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A framework for computing power consumption scheduling functions under uncertainty

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Abstract—One of the goals of this paper is to make a step further toward knowing how an electrical appliance should exploit the available information to schedule its power consumption; mainly, this information corresponds here to an imperfect forecast of the non-controllable (exogenous) load or electricity price. Reaching this goal led us to three key results which can be used for other settings which involve multiple agents with partial information: 1. In terms of modeling, we exploit the principal component analysis to approximate the exogenous load and show its full relevance; 2. Under some reasonable but improvable assumptions, this work provides a full characterization of the set of feasible payoffs which can be reached by a set of appliances having partial information; 3. A distributed algorithm is provided to compute good power consumption scheduling functions. These results are exploited in the numerical analysis, which provides several new insights into the power consumption scheduling problem. XXX completer en tant un plus specifique: choix du cout, un ou deux messages forts issus des simulations.

I. INTRODUCTION

An important problem for modern electrical networks is to design an electrical vehicle (EV) battery charging device which is able to exploit the knowledge it has about the non-EV demand or the electricity price to reach a certain objective. The objective can be to reduce the impact of charging operations on the distribution network or to minimize the monetary charging cost paid by the EV user. The most standard approach is to design a charging scheme which assumes a perfect knowledge of the non-EV demand (or price) and evaluate the performance of the corresponding algorithm by feeding the latter with a forecast or noisy version of the non-EV demand. Illustrative and recent examples of this approach are given e.g., by [?] XXX-shinwari-2012, XXX-gan-2013, [?]. In the quoted references, the energy need of a given user is computed by assuming perfect knowledge of the electricity price or the exogenous demand namely, the part of the demand which is not controlled by the smart consuming devices. The obtained power scheduling schemes have essentially or exactly the water-filling structure i.e., that holes in terms of price or demand are exploited in the first place. One of the drawbacks of this approach is the potential lack of robustness of the designed algorithm to imperfect forecast.

Among existing works which take uncertainty into account in the design of power scheduling scheme we find in particular [?]. Therein the authors propose a threshold-based scheduling policy which accounts for past and current prices and the statistics of future prices; each appliance consumes according to a rectangular profile and starts consuming when the price

is below a time-varying threshold. In [?] the price values are assumed independent from the load level in each time slot while this assumption is relaxed in XXX-samadi-mohsenian-rad. Additionally, in the latter reference the authors also considers uncertainty in the algorithm design part but uncertainty concerns load and user energy consumption needs and not prices as in [?]. Another example of relevant work where price uncertainty is considered in real-time demand response model is [?]; therein robust optimization is exploited. In [?], the problem is addressed by stochastic gradient based on the statistical knowledge of future prices.

To our knowledge, there is no contribution in the literature related to the present work which treats the problem of optimality of a power consumption scheduling scheme under given arbitrary imperfect observation or forecast. In fact, to characterize the performance achievable when exploiting optimally the available information, we resort to a very recent result in information theory [?] (Sec. ??). In Sec. ??, we exploit the corresponding theorem and the model proposed in Sec. ?? to build a convergent algorithm which allows suboptimal but typically good power consumption scheduling schemes to be determined numerically. This algorithm is exploited in Sec. ?? to conduct the provided numerical analysis. In Sec. ??, the main assets of the proposed approach are summarized and several extensions to address its limitations are provided.

II. PROPOSED SYSTEM MODEL

We consider a set of $K \geq 1$ smart electrical appliances. Each appliance aims at scheduling its power consumption to maximize a certain payoff function, which is provided further. To this end, it exploits the available knowledge about a state which affects its payoff. In this paper, this state is the *exogenous load* namely, the part of the load which is not controlled by the smart consuming devices but the proposed model and derived results can be directly used for other types of states such as the electricity price. To define the exogenous load and other key quantities such as the power consumption vectors, we need to specify the timing aspect. Time is assumed to be slotted in stages $t \in \{1, \dots, T\}$. Typically, a stage may represent a day and T may represent the number of days over which the payoff is averaged. At the beginning of *stage* t , appliance k has to choose a power consumption vector¹

¹For example, $N = 24$ if a stage comprises 24 time-slots whose duration is one hour.

$\underline{x}_k = (x_{k,1}, \dots, x_{k,N})$ by exploiting perfect observations of the past exogenous load vectors $\underline{x}_0(1), \dots, \underline{x}_0(t-1)$ and a signal which is an image or forecast of the system state and appliances actions at stage t ; this signal is denoted by $\underline{s}_k \in \mathcal{S}_k$. For example, such a signal may be a forecast of the exogenous load or the total load, the total load being equal to the sum $x_0(t) + \sum_{k=1}^K x_k(t)$. For $k \in \{0, 1, \dots, K\}$, the set in which \underline{x}_k lies is assumed to be discrete and is denoted by \mathcal{X}_k . This choice is not only motivated by the fact that both power and time can be discrete in real systems such as electrical vehicle battery charging systems but also to obtain a solution which is robust against forecasting noise; the latter issue has been addressed recently in XXX-tsg1-olivier where rectangular consumption profiles typically perform better than continuous profiles.

The computational complexity of the algorithm proposed in Sec. XXX depends on the cardinality of the set \mathcal{X}_0 . In general, assuming \mathcal{X}_0 to be discrete amounts to approximating the exogenous load vectors. To obtain a good approximation, we propose to apply the principal component analysis (PCA) XXX-book-mallat on exogenous load vectors. The exogenous load vector for stage t is approximated as follows:

$$\widehat{\underline{x}}_0(t) = \widehat{\underline{\mu}}_L + \sum_{i=1}^M a_i(t) v_i \quad (1)$$

where $\widehat{\underline{\mu}}_L$ is defined by

$$\widehat{\underline{\mu}}_L = \frac{1}{L} \sum_{t \in \mathcal{L}} \underline{x}_0(t), \quad (2)$$

the set \mathcal{L} represents a set of L samples which is available to estimate $\widehat{\underline{\mu}}_L$; typically, it may correspond to data measured during the preceding year. The vectors v_i are the eigenvectors of the following matrix

$$\widehat{\mathbf{R}}_L = \frac{1}{L} \sum_{t \in \mathcal{L}} \left[\underline{x}_0(t) - \widehat{\underline{\mu}}_L \right] \left[\underline{x}_0(t) - \widehat{\underline{\mu}}_L \right]' \quad (3)$$

where the notation $(\cdot)'$ stands for transpose. One of the advantages of using such a decomposition is that for a given number of vector basis M , the quality of approximation is maximized; more specifically, the expected distortion $\mathbb{E} \left\| \widehat{\underline{X}}_0 - \underline{X}_0 \right\|^2$ is minimized. To minimize the latter quantity we will exploit the Lloyd-Max algorithm XXX in the numerical analysis; it will be applied to the vector (a_1, \dots, a_K) .

The *stage or instantaneous payoff* function of appliance k is denoted by $u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$. In the simulations we will assume that u_k writes as $u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) = \sum_{n=1}^N f_{k,n}(x_{0,n} + \sum_{k=1}^K x_{k,n})$. The function $f_{k,n}$ can e.g., represent the price charged to the consumer, Joule losses, battery aging, or distribution transformer aging on time-slot n . To define the average payoff of appliance k which is the function to be maximized by appliance k , the key notion of power consumption scheduling strategies needs to be defined.

A *strategy* for appliance k is a sequence of mappings which is defined by:

$$\sigma_{k,t} : \begin{array}{l} \mathcal{X}_0^{t-1} \times \mathcal{S}_k \rightarrow \mathcal{S}_k \\ (\underline{x}_0(1), \dots, \underline{x}_0(t-1), \underline{s}_k(t)) \mapsto x_k(t) \end{array} \quad (4)$$

The *average payoff* of appliance k is then defined by

$$U_k(\sigma_1, \dots, \sigma_K) = \mathbb{E} \left[\frac{1}{T} \sum_{t=1}^T u_k(X_0(t), X_1(t), \dots, X_K(t)) \right]. \quad (5)$$

The expectation operator is used since the load is typically considered as a random process, which implies that in full mathematical generality, the appliances decisions have also to be considered as random processes. The main two technical problems we address in this paper is the characterization of achievable average payoffs when T grows large and the determination of a practical scheme which provides good performance. A key technical point here is that we will assume a general structure for the knowledge \underline{s}_k . Indeed, we assume that $(\underline{s}_1, \dots, \underline{s}_K)$ are the outputs of a general discrete memoryless channel (see e.g., XXX-cover-book for more details) whose conditional probability is $\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0)$.

III. MAIN ANALYTICAL RESULTS

A. Limiting performance characterization

XXX Attention j ai pris un Gamma plus general que celui de depart, c est mieux, on revendique un nouveau theoreme du coup, certes seulement un peu nouveau par rapport a ITW. Dans les simulations faudra preciser quon regarde un Gamma particulier.

Theorem 1: Assume the random process $\underline{X}_0(t)$ to be i.i.d. with a probability distribution ρ and the available load forecast \underline{S}_k to be the output of a discrete memoryless channel whose conditional probability is $\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$. An expected payoff vector $(\overline{U}_1, \dots, \overline{U}_K)$ is achievable in the limit $T \rightarrow \infty$ if and only if it writes as:

$$\overline{U}_k = \sum_{\substack{\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K, \underline{w}, \underline{s}_1, \dots, \underline{s}_K \\ \left(\prod_{k=1}^K P_{\underline{X}_k | \underline{S}_k, \underline{W}}(\underline{x}_k | \underline{s}_k, \underline{w}) \right)}} \rho(\underline{x}_0) P_{\underline{W}}(\underline{w}) \Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0) \times u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) \quad (6)$$

where \underline{W} is an auxiliary variable which can be optimized.

The proof of this theorem is omitted here. It consists of a generalization of a result given in XXX-larrousse-ITW-2015. Therein, the authors characterize implementable distributions for the case $\sigma_{k,t}(S_k)$, S_k being the output of the simplified channel $\Upsilon(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0)$ namely, the output signal only depends on the state.

One of the merits of Theorem 1 is to provide the best performance achievable in terms of average payoffs when the appliances have an arbitrary information structure (as long as it is of the form given by (4)). This requires the knowledge of the distribution of the exogenous load i.e., ρ and the condition distribution Γ . However, Theorem 1 does not provide practical strategies which would allow a given payoff

vector to be reached. Finding "optimal" scheduling strategies consists in finding good sequences of functions structured according to (4)), which is an open and promising direction to be explored. More pragmatically, we restrict our attention to finding stationary strategies which are merely scheduling functions of the form $f_k : \mathcal{S}_k \rightarrow \mathcal{X}_k$. This choice is motivated by practical considerations such as computational complexity and it also coherent with the current state of the literature. The water-filling solution is a special instance of this class of strategies. To find good scheduling functions, the idea we propose is to exploit the expected payoff given by Theorem 1. This is the purpose of the next section.

B. Procedure for determining power consumption scheduling functions

The first observation we make is that the best performance only depends on the vector of conditional probabilities $(P_{\mathcal{X}_1|\mathcal{S}_1, \mathcal{W}}, \dots, P_{\mathcal{X}_K|\mathcal{S}_K, \mathcal{W}})$. It is therefore relevant to try to find an optimum vector of lotteries and use it to take decisions. Since this task is typically computationally demanding, a sub-optimal approach consists in applying a distributed algorithm to maximize the average payoff. The procedure we propose here is to use the sequential best response dynamics (see e.g., XXX-book-samson). XXX je vous laisse expliquer cette procedure, on l a deja fait ailleurs avec Olivier. The key observation we make is then to see that when the lotteries of the other appliances are fixed, the best lottery for appliance k boils down to a function of \mathcal{S}_k and \mathcal{W} . The choice of power scheduling functions we propose precisely corresponds to running the algorithm provided below several times, convergence being obtained after a few iterations typically.

A sufficient condition for convergence of the algorithm is that the stage payoff function writes as $u_k(\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_K) = \alpha_k u(\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_K)$. This special case will be assumed in the simulations and has been shown to be very relevant in the smart grid literature. For instance, in XXX-mohsenian-rad u is a function which represents the total cost associated with operations such as charging operations and $\alpha_k = \frac{C_k}{\sum_{j=1}^K C_j}$ is a constant which represents the way this cost is shared among the consumers, C_j being the energy need of user j .

XXX Il faut ecrire l algo tres proprement, eviter de l appeler BRD.

XXX Je donnerais l algo avec les u_k

IV. NUMERICAL ANALYSIS

XXX URGENT : detaillier les simulations qu on attend, setup, ce qu on represente etc.

XXX Penser a definir un profil rectangulaire sous forme de vecteur et expliquer que l optim revient a choisir un scalaire.

V. CONCLUSION

Numerical results show the full relevance of the proposed PCA-based model. Remarkably, an accurate approximation can be obtained by using only a few eigenvectors and applying

the Lloyd-Max algorithm on the weights to be applied to these eigenvectors. The proposed framework to characterize the best performance of power consumption scheduling exploits very recent in information theory and also allows to derive robust scheduling functions. Simulations show that in the presence of uncertainty on the exogenous load forecast, the obtained functions outperform such as iterative water-filling based schemes.

The proposed framework can be extended. While the discrete alphabet assumption seems a good assumption to obtain robust scheduling schemes, the i.i.d assumption on the exogenous load would need to be relaxed e.g., into a milder assumption such as a Markovian process, the i.i.d. assumption being made for the performance characterization theorem. Another direction to further improve performance is maximize the expected payoff jointly and not by using a distributed algorithm. The obtained schemes would still be distributed decision-wise. As another extension, a stage might be a time-slot instead of a day. Indeed, with our approach, the scheduling decision is taken once and for all at the beginning of each stage while it might be updated during the day.

XXX Finding "optimal" scheduling strategies consists in finding good sequences of functions structured according to (4)), which is an open and promising direction to be explored.

One strong asset of the proposed framework is that it can be applied to a broad class of forecast model and payoff functions. discrete distributed

Strengths: general methodology, new model (PCA), we establish a link with info theo, limiting perf are characterized, complexity is reasonable (XXX-incontrastwith-kim-poor)

Nous on modlise aussi, deuxime contrib. Et on maximiser sur l'annee. Notre approche et gnrique et en lien avec la thorie de l info

kim poor: Although using stochastic dynamic programming can derive the optimal policies, it results in very high computational complexity and is generally prohibitive to be explicitly solved.

List of limitations or our approach and extensions. IID state (ext=Markov state), DMC obs. On ne met pas a jour au cours de la journee. Optimality (multi-linear program). Multi-objective and algo convergence.

extension revue: and possibly an image of past consumption decisions taken by the other appliances.

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- [1] H. Kopka and P. W. Daly, *A Guide to L^AT_EX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.