New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we knew the heart potential we can compute the electrical potential
\[
\begin{align*}
\div (\sigma_r \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\frac{\sigma_r \nabla u_T}{n} &= 0, \text{ on } \Gamma_{ext}, \\
u_T &= u_e, \text{ on } \Sigma.
\end{align*}
\]

Inverse problem
If we knew the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[
\begin{align*}
\div (\sigma_r \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\sigma_r \nabla u_T \cdot n &= 0, \text{ and } u_T = \frac{\mathcal{H}}{\Omega_{1}}, \text{ on } \Gamma_{ext}, \\
u_T &= 7, \text{ on } \Sigma.
\end{align*}
\]

MFS approach
Solve the linear system
\[
\mathbf{A} \mathbf{a} = \mathbf{b}
\]
\[
\mathbf{A} = \begin{bmatrix}
\frac{1}{\Omega_{1}} & \cdots & \frac{1}{\Omega_{1}} \\
\vdots & \ddots & \vdots \\
\frac{1}{\Omega_{1}} & \cdots & \frac{1}{\Omega_{1}}
\end{bmatrix}
\]
\[
\mathbf{b} = \begin{bmatrix}
\frac{w_{1}(x_{1})}{\Omega_{1}} \\
\vdots \\
\frac{w_{j}(x_{j})}{\Omega_{1}} \\
\frac{w_{T}(x_{T})}{\Omega_{1}}
\end{bmatrix}
\]
\[
\mathbf{a} = \begin{bmatrix}
y_{1} \\
\vdots \\
y_{j} \\
y_{T}
\end{bmatrix}
\]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional
\[
J(\mathbf{a}) = \frac{1}{2} \int_{\Omega_{1}} (\nabla u_{M}(\mathbf{a}) - \nabla u_{G}(\mathbf{a}))^2 d\Omega
\]

Subject to
\[
\begin{align*}
\div (\sigma_r \nabla u_{M}(\mathbf{a})) &= 0, \text{ in } \Omega_{1}, \\
u_{M}(\mathbf{a}) &= \frac{\mathcal{H}}{\Omega_{1}}, \text{ on } \Gamma_{ext}, \\
u_{G}(\mathbf{a}) &= \lambda, \text{ on } \Sigma.
\end{align*}
\]

Descent gradient methods
\[
\nabla J(\mathbf{a}) = \sigma_r (\nabla u_{M}(\mathbf{a}) - \nabla u_{G}(\mathbf{a})) n_{\Sigma}
\]

Discretization with Finite elements method.

In silico gold standard

Anatomical data
Computational heart and torso anatomical models + electrodes position

Simulated cases
- 8 single and double stimuli
- 14 entry cases

Torso Heterogeneity effect

Comparison of the optimal control solution for heterogeneous (blue) and homogeneous (green) torso conductivities

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C integrated</th>
<th>O.C refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimuli (6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>CC</td>
<td>0.75±0.07</td>
<td>0.74±0.08</td>
<td>0.84±0.04</td>
</tr>
<tr>
<td>All 20 cases</td>
<td>CC</td>
<td>0.79±0.04</td>
<td>0.69±0.04</td>
<td>0.65±0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced.
- The error is more important in the left ventricle.

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control.
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient: RE was improved from 0.79±0.06 to 0.59±0.05
- CC was improved from 0.59±0.07 to 0.84±0.04.
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution.

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