

Phantom project

Alexandre Ancel ² Alexandre Fortin ¹ Simon Garnotel ³ Olivia Miraucourt ¹
Stéphanie Salmon ¹ Ranine Tarabay ²

¹University of Reims Champagne-Ardenne, Reims, France

²University of Strasbourg, IRMA / UMR 7501, Strasbourg, France

³University of Picardie Jules Verne, BioFlowImage laboratory, Amiens, France

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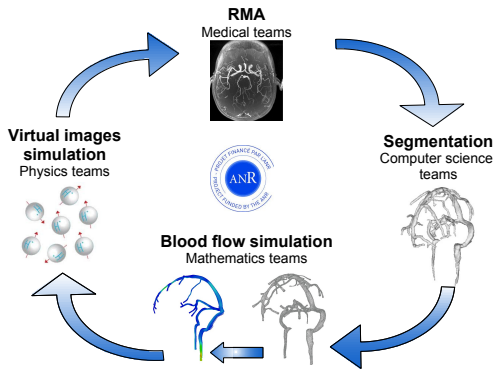


Figure : The VIVABRAIN project task loop

Phantom project goal:

- Validation of the CFD simulations:
 - Cross validation Feel++/FreeFem++
 - Validation with experimental data
- Validation of the MRI simulations
 - Comparison with the initial MRI
- Apply the task loop on the Phantom

Phantom

What is a phantom ? A device designed to reproduce some features of flows, compatible with the MRI.



Figure : Physical phantom for cerebral arteries.

Outline

- 1 Feel++/FreeFem++ cross validation
 - Numerical methods
 - Fluid simulation Results
- 2 Feel++ and FreeFem++ comparison with experimental data
- 3 MRI simulation results
- 4 AngioTk pipeline
- 5 Conclusion

Mathematical model for blood flow simulations

Blood : homogeneous, incompressible fluid, with “standard” Newtonian behaviour,

Mathematical model: unsteady Navier-Stokes equations:

$$\begin{aligned} \rho \frac{\partial \mathbf{u}}{\partial t} - 2\nabla \cdot (\mu \mathbf{D}(\mathbf{u})) + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p &= \mathbf{f}, \quad \text{dans } \Omega \times I \\ \nabla \cdot \mathbf{u} &= 0, \quad \text{dans } \Omega \times I \\ &+ \text{initial conditions} \\ &+ \text{boundary conditions} \end{aligned}$$

- $\Omega \subset \mathbf{R}^d (d \geq 2)$: domain
- \mathbf{u} : viscosity of the fluid ;
- p : pressure of the fluid ;
- $\mathbf{D}(\mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$: deformation tensor ;
- ρ and μ density and dynamic viscosity of the fluid.

Feel++ FE method: Oseen scheme

Let's consider a Dirichlet condition at the inflow and a Neumann BC at the outflow:

$$\mathbf{u} = \mathbf{u}_{in} \quad \text{on } \Gamma_{in}, \quad (1)$$

$$\boldsymbol{\sigma}(\mathbf{u}, p)\mathbf{n} = \mathbf{g} \quad \text{on } \Gamma_{out}. \quad (2)$$

For $\mathbb{V} = \{\mathbf{v} \in [H^1(\Omega)]^d \mid \mathbf{v} = \mathbf{0} \text{ on } \Gamma_w, \mathbf{v} = \mathbf{u}_{in} \text{ on } \Gamma_{in}\}$ and $\mathbb{M} = L_0^2(\Omega)$ the variational formulation reads: find $(\mathbf{u}, p) \in \mathbb{V} \times \mathbb{M}$ such that $\forall q \in \mathbb{M}$,

$\forall \mathbf{v} \in \{\mathbf{v} \in [H^1(\Omega)]^d \mid \mathbf{v} = \mathbf{0} \text{ on } \Gamma_w \cup \Gamma_{in}\}$, we have:

$$\int_{\Omega} \rho \frac{\partial \mathbf{u}}{\partial t} \cdot \mathbf{v} + \int_{\Omega} \rho (\mathbf{u} \cdot \nabla \mathbf{u}) \cdot \mathbf{v} + 2\mu \int_{\Omega} \mathbf{D}(\mathbf{u}) : \mathbf{D}(\mathbf{v}) \, dx - \int_{\Gamma_{out}} \mathbf{g} \mathbf{n} \cdot \mathbf{v} \, ds - \int_{\Omega} p \operatorname{div}(\mathbf{v}) \, dx =$$

$$\int_{\Omega} q \operatorname{div}(\mathbf{u}) \, dx =$$

Space discretisation: Taylor-Hood finite element $[P_c^{N+1}(\Omega_{(h,k_{geo})})]^d \times P_c^N(\Omega_{(h,k_{geo})})$

Time discretisation: Finite difference order 2

Convective term treatment: Extrapolation of order 2

FreeFem++ FE method: Method of characteristics

For every particle, we write:

$$\begin{cases} \frac{dX}{dt}(x, s; t) = u(t, X(x, s; t)) \\ X(x, s; s) = x \end{cases} \quad (5)$$

where $X(x, s; t)$ is the particle position at time t who was in x at time s .
That gives:

$$(\partial_t u + (u \cdot \nabla)u)(t_n, x) \sim \frac{u(t_{n+1}, x) - u(t_n, X^n(x))}{dt} \quad (6)$$

with $X^n(x) = x - u(t_n, x) \cdot dt + \mathcal{O}(dt^2)$.

We finally have:

$$\frac{\rho}{dt}(u^{n+1} - u^n \circ X^n) - \mu \Delta u^{n+1} + \nabla p^{n+1} = f^{n+1} \quad (7a)$$

$$\operatorname{div}(u^{n+1}) = 0 \quad (7b)$$

This method is implemented using the *convect* operator of FreeFem++.

Benchmark setup

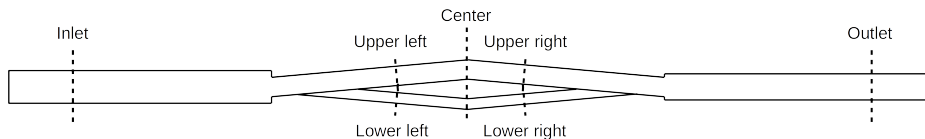


Figure : Radial slices where the velocity profiles are plotted

Geometry	h_{min}	h_{max}	h_{avrg}	Tetrahedrons	DOF
M1	$2 \cdot 10^{-1}$	$5 \cdot 10^{-1}$	$3 \cdot 10^{-1}$	157,245	769,662
M2	$2.5 \cdot 10^{-1}$	$6.25 \cdot 10^{-1}$	$3.75 \cdot 10^{-1}$	93,655	469,008
M3	$3 \cdot 10^{-1}$	$7.5 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	60,349	307,510

Table : The characteristics of the three types of geometries

Constant Poiseuille flow

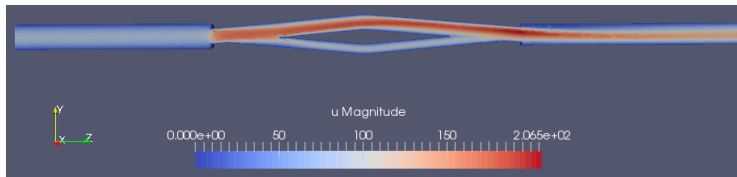
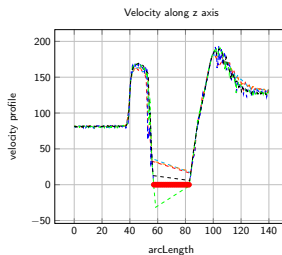
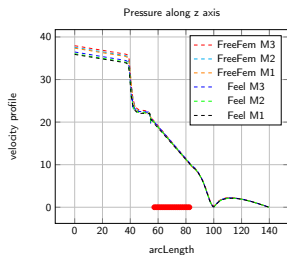


Figure : Velocity magnitude, M3 mesh, constant flow

Velocity and pressure magnitude along the centerline:



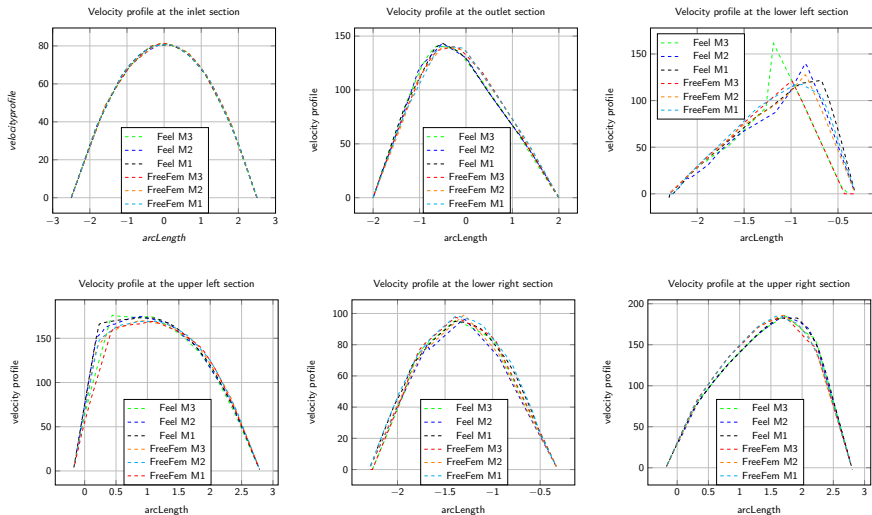
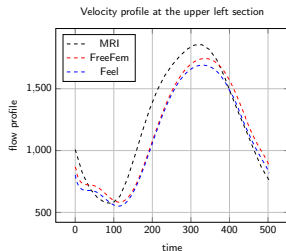
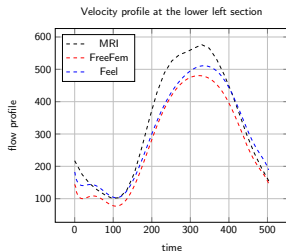
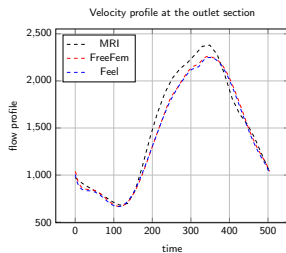
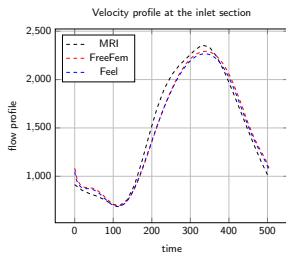


Figure : Feel++ vs FreeFem++ comparison on the M3 mesh with a constant flow (V_{min}), P2P1: Velocity profile at the right and left sections in the upper and lower channels

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Comparison of the numerical outputs with respect to the experimental measurements: (pulsatile flow)



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Mathematical model for MRI simulations

In MRI, the signal collected over time is generated by the temporal variations of the macroscopic magnetization of tissues. This signal contains all the information needed to reconstruct the final image. The most popular technique for MRI simulation is isochromat summation. The sample to be imaged is divided into equal subvolumes called isochromats, see Fig. 8. Those subvolumes are supposed to possess uniform physical properties: spin relaxation times T_1 , T_2 , T_2^* , equilibrium magnetization M_0 and magnetic susceptibility χ .

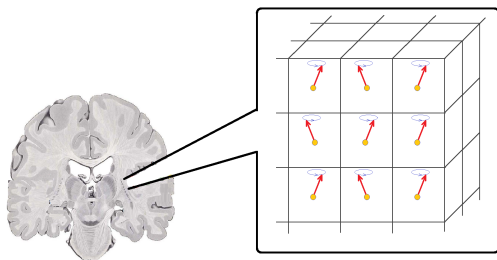


Figure : Cutting the sample into isochromats. A magnetization vector is associated to each isochromat and its evolution is monitored during the acquisition sequence. The collected MR signal corresponds to the transverse component of the magnetization.

Mathematical model: Bloch equations:

Bloch equations : temporal evolution of magnetization

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} - \hat{\mathbf{R}}(\mathbf{M} - \mathbf{M}_0) \quad (8)$$

where

- \mathbf{M} is the magnetization vector of the tissue,
- γ is the gyromagnetic ratio of hydrogen,
- \mathbf{B} is the external magnetic field
- $\hat{\mathbf{R}}$ the relaxation matrix containing $T1$ and $T2$.

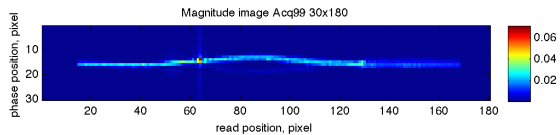
The magnetic field term $\mathbf{B}(\mathbf{r}, t)$ contains all the MR sequence elements (gradients and RF pulses). Its expression is given by:

$$\mathbf{B}(\mathbf{r}, t) = [\mathbf{G}(t) \cdot \mathbf{r} + \Delta B(\mathbf{r}, t)] \cdot \mathbf{e}_z + \mathbf{B}_1(\mathbf{r}, t) \quad (9)$$

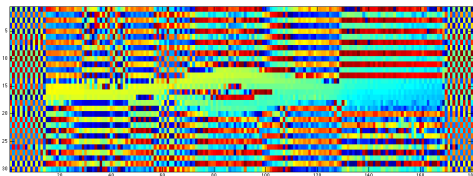
where

- $\mathbf{G}(t)$ is the gradients sequence,
- \mathbf{r} is the isochromat position,
- $\Delta B(\mathbf{r}, t)$ is the field inhomogeneities due to off-resonance and non uniform gradients,
- $\mathbf{B}_1(\mathbf{r}, t)$ the RF pulses sequence.

JEMRIS results



(a) Amplitude



(b) Phase

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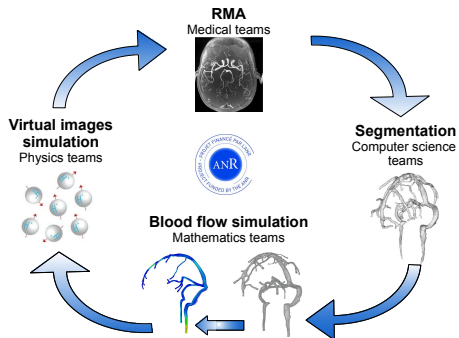


Figure : The VIVABRAIN project task loop

Reproduce the vivabrain loop with a collection of software:

- Filtering: algorithms developed in the project
- Segmentation: algorithms developed in the project
- Mesh generation: VTK, vmtk
- Simulation: Feel++, FreeFem++
- MRA simulation: JEMRIS

Input data

- Input data: Images in the DICOM format

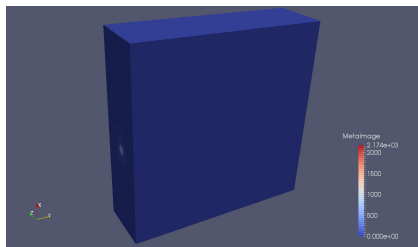


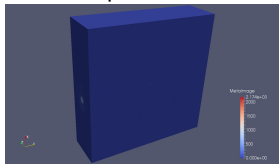
Figure : DICOM image stack



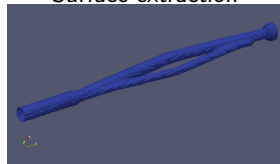
Figure : Volume rendering of DICOM data

AngioTK Pipeline: Mesh processing

Input Data



Surface extraction



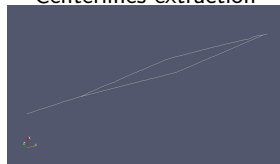
Mesh refinement



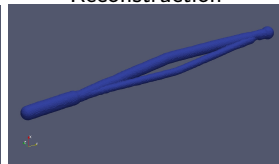
Mesh smoothing



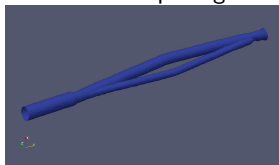
Centerlines extraction



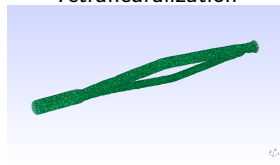
Reconstruction



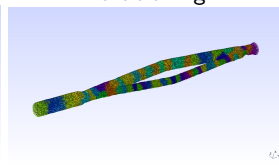
Surface opening



Tetrahedralization



Partitioning





(a) Thresholded MRI



(b) Segmented MRI

Figure : Comparison of the realistic geometry with MRI segmentations, the first is obtained by a simple threshold and the second by the classical method of snake.

Radial Slices	IN	UL	LL	UC	LC	UR	LR	OUT
Realistic mesh	5.00	3.00	2.00	3.00	2.00	3.00	2.00	4.00
Thresholded MRI	3.12	2.70	1.23	2.45	1.73	2.70	1.97	3.36
Segmented MRI	4.30	5.23		3.30	2.20	5.28		5.04

Table : Comparison of the diameters (mm) at the radial slices : IN=inlet, UL=upper left, LL=lower left, UC=upper center, UL=upper left, UR=upper right, LR=lower right and OUT=outlet. As the UL and LL parts and respectively the UR and LR parts are stuck together in the segmented IRM, we compute the diameters on the entire left slice and respectively on the entire right slice which are theoretically equal to 5.11mm.

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Conclusion

- Cross validation Feel++/FreeFem++: comparable results qualitatively and quantitatively
- Feel++/FreeFem++ vs experimental data: qualitatively ok, but not quantitatively
 - Improve MRA acquisition
- First, virtual images from simulation
 - Geometry OK, but still issues with velocities
- AngioTk pipeline: from MRA to computational meshes
 - Need tuning for the simulation step