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► **To cite this version:**

Philippe Réfrégier, Antoine Roueff, Aurélien Arnaubec, Pascale Dubois-Fernandez. Invariant Contrast Parameters of PolInSAR Homogenous RVoG Model. IEEE Geoscience and Remote Sensing Letters, 2014, 11 (8), p. 1414-1417. 10.1109/LGRS.2013.2294343 . hal-01221764

HAL Id: hal-01221764

<https://hal.science/hal-01221764>

Submitted on 16 Jul 2021

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Invariant Contrast Parameters of PolInSAR Homogenous RVoG Model

Philippe Réfrégier, Antoine Roueff, Aurélien Arnaubec, and Pascale C. Dubois-Fernandez, *Senior Member, IEEE*

Abstract—It has been shown that the Cramer–Rao bound (CRB) can be helpful to characterize vegetation and ground height estimations based on the homogenous random volume over ground (RVoG) model and polarimetric interferometric SAR techniques. However, this model is a function of 20 unknown parameters, which makes the performance analysis a tedious task. We show that the group invariance property of the RVoG model can greatly reduce the complexity of the analysis since the CRB of the vegetation and ground heights only depends on four unknown parameters instead of 20. Furthermore, for the considered situations analyzed in this letter, only three of these four parameters have a nonnegligible influence and can be interpreted as contrast parameters.

Index Terms—Cramer–Rao bound (CRB), polarimetric interferometric SAR (PolInSAR), random volume over ground (RVoG) model.

I. INTRODUCTION AND BACKGROUND

POLARIMETRIC interferometric SAR (PolInSAR) techniques consist in illuminating a scene with electromagnetic waves with two different polarization states and receiving the backscattered wave at two close locations [1], [2]. Let S_j^{XY} denote the measured amplitude along the polarization Y of the receiver number j when the scene is illuminated with a transmit polarization X . Assuming that the reciprocity principle holds, the component S_j^{YH} is equal to S_j^{HV} and is thus not mentioned [1], [2]. With two orthogonal transmit polarization states (H, V), it is possible to obtain the vector response $\mathbf{u}_j^T = (S_j^{HH}, \sqrt{2}S_j^{HV}, S_j^{VV})$, where T denotes the transpose. With two polarimetric receivers, the response can be written with the complex vector $\mathbf{k} = (\mathbf{u}_1^T, \mathbf{u}_2^T)^T$. This vector \mathbf{k} is assumed to

be distributed with a circular zero-mean Gaussian distribution characterized by its covariance matrix Υ , i.e.,

$$\Upsilon = \begin{pmatrix} \mathbf{T}_1 & \boldsymbol{\Omega} \\ \boldsymbol{\Omega}^\dagger & \mathbf{T}_2 \end{pmatrix} \quad (1)$$

where \dagger corresponds to the transpose complex conjugate, where $\mathbf{T}_j = \langle \mathbf{u}_j \mathbf{u}_j^\dagger \rangle$ for $j = 1, 2$, $\boldsymbol{\Omega} = \langle \mathbf{u}_1 \mathbf{u}_2^\dagger \rangle$, and where $\langle \cdot \rangle$ is the statistical average.

The random volume over ground (RVoG) model [3] has been proposed to analyze the vegetation response. The interaction of the incident wave with the vegetation volume (respectively, the ground) is thus described with the covariance matrix \mathbf{T}_{vol} (respectively, \mathbf{T}_{gro}) and the attenuation σ_v . The PolInSAR system geometry is characterized by the height sensitivity k_z and the incident angle β . The RVoG model can thus be written as [3]–[6]

$$\begin{aligned} \mathbf{T} &= \mathbf{T}_1 = \mathbf{T}_2 = I_1 \mathbf{T}_{\text{vol}} + a \mathbf{T}_{\text{gro}} \\ \boldsymbol{\Omega} &= e^{ik_z z_g} (I_2 \mathbf{T}_{\text{vol}} + a \mathbf{T}_{\text{gro}}) \end{aligned} \quad (2)$$

with $I_1 = (1 - e^{-\alpha h_v})/\alpha$, $I_2 = (e^{ik_z h_v} - e^{-\alpha h_v})/(ik_z + \alpha)$, $a = e^{-\alpha h_v}$, and $\alpha = 2\sigma_v/\cos \beta$. In this letter, we consider single baseline P-band PolInSAR systems, and we assume as in [7] that the attenuation σ_v is polarization independent and *a priori* known. The nine parameters of \mathbf{T}_{vol} , noted $t_{\text{vol},1}, \dots, t_{\text{vol},9}$, and the nine parameters of \mathbf{T}_{gro} , noted $t_{\text{gro},1}, \dots, t_{\text{gro},9}$, are assumed unknown. With these assumptions, the vegetation height h_v and the ground height z_g can be estimated [7], [8], with the estimation of Υ in homogenous regions, so that the elements of the set $\mathcal{K} = \{\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_N\}$ of N measured vectors \mathbf{k}_j can be considered distributed with the same probability density. Although the random volume model leads to a constrained parametric form of \mathbf{T}_{vol} [2], no constraint are assumed in this letter on this matrix because such a constraint is usually not taken into account for the estimation of h_v and z_g [5]–[7].

The Cramer–Rao bounds $\text{CRB}[h_v]$ and $\text{CRB}[z_g]$ [8], [9] correspond to a lower bound for the variance of the estimation of h_v and z_g , respectively, with unbiased estimators (i.e., such that the mean value of the estimated quantity is equal to the true value) [10]. These CRBs are function of the 20 scalar unknown parameters $\boldsymbol{\Theta}^T = (\theta_1, \theta_2, \dots, \theta_{20})$, where $\boldsymbol{\Theta}^T = (h_v, z_g, t_{\text{vol},1}, \dots, t_{\text{vol},9}, t_{\text{gro},1}, \dots, t_{\text{gro},9})$. This dependence of the CRB on a large number of parameters is a source of complexity. Indeed, for a fixed PolInSAR configuration (specified by k_z and β) and for a known value of σ_v , a

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full analysis of the precision on h_v and z_g would require to examine $\text{CRB}[h_v]$ and $\text{CRB}[z_g]$ in a mathematical space of dimension 20. However, the analysis of the estimations of h_v and z_g based on the representation of the region of coherence [1], [2], [5]–[7] corresponds to a parametric model with less than 20 parameters, and the demonstration that this approach can be efficient [8], [9] suggests that the rigorous dimension of the problem may be indeed lower than 20. As shown in the next section, group invariance properties can be helpful to rigorously show that $\text{CRB}[h_v]$ and $\text{CRB}[z_g]$ only depend on four unknown parameters, which greatly simplify the analysis.

II. INVARIANT PARAMETERS

Statistical invariance properties have been widely used in statistics [11], [12] and more recently for the analysis of polarimetric image processing performance [13]–[15]. Let us consider that the N measurements of the set \mathcal{K} have been transformed into $\mathbf{k}'_j = \mathbf{M}\mathbf{k}_j \forall j \in \{1, \dots, N\}$, where

$$\mathbf{M} = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & e^{ik_z z} \mathbf{B} \end{bmatrix} \quad (3)$$

where z is a real scalar, $\mathbf{0}$ is the 3×3 zero matrix, and \mathbf{B} is a nonsingular 3×3 matrix. It is easy to see that the new set of measurements $\mathcal{K}' = \{\mathbf{k}'_1, \dots, \mathbf{k}'_N\}$ could have been also obtained with an RVoG model of parameters $h'_v = h_v$, $z'_g = z_g - z$, $\mathbf{T}'_{\text{vol}} = \mathbf{B}\mathbf{T}_{\text{vol}}\mathbf{B}^\dagger$, and $\mathbf{T}'_{\text{gro}} = \mathbf{B}\mathbf{T}_{\text{gro}}\mathbf{B}^\dagger$. Thus, the application of (3) transforms an RVoG problem into another one. This property thus shows that transformations described by (3) leave invariant the family of RVoG models [11], [12]. It can be seen (see the Appendix) that the CRBs for the estimation of h_v and z_g are equal for the RVoG model with parameters $h_v, z_g, \mathbf{T}_{\text{vol}}$, and \mathbf{T}_{gro} and with parameters $h_v, z_g - z, \mathbf{T}'_{\text{vol}}$, and \mathbf{T}'_{gro} . Since z is deterministic and can have any values (in the limit of nonambiguous values), the CRB is expected not to depend on z_g . Furthermore, assuming that \mathbf{T}_{vol} is nonsingular, the choice $\mathbf{B} = \mathbf{U}\mathbf{T}_{\text{vol}}^{-1/2}$, where \mathbf{U} is the unitary matrix that diagonalizes the Hermitian matrix $\mathbf{T}_{\text{vol}}^{-1/2}\mathbf{T}_{\text{gro}}\mathbf{T}_{\text{vol}}^{-1/2}$, leads to $\mathbf{T}'_{\text{vol}} = \mathbf{I}_d$, where \mathbf{I}_d is the 3×3 identity matrix and $\mathbf{T}'_{\text{gro}} = \text{Diag}[\lambda_1, \lambda_2, \lambda_3]$, where $\text{Diag}[\lambda_1, \lambda_2, \lambda_3]$ is the diagonal matrix with positive diagonal values $\lambda_1, \lambda_2, \lambda_3$ that are the eigenvalues of $\mathbf{T}_{\text{vol}}^{-1/2}\mathbf{T}_{\text{gro}}\mathbf{T}_{\text{vol}}^{-1/2}$.

The obtained RVoG model, which corresponds to parameters $h'_v = h_v, z'_g = 0, \mathbf{T}'_{\text{vol}} = \mathbf{I}_d$, and $\mathbf{T}'_{\text{gro}} = \text{Diag}[\lambda_1, \lambda_2, \lambda_3]$, has only four independent parameters, and it is thus expected that $\text{CRB}[h_v]$ and $\text{CRB}[z_g]$ depend only on these four parameters. Thus, it is sufficient to explore a mathematical space of dimension 4 instead of 20 in order to analyze the performance of the estimation of h_v and z_g with a homogenous RVoG model and PolInSAR systems. An illustration of the invariance of $\text{CRB}(h_v)$ and $\text{CRB}(z_g)$ for different z_g values can be obtained with the region of coherence [6] since changing z_g corresponds to a rotation of this region without modifying its shape.

The property of invariance is illustrated in Fig. 1(a), where the CRB values are shown when the sample size is $N = 100$.

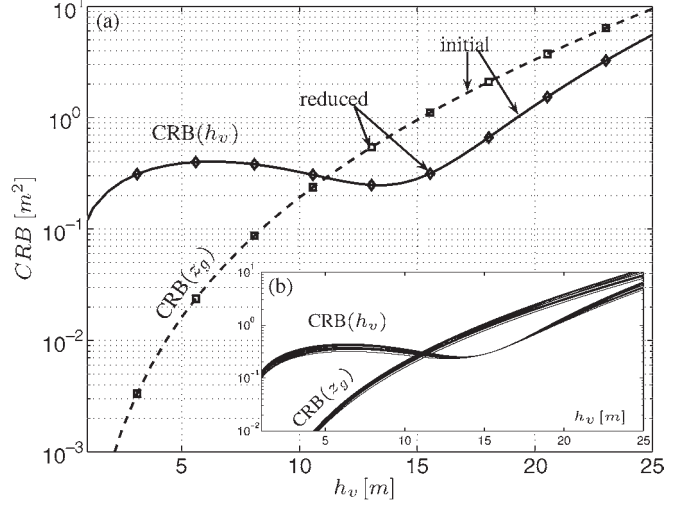


Fig. 1. (a) Continuous: CRB for h_v with the initial model. Diamonds: CRB for h_v with the reduced model. Dashed: CRB for z_g with the initial model. Squares: CRB for z_g with the reduced model. (b) Inset: CRB as a function of h_v for different $x \in [0, 1]$ when $\lambda_2(x) = x\lambda_1(x) + (1-x)\lambda_3(x)$, $\mathcal{A} = 0.59$, and $\mathcal{E} = 71.2$.

Matrices \mathbf{T}_{gro} and \mathbf{T}_{vol} (the physical unit of \mathbf{T}_{vol} is m^{-1} ; see [8]) were equal to

$$\mathbf{T}_{\text{gro}} = \begin{pmatrix} 5.43 & 2.03+i1.06 & 1.06+i0.318 \\ 2.03-i1.06 & 4.94 & 0.0886+i0.452 \\ 1.06-i0.318 & 0.0886-i0.452 & 2.17 \end{pmatrix}$$

$$\mathbf{T}_{\text{vol}} = 0.01 \begin{pmatrix} 23.9 & -3+i0.793 & 3.59+i1.27 \\ -3-i0.793 & 16.8 & -0.582+i2.2 \\ 3.59-i1.27 & -0.582-i2.2 & 13.7 \end{pmatrix}$$

and correspond to the initial considered model with $k_z = 0.1 \text{ m}^{-1}$; $\sigma_v = 0.0345 \text{ m}^{-1}$, $\beta = \pi/4$ rd, and $z_g = -1.74$ m. The results are also reported for the reduced model described above that leads to $\mathbf{T}'_{\text{vol}} = \mathbf{I}_d$ and $\mathbf{T}'_{\text{gro}} = \text{Diag}[43.8, 16.16, 11.24]$ and $z_g = 0$. The same CRB can be observed for the initial and the reduced models. Other examples with different \mathbf{T}_{vol} , \mathbf{T}_{gro} , and z_g but with identical λ_1, λ_2 , and λ_3 have led to the same perfect superposition and have shown that the value of z_g has no influence on the CRB in the range of nonambiguous values. The importance of the eigenvalues of the matrix $\mathbf{T}_{\text{vol}}^{-1}\mathbf{T}_{\text{gro}}$ (which are equal to $(\lambda_j)_{j=1,2,3}$) has been already mentioned in [6], and the present result shows that, with h_v , they rigorously constitute invariant parameters for the estimation precision characterized by the CRB of h_v .

III. APPROXIMATE INVARIANT PARAMETERS

Assuming that the convention $\lambda_1 \geq \lambda_2 \geq \lambda_3$ has been adopted, the four parameters of the RVoG model can be chosen as $h_v, \mathcal{A} = (\lambda_1 - \lambda_3)/(\lambda_1 + \lambda_3)$, $\mathcal{E} = \lambda_1 + \lambda_2 + \lambda_3$, and λ_2 . Parameter \mathcal{A} can be related to the difference of polarimetric behavior of the vegetation and of the ground, whereas \mathcal{E} can be related to the ratio of the energies scattered by the ground and the volume. Indeed, if $\mathbf{T}_{\text{gro}} = y\mathbf{T}_{\text{vol}}$, where y is a positive real value, then $\mathcal{A} = 0$ and $\mathcal{E} = 3y$. Furthermore, if $\mathbf{T}_{\text{vol}} = y\mathbf{I}_d$, then \mathcal{A} is related to the polarimetric response of the ground.

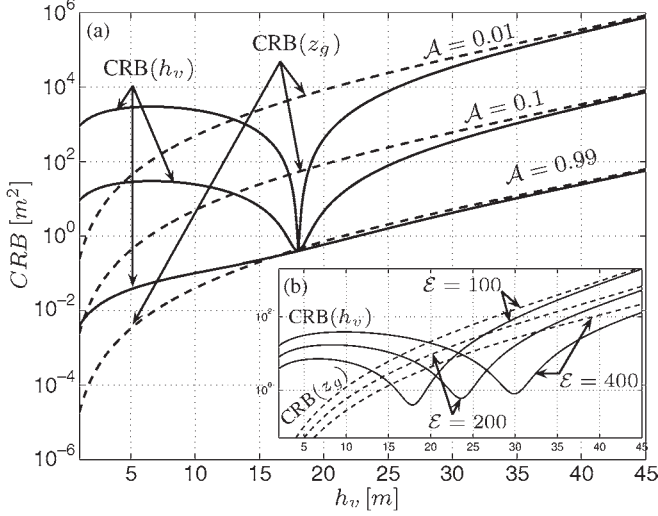


Fig. 2. (a) Same as Fig. 1 but for different values of $\mathcal{A} = (\lambda_1 - \lambda_3)/(\lambda_1 + \lambda_3)$ when $\mathcal{E} = 100$ and for $x = 0.2$. (b) Inset: Same as (a) but when $\mathcal{A} = 0.2$ and for different values of \mathcal{E} .

The influence of λ_2 when $\lambda_2(x) = x\lambda_1(x) + (1-x)\lambda_3(x)$ with $x \in [0, 1]$ is illustrated in Fig. 1(b) for the same values of k_z , β , and σ_v as in Fig. 1(a). For each value of x , $\lambda_1(x)$ and $\lambda_3(x)$ are determined so that \mathcal{A} and \mathcal{E} remain unchanged. It can be observed, for the considered example, that the influence of λ_2 is low in comparison to the influence of h_v . An appropriate approximation thus consists in considering that the CRBs of h_v and z_g are essentially functions of the unknown parameters h_v , \mathcal{A} , and \mathcal{E} and of the known parameters k_z and $\alpha = 2\sigma_v/\cos\beta$.

Fig. 2 reports the influence of \mathcal{A} and \mathcal{E} . The influence of \mathcal{A} is crucial on the CRB values since, for example, when $h_v = 35$ m, the CRB is approximately multiplied by a factor 100 when \mathcal{A} is divided by a factor 10.

The value h_v for which the CRB is minimal has been analyzed in [9] and roughly corresponds to the vegetation height for which the same energy is backscattered by the vegetation and by the ground. It can be seen in Fig. 2(b) that this value of h_v is an increasing function of \mathcal{E} .

The influence of the height sensitivity k_z and of the incident angle β is illustrated in Fig. 3. The influence of k_z is low, as long as h_v is smaller than the ambiguity height $2\pi/k_z$ with the RVoG model that does not take into account instrumental phase noise. Furthermore, the influence of β is essentially to modify the value h_v for which the CRB h_v is minimal.

IV. CONCLUSION

The group invariance of the homogenous RVoG model shows that four reduced parameters instead of the 20 initial unknown parameters characterize the difficulty of the vegetation and ground height estimation. Moreover, on the considered examples in this letter, only three of them have a nonnegligible influence. The influence of the first parameter h_v , the vegetation height, was previously analyzed in [9]. This letter shows that the second parameter \mathcal{A} can have a strong influence on the CRB. As illustrated in the proposed examples, high precision can be expected for forest with $\mathcal{A} \simeq 1$, whereas low performance is

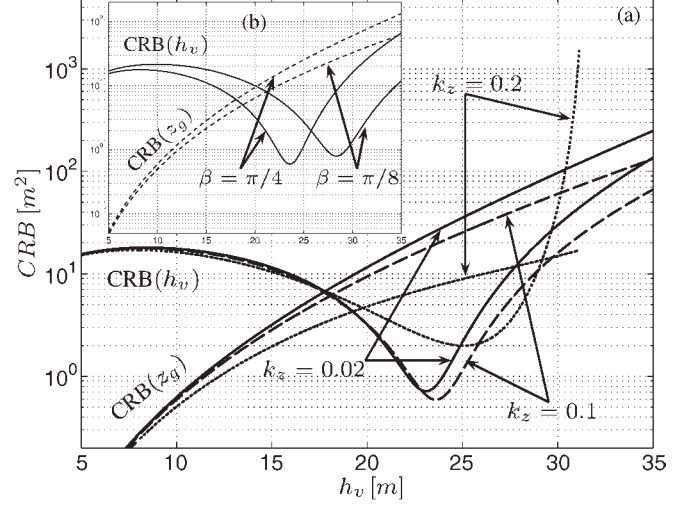


Fig. 3. (a) Same as Fig. 1 ($x = 0.2$, $\beta = \pi/4$, $\mathcal{A} = 0.2$, and $\mathcal{E} = 200$) but for different values of k_z . (b) Inset: Influence of β .

obtained if $\mathcal{A} \simeq 0$. Indeed, \mathcal{A} can be related to the difference of the polarimetric response between the ground and the volume. The third parameter \mathcal{E} is related to the ratio of the energy scattered by the ground and the volume. The influence of \mathcal{E} is thus important. In particular, it modifies the value of the vegetation height for which the CRB on h_v has a local minimum. In the framework of the model, where the instrumental phase noise is not taken into account, the height sensitivity k_z seems to have a low influence as long as the vegetation height is smaller than the ambiguity height $2\pi/k_z$, whereas β leads to an analogous behavior to the one of \mathcal{E} . Any bijection function of the invariant parameters also defines invariant parameters that characterize the difficulty of the estimation task, which can be a useful property to define a mean volume-to-ground ratio that constitutes a rigorous contrast parameter.

APPENDIX

Since the vectors \mathbf{k}_j are assumed distributed with a circular zero-mean Gaussian distribution, the vectors $\mathbf{k}'_j = \mathbf{M}\mathbf{k}_j$ are also distributed with a circular zero-mean Gaussian distribution but with a different unknown parameter vector that can be written as $\Theta' = g_M(\Theta)$. Thus, the log likelihood of observing $\mathcal{K}' = \{\mathbf{k}'_1, \dots, \mathbf{k}'_N\}$ can be written as $L(\mathcal{K}'|\Theta') = L(\mathcal{K}'|g_M(\Theta))$. However, a direct calculus shows that $L(\mathcal{K}'|\Theta') = F(\mathbf{M}) + L(\mathcal{K}|\Theta)$, where $F(\mathbf{M})$ is a function of \mathbf{M} but independent of \mathcal{K}' and Θ . Thus, if $\hat{\Theta} = (\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_{20})$ maximizes $L(\mathcal{K}|\Theta)$ and if $\hat{\Theta}' = (\hat{\theta}'_1, \hat{\theta}'_2, \dots, \hat{\theta}'_{20})$ maximizes $L(\mathcal{K}'|\Theta')$, then, when there is no estimation ambiguity, $\hat{\Theta}' = g_M(\hat{\Theta})$.

The considered transformation $\mathbf{k}'_j = \mathbf{M}\mathbf{k}_j$ does not modify h_v and translates z_g with a deterministic value, which implies that $\hat{\theta}'_1 = \hat{\theta}_1$ and $\hat{\theta}'_2 = \hat{\theta}_2 - z$. The variance of these estimators is thus the same with the samples \mathcal{K}' and \mathcal{K} , i.e., when the RVoG model corresponds to parameter Θ or $\Theta' = g_M(\Theta)$.

Finally, since maximum-likelihood estimators are asymptotically efficient (i.e., with a variance that reaches the CRB), the CRBs of h_v and z_g are also invariant with transformations \mathbf{M} .

ACKNOWLEDGMENT

The authors are grateful to the Conseil Régional Provence-Alpes-Côte d'Azur (PACA) for taking part in the funding of the Ph.D. of A. Arnaubec.

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