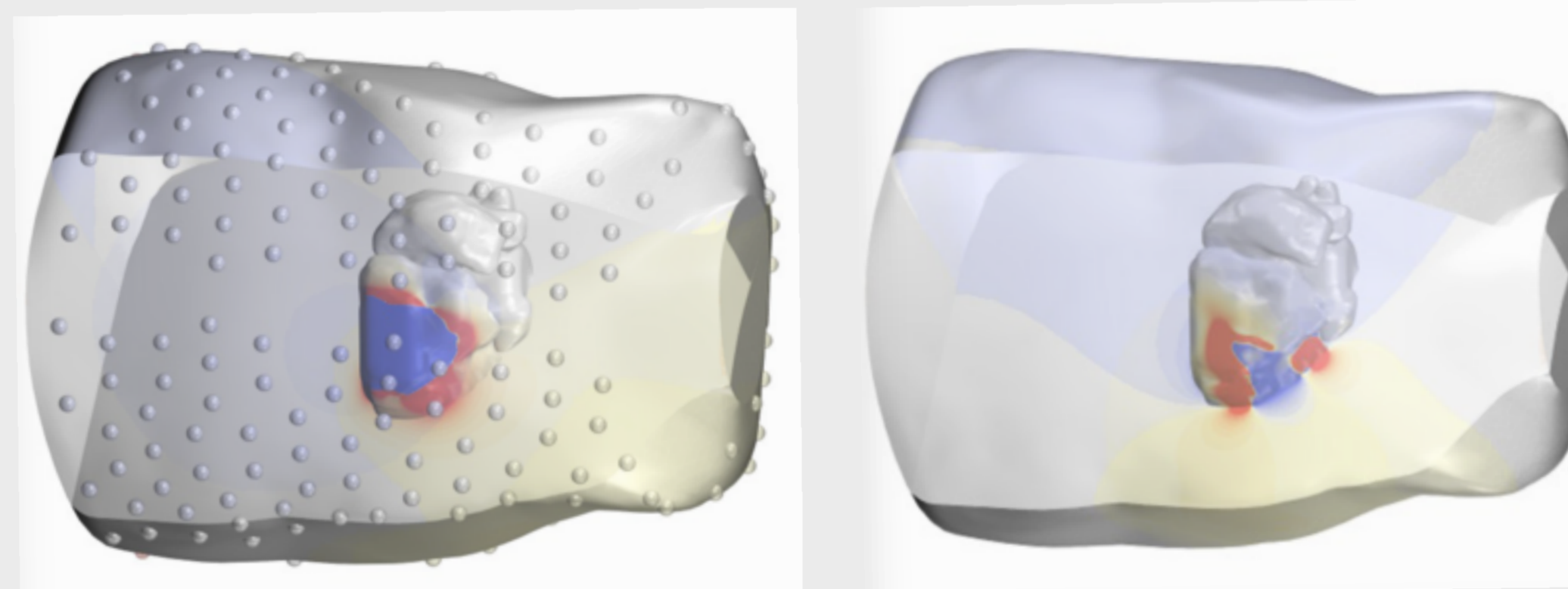
Computational heart and torso anatomical models + electrodes position



Computational torso meshes: 250 nodes mesh (blue). More accurate FE mesh with 6400 nodes (green)

### Simulated cases

- 6 single and double stimuli
- 14 reentry cases



## Results

### Relative error and correlation coefficient

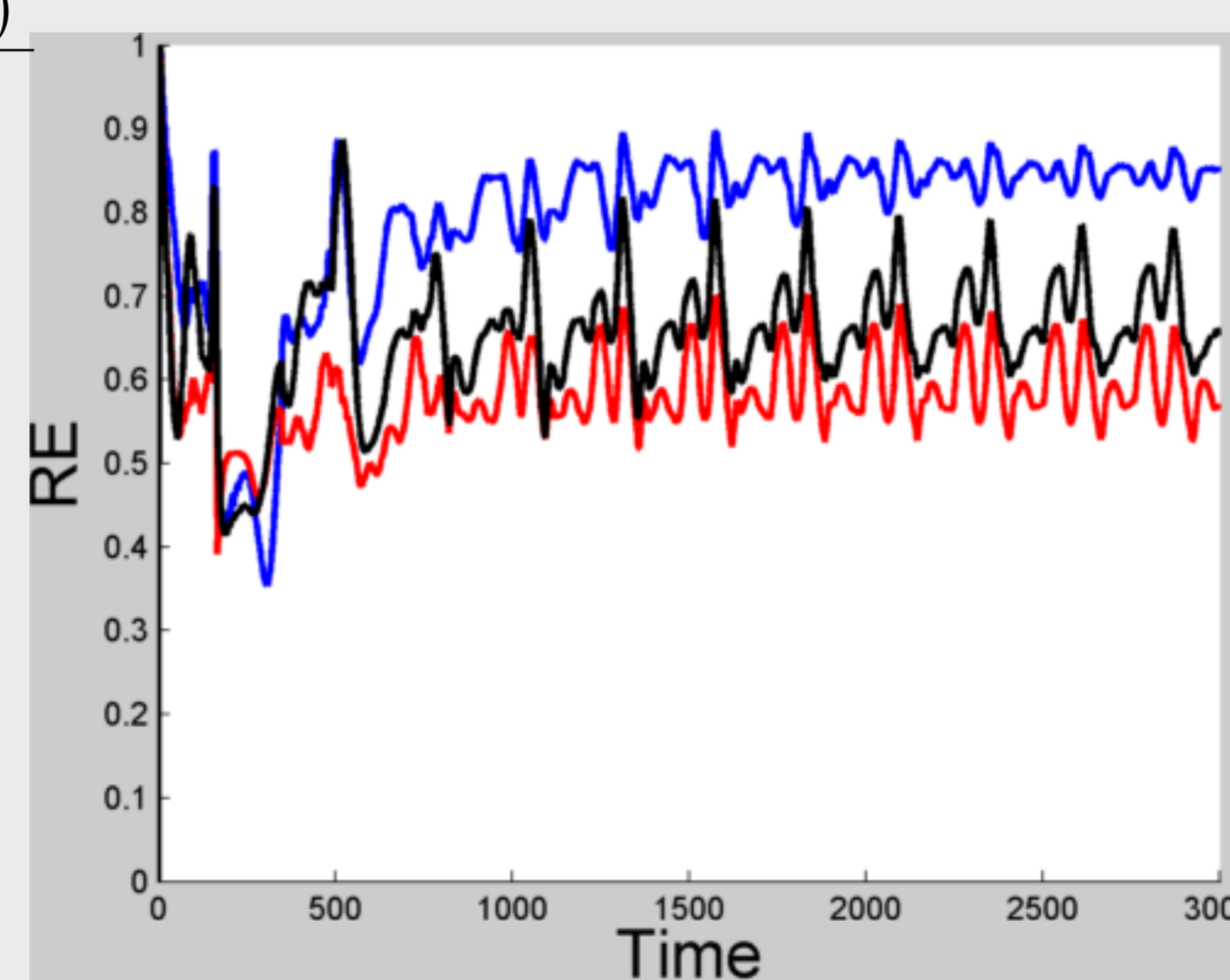
Cases	metric	MFS + CRESO	O.C interpolated	O.C refined data
Single and double stimulus (6 cases)	RE	0.81±0.04	0.71±0.02	0.59±0.06
	CC	0.57±0.07	0.7±0.03	0.8±0.04
Re-entry (VT) (14 cases)	RE	0.78±0.06	0.67±0.04	0.59±0.05
	CC	0.6±0.08	0.73±0.04	0.83±0.04
All 20 cases	RE	0.79±0.06	0.69±0.04	0.59±0.05
	CC	0.59±0.07	0.72±0.04	0.82±0.04

$$RE = \sqrt{\frac{\sum_{i=1}^L (V_i^C - V_i^M)^2}{\sum_{i=1}^L (V_i^M)^2}}$$

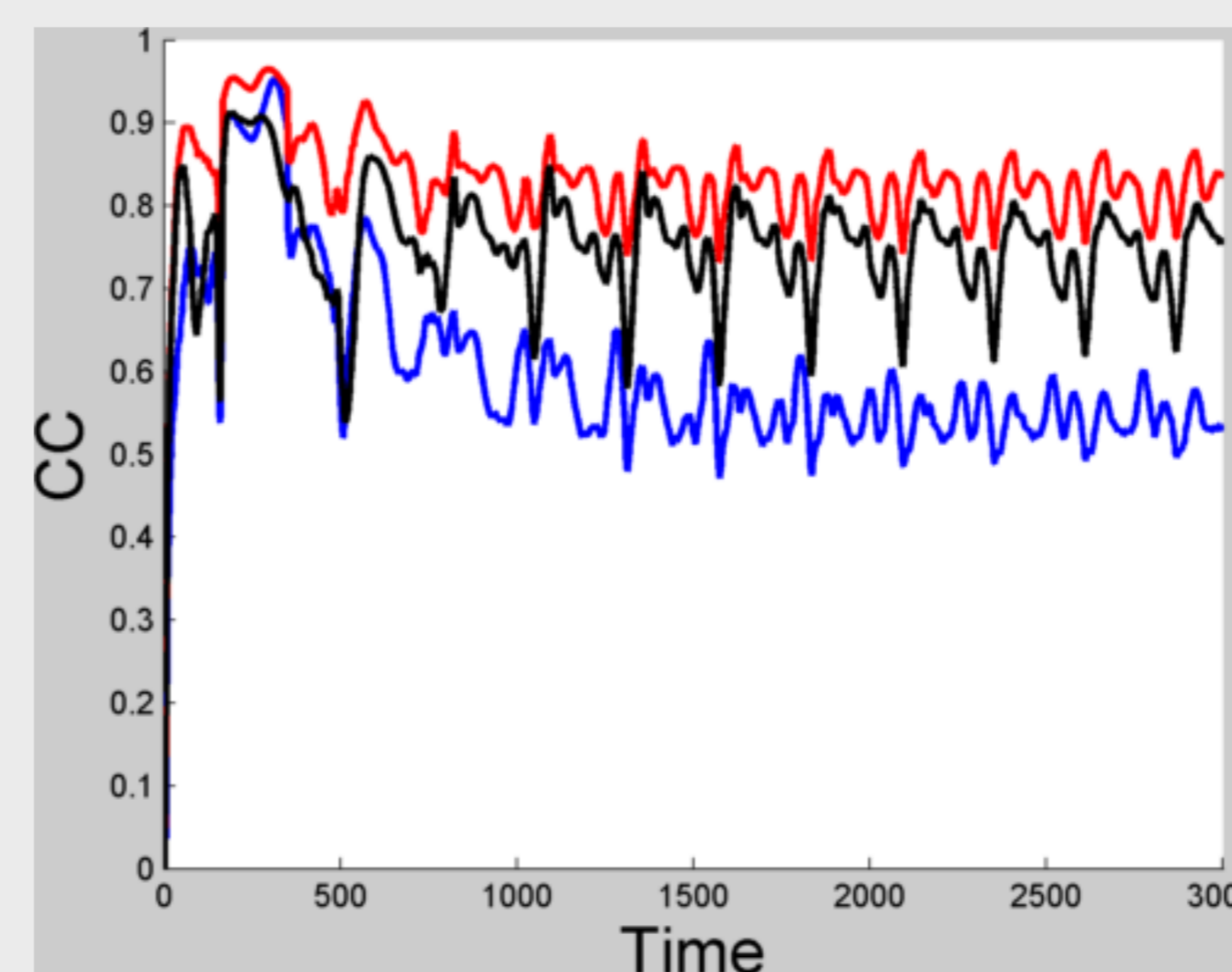
- MFS + CRESO (blue line)

- Optimal Control + Tikhonov data from vest interpolated on Finer mesh (black line)

- Optimal Control + Tikhonov more rich data from Finer mesh (red line)

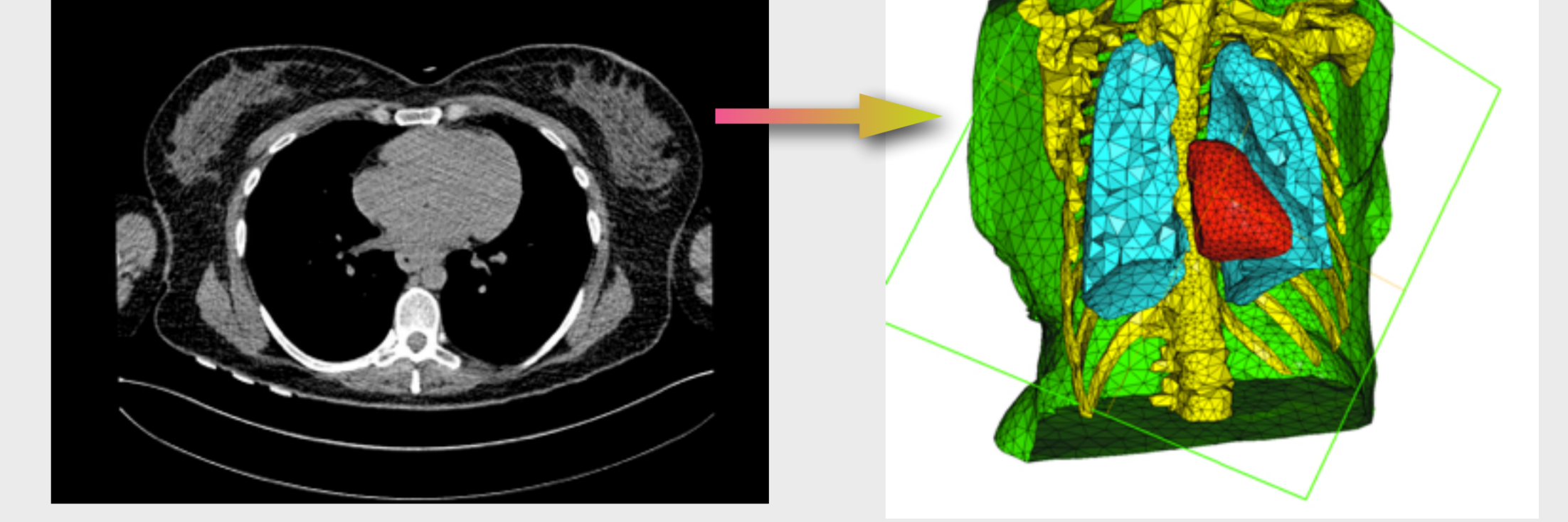


$$CC = \frac{\sum_{i=1}^L (V_i^M - \bar{V}^M)(V_i^C - \bar{V}^C)}{\sqrt{\sum_{i=1}^L (V_i^M - \bar{V}^M)^2} \sqrt{\sum_{i=1}^L (V_i^C - \bar{V}^C)^2}}$$

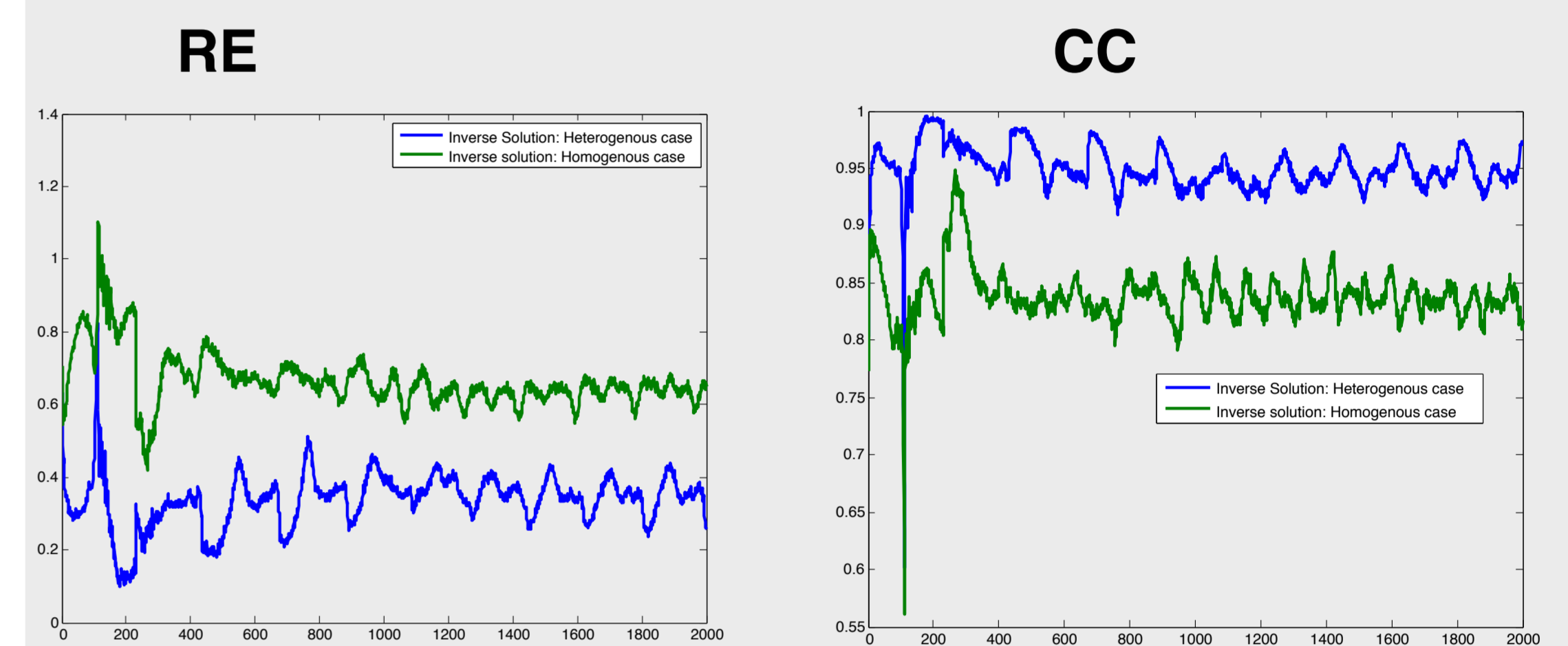


## Torso Heterogeneity effect

### Anatomical data

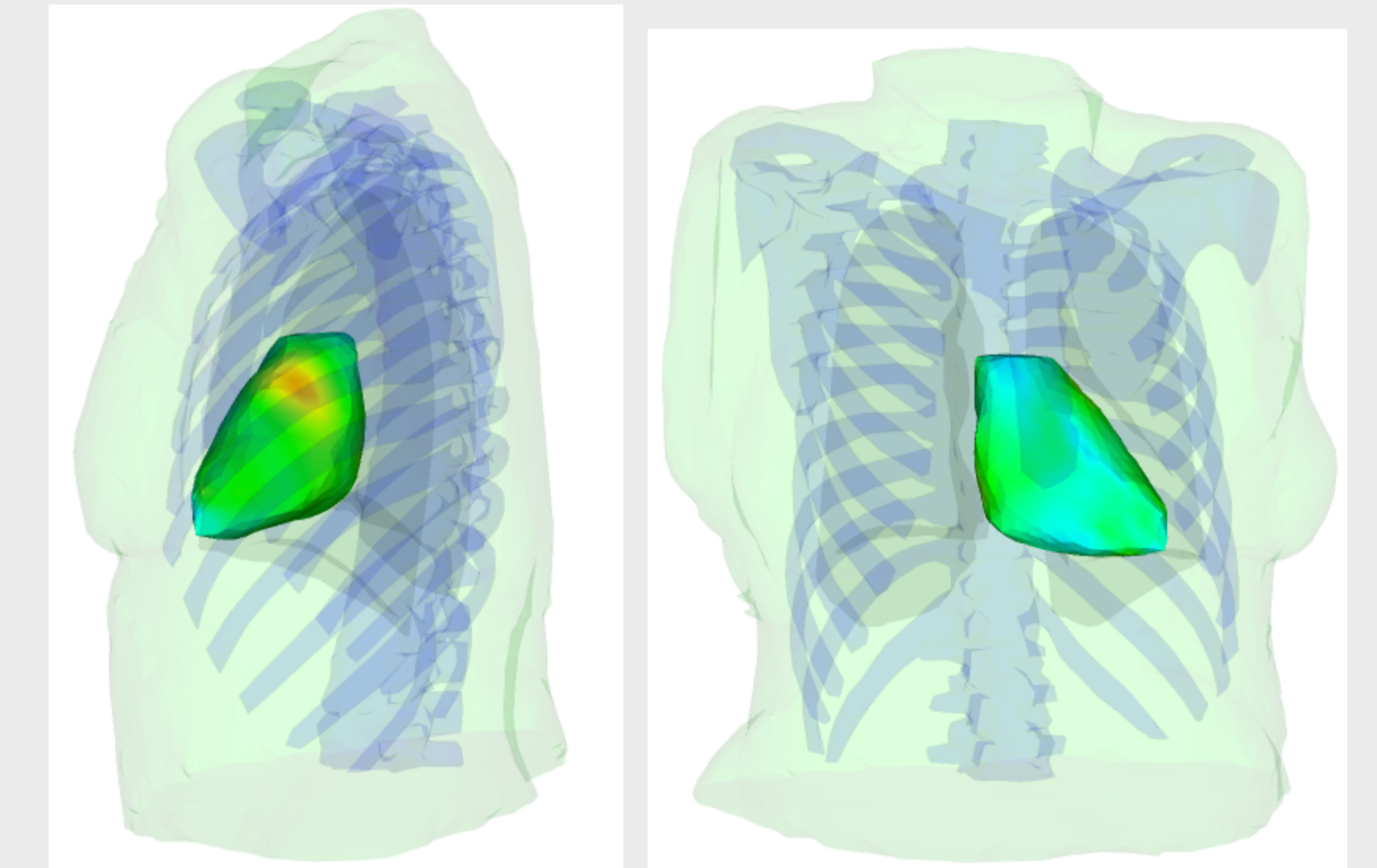


CT scan of a 43 years old women (left) and computational mesh obtained after segmentation (right)



Comparison of the optimal control solution for heterogeneous (bleu) and homogeneous (green) torso conductivities

### Space distribution of the error



Space distribution of the RE over time: Left (left) and right (right) ventricles views

### Remarks

- Introducing the torso heterogeneity is natural with FEM. also anisotropy could be introduced
- The error is more important in the left ventricle

## Conclusions

### Main results and perspectives

- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.82±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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